BACK-STEPPING CONTROL FOR ROTARY INVERTED PENDULUM

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ABSTRACT

Control of Rotary Inverted Pendulum (RIP) is a classic control problem and is considered a benchmark in control theory problems. It represents a broader class of under-actuated systems. Rotary inverted pendulum complex system, widely studied in many areas due to its complexity, nonlinearities and non-minimum-phase system. It relates to a class of real world system such as cart and pole, pendubot, missile launchers, human balance, bipedal robot, metronome and many more. The problems associated with it are always interesting topics in control engineering. This paper presents a back-stepping technique to control this system balancing at a vertical upright position-the unstable equilibrium point. Lyapunov-based back-stepping is proposed for a rotary inverted pendulum in this research. Both the simulation and experimental results on controllers-swing up and balancing- and system responses show the effectiveness of this method. And two methods swing-up will be proposed in this article are Furuta method and exponentiation of the pendulum position method.

Keywords: *Rotary inverted pendulum; Back-stepping control; Furuta method; Exponentiation of the pendulum position; Balancing control.*

1. INTRODUCTION

A rotary inverted pendulum system (RIP) is the most typical representation for an under-actuated system. Because of its nonlinear model and simplicity in mechanical structure, this model is very popular in research and technical education. The system has two degrees of freedom where on the first one exciting torque is applied. This system has two links, one mounted horizontally and another one attached to it in a vertical direction representing the pendulum. Different control algorithms are applied by many researchers, from classical PID controller, LQR controller [1, 2] to advanced controller fuzzy logic controller [3], neural networks, genetic algorithms (GA) [4], particle swarm optimization (PSO) [4], ant colony optimization (ACO) methods [4], sliding mode controller [5], back-stepping controllers [6, 7, 8]. However, these back-stepping methods are applied on model models in simulation only and the input signal is the moment of the motor. This assumption makes the experiment does not suit the simulation. Therefore, in this research, a back-stepping controller which can be both used in simulation and experiment to implement the knowledge in this direction.

Back-stepping controller was first introduced by Petar V. Kokotovic in 1990. Backstepping control technique is a Lyapunov based nonlinear robust technique which is applicable to only a strict feedback system [9]. Backstepping method is used in the first subsystem to design a virtual variable control for the next subsystem and the design is implemented in the next subsystem to design another virtual variable control until the dynamics of the subsystem contain the control input variable. This method was applied to balancing the RIP and the response of the system is effective.

In this article, the authors will design a block of backstepping to balance the RIP. And this control law will be applied on real model to evaluate with simulation result. And the swing-up controller will be represented in this paper are Furuta method and exponentiation of the pendulum position method.

The paper consists of five sections. Part 1 introduces the model and control method. Part 2 presents the dynamic equation. Part 3 shows how to use a control algorithm that is based on a standard model for a survey. The simulation and experimental results are presented in part 4. Finally, the conclusion is mentioned in part 5.

2. SYSTEM MODEL

2.1 System model of RIP

Structure of RIP is shown in Figure 1. It contains two links. The first link is pendulum and the second link is arm. Angle of pendulum and arm are α and β, respectively. The model's parameters are shown in Table 2.

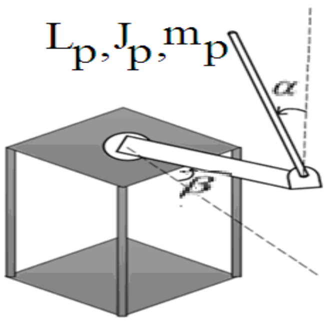

Figure 1. *RIP*

Total kinetic energy of system is

$$K=\frac{1}{2}J_r\dot{\beta}^2+\frac{1}{2}J_p\dot{\alpha}^2 +\frac{1}{2}m_p\left\{L_r^{\,2}\dot{\beta}^2+\frac{1}{4}L_p^{\,2}\left(\dot{\beta}^2-\dot{\beta}^2\cos^2\alpha\right)\right\}-\frac{1}{2}L_rL_pm_p\dot{\beta}\dot{\alpha}\cos\alpha \quad (1)$$

Total potential energy of system is

$$V=m_pgL_p\left(1-\cos\alpha\right) \quad (2)$$

Lagrangian operator is

$$L=K-V \quad (3)$$

By Euler-Lagrange method, dynamic equations of system are:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{\beta}}\right)-\frac{\partial L}{\partial\beta}=\tau-B_r\dot{\beta} \quad (4)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{\alpha}}\right)-\frac{\partial L}{\partial\alpha}=-B_p\dot{\alpha} \quad (5)$$

Solve (4) and (5), the dynamic equations of the Furuta pendulum are obtained as [10]:

$$\left(m_pL_r^2+\frac{1}{4}m_pL_p^2-\frac{1}{4}m_pL_p^2\cos^2\alpha+J_r\right)\ddot{\beta}+ -\frac{1}{2}m_pL_pL_r\cos\alpha.\ddot{\alpha}+\frac{1}{2}m_pL_p^2\sin\alpha\cos\alpha.\dot{\beta}\dot{\alpha}+ +\frac{1}{2}m_pL_pL_r\sin\alpha.\dot{\alpha}^2=\tau-B_r\dot{\beta} \quad (6)$$

$$-\frac{1}{2}m_pL_pL_r\cos\alpha.\ddot{\beta}+\left(J_p+\frac{1}{4}m_pL_p^2\right)\ddot{\alpha}+ -\frac{1}{4}m_pL_p^2\cos\alpha\sin\alpha.\dot{\beta}^2-\frac{1}{2}m_pL_pg\sin\alpha=-B_p\dot{\alpha} \quad (7)$$

However, for the convenience of adjusting the motor as well as applying the controller to the real model, the authors transform the control signal from moment of DC motor to voltage that is applied to DC servo motor by formula (8):

$$\tau=-k_3\ddot{\beta}-k_2\dot{\beta}+k_1e \quad (8)$$

where $k_1=\frac{K_t}{R_m};k_2=C_m+\frac{K_t}{R_m}K_b;k_3=J_m$

According to [10] and [11], the parameter identification method is applied in this model. These parameters are listed in Table 1:

Table 1. *Motor parameters*

L_m (H)	0.1756
K_b (V / (rad / sec))	0.0531
R_m (Ω)	11.7356
$\lvert T_f\rvert$ (N.m)	0.0014
J_m (kg.m^2)	0.0195
C_m ((N.m / (rad / sec))	3.7757e-05

Combining (6), (7), (8), we have the kinetic equations of the RIP:

$$D(\alpha,\beta).(\ddot{\alpha},\ddot{\beta})+C(\alpha,\dot{\alpha},\beta,\dot{\beta}).(\dot{\alpha},\dot{\beta})+G(\alpha,\beta)=v \quad (9)$$

where

$$D(\alpha,\beta)=\begin{bmatrix} & m_pL_r^2+\frac{1}{4}m_pL_p^2 \\ -\frac{1}{2}m_pL_pL_r\cos\alpha & -\frac{1}{4}m_pL_p^2\cos^2\alpha \\ & +J_r+k_3 \\ \left(J_p+\frac{1}{4}m_pL_p^2\right) & -\frac{1}{2}m_pL_pL_r\cos\alpha \end{bmatrix};$$

$$C(\alpha,\dot{\alpha},\beta,\dot{\beta})=\begin{bmatrix} \left(\frac{1}{2}m_pL_pL_r\sin\alpha\right)\dot{\alpha} & \left(\frac{1}{2}m_pL_p^2\sin\alpha\cos\alpha\right)\dot{\alpha} \\ & +B_r+k_2 \\ B_p & -\frac{1}{4}m_pL_p^2\cos\alpha\sin\alpha\dot{\beta}^2 \end{bmatrix};$$

$$G(\alpha,\beta)=\begin{bmatrix} 0 \\ -\frac{1}{2}m_pL_pg\sin(\alpha) \end{bmatrix};\quad v=\begin{bmatrix} k_1e \\ 0 \end{bmatrix}$$

After identification, the motor parameters are shown in Table 1. Then, system parameters are shown in Table 2.

Table 2. *Parameters of system*

No	Para-meter	Description	Unit	Value
1	m_p	Mass of pendulum	kg	0.027
2	L_p	Length of pendulum	m	0.328
3	J_p	Inertial moment of pendulum	kg.m	0.0046617
4	L_r	Length of arm	m	0.205
5	J_r	Inertial moment of arm	kg.m	0.0019
6	g	Gravitation acceleration	m/s^2	9.81
7	B_r	Friction of arm	NA	0.0017
8	B_p	Friction of pendulum	NA	≈0

In order to linearize the model in State Space Form, we assume that $\sin\alpha\approx\alpha$, $\sin\beta\approx\beta$, $\cos\alpha=1$, $\cos\beta=1$.

Defining the state variables as below:

$$x_1=\alpha; x_2=\dot{\alpha}; x_3=\beta; x_4=\dot{\beta}$$

$$x=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T;\ y=\begin{bmatrix} x_1 & x_3 \end{bmatrix}^T$$

The system in state space equation is

$$\dot{x}=Ax+Be; y=Cx \tag{10}$$

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}; B=\begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix} \tag{11}$$

where

$$a_{21}=\frac{\partial\ddot{\alpha}}{\partial x_1};\ a_{22}=\frac{\partial\ddot{\alpha}}{\partial x_2};\ a_{23}=\frac{\partial\ddot{\alpha}}{\partial x_3};\ a_{24}=\frac{\partial\ddot{\alpha}}{\partial x_4};\ a_{41}=\frac{\partial\ddot{\beta}}{\partial x_1};$$

$$a_{42}=\frac{\partial\ddot{\beta}}{\partial x_2};\ a_{43}=\frac{\partial\ddot{\beta}_4}{\partial x_3};\ a_{44}=\frac{\partial\ddot{\beta}_4}{\partial x_4};\ b_2=\frac{\partial\ddot{\alpha}}{\partial e};\ b_4=\frac{\partial\ddot{\beta}}{\partial e}$$

From (6), (7), (8), (9), operators $\ddot{\alpha}$, $\ddot{\beta}$ are found and then substituted into (11) to estimate matrix A, B in (12) (At equilibrium point, we consider $\alpha,\dot{\alpha},\beta,\dot{\beta},e$ are 0).

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 47.1570 & -0.1086 & 0 & -0.6303 \\ 0 & 0 & 1 & 1 \\ 14.1057 & -0.0325 & 0 & -0.8279 \end{bmatrix};\ B=\begin{bmatrix} 0 \\ 1.4698 \\ 0 \\ 1.9306 \end{bmatrix} \tag{12}$$

2.2 Swing up controller

In this article, the authors recommend two methods to swing-up the pendulum from a pending position to the vertical upward point. Swing-up the pendulum can be achieved by using an energy method. At the upward position, another controller will be applied to balance the pendulum. Swing-up by energy control is the classical method whereas the second method determines the control input by exponentiation of the pendulum position.

2.2.1 Swing-up by energy control [12]

Swinging pendulum from a downward position to upward position, energy control is used to implement energy to make pendulum to vertical upward position. The movement of arm relies on the position and the velocity of the pendulum. The normalized energy can be then written as below:

$$E = ml^2\dot{\alpha}^2/2 + mgl\cos(\alpha) \quad (13)$$

The pendulum link is swing from downward position to upward position, so the energy of the system is $-\mathrm{mgl}$ to mgl. Thence, expected energy of system is

$$\mathrm{E}_0 = mgl \quad (14)$$

The relationship of control input e and energy E is shown as (15)

$$\dot{E} = mgle\dot{\alpha}\cos(\dot{\alpha}) \quad (15)$$

The energy of RIP is grown up if $e\dot{\alpha}\cos(\dot{\alpha}) > 0$. Therefore, in order to make the pendulum swing-up to vertical upward point, the control input e will be choosen so that $(\mathrm{E}-\mathrm{E}_0)^2$ decreases,

$$e = -e_{\max}\,\mathrm{sgn}((E-E_0)\dot{\alpha}\cos(\alpha)) \quad (16)$$

$$e = \begin{cases} e_{\max} & if\,(E-E_0)\dot{\alpha}\cos\alpha < 0 \\ -e_{\max} & if\,(E-E_0)\dot{\alpha}\cos\alpha > 0 \end{cases} \quad (17)$$

2.2.2 Swing-up by exponentiation of the pendulum position [13]

The main idea of this method is the energy supplied for the system depends on the pendulum position. In view of the preferred progress of the exponential function, when the pendulum from a pendant position to vertical upward position, the energy at upward point is lowest. In contrast, at a downward point, the energy is highest. The control law is described in (18)

$$e = k_v\,|\alpha^n|\,\mathrm{sgn}(\alpha\cos(\dot{\alpha})) \quad (18)$$

where: k_v determines how fast the pendulum reaches the vicinity of the unstable equilibrium; n is constant.

The Strong point of this method does not need to pre-calculate as in the case of swing-up with energy control, where it is vital to know the energy of the pendulum at an upright position.

3. BALANCING AND SWING UP CONTROL

3.1 Linear Back-stepping control

Step 1: First of all, the new control variable is defined as

$$z_1 = x_1 - k_1x_3 \quad (19)$$

Where k_1 is design constant. Therefore, the derivative of z_1 is calculated as

$$\dot{z}_1 = \dot{x}_1 - k_1\dot{x}_3 = x_2 - k_1x_4 \quad (20)$$

We need to find control law to converge z_1 to zero. We consider x_2 is a virtual control variable. A positive definite Lyapunov function is selected as

$$V_1 = z_1^{\,2}/2 > 0 \quad (21)$$

By Lyapunov criteria, a suitable stabilizing function should be selected to find out the desired value of the virtual input for the first subsystem to satisfy:

$$\dot{V}_1 = -c_1z_1^{\,2} < 0 \quad (22)$$

From (A.3) in Appendix A, we obtain stabilizing function

$$\alpha_1 = k_1x_4 - c_1z_1 \quad (23)$$

where c_1 is designed positive constant

Step 2: The corresponding error variable is defined as

$$z_2 = x_2 - \alpha_1 \quad (24)$$

The derivative of z_2 is calculated as

$$\dot{z}_2 = \dot{x}_2 + c_1\dot{z}_1 - k_1\dot{x}_4 \quad (25)$$

Replacing equation (10) into (24), we have:

$$\begin{aligned}\dot{z}_2 &= \left(a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + b_2e\right) + \\ &+ c_1\dot{z}_1 - k_1\left(a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + b_4e\right)\end{aligned} \quad (26)$$

Replacing equation (19) into (25)

$$\begin{aligned}\dot{z}_2 &= \left(a_{21} - k_1a_{41}\right)x_1 + \left(a_{22} - k_1a_{42} + c_1\right)x_2 \\ &+ \left(a_{23} - k_1a_{43}\right)x_3 + \left(a_{24} - k_1a_{44} - c_1k_1\right)x_4 \\ &+ \left(b_2 - k_1b_4\right)e\end{aligned} \quad (27)$$

We set:

$$d_1 = (a_{21} - k_1 a_{41})\ ;\ d_2 = (a_{22} - k_1 a_{42} + c_1)\ ; \quad (28)$$
$$d_3 = (a_{23} - k_1 a_{43})\ ;\quad d_4 = (a_{24} - k_1 a_{44} - c_1 k_1)\ ;$$
$$d_5 = (b_2 - k_1 b_4)$$

So, equation (26) can be rewritten as:

$$\dot{z}_2 = d_1 x_1 + d_2 x_2 + d_3 x_3 + d_4 x_4 + d_5 e \qquad (29)$$

Analyzing stability of the system, we have

$$V_2 = \left(z_1^2 + z_2^2\right)/2 \qquad (30)$$

From equation (19) and (20), we have:

$$\dot{z}_1 = z_2 - c_1 z_1 \qquad (31)$$

Derivative (30), it yields:

$$\dot{V}_2 = z_1 \dot{z}_1 + z_2 \dot{z}_2 \qquad (32)$$

Replacing (31) into (32), we have:

$$\dot{V}_2 = z_1(z_2 - c_1 z_1) + z_2 \dot{z}_2 = -c_1 z_1^2 + z_2(z_1 + \dot{z}_2) \qquad (33)$$

In order to $\dot{V}_2$ negative definite, the desired dynamics of z_2 can be defined:

$$\dot{z}_2 = -z_1 - c_2 z_2 \qquad (34)$$

Where c_2 is designed positive constant

Replacing (19), (24) and (29) into (34), we have the control law as below:

$$e = \frac{(-x_1 + k_1 x_3) - c_2\left[x_2 + c_1(x_1 - k_1 x_3) - k_1 x_4\right] - \sum_{i=1}^{4} d_i x_i}{d_5} \qquad (35)$$

where c_1, c_2, k_1 are chosen by trial-and-error test and listed in Table 3 below.

Table 3. *Parameters of back-stepping control*

c_1	c_2	k_1
98.8100	95.6400	0.0200

3.2 Swing-up control

Swing-up is used to move the pendulum from initial downward position π to balancing position, another controller is used to stabilize the pendulum. In this study, back-stepping technique is applied to stabilize the pendulum.

Swing-up pendulum link by Furuta method is shown in (17). Following this method, we recognize that the arm rotates in a positive direction when $e = e_{max}$ and in a negative direction when $e = -e_{max}$.

Another method is exponentiation of the pendulum position. From (18), we recognize that the upward position of the pendulum is set $\alpha = 0$, the dynamics of the swing-up is influenced by a suitable exponentiation of the position α^n, so that at nearly upward position, the pendulum link is supplied less energy.

4. SIMULATION AND EXPERIMENT

4.1 Simulation result

4.1.1 Stabilization using back-stepping

Stabilization of RIP is shown from Fig 2. to Fig 5. The parameters for this simulation follow Table 1., Table 2. And Table 3. Initial values of system are chosen as

$$x = [0.1 \quad 0 \quad -0.1 \quad 0]^T \qquad (36)$$

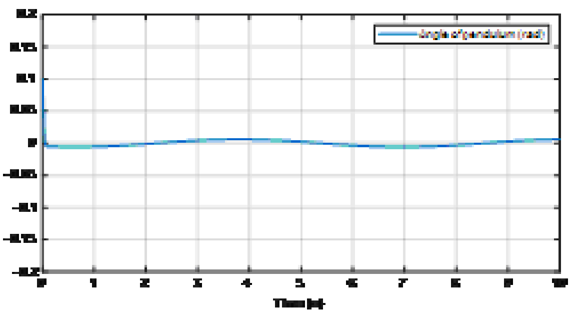

Figure 2. *Angle of pendulum (rad)*

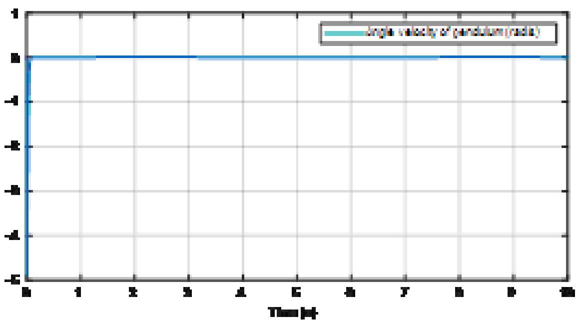

Figure 3. *Angle velocity of pendulum (rad/s)*

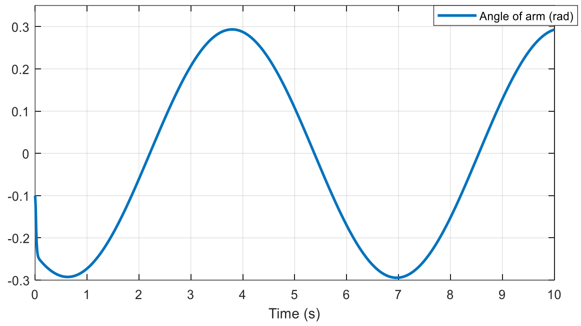

Figure 4. *Angle of arm (rad)*

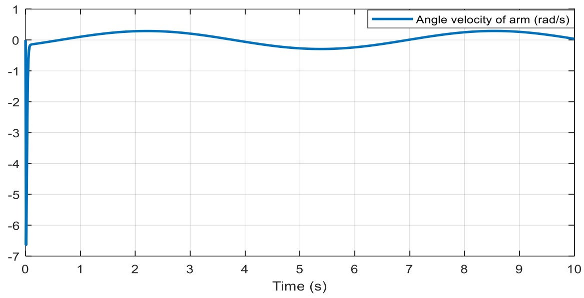

Figure 5. *Angle velocity of arm (rad/s)*

According to Fig 2, under control law (35), controller can balance the pendulum at stabilization point. The linear back-stepping control can keep the pendulum at upright position. Therefore, a back-stepping controller can acquire our control objective within the range of equilibrium zone rapidly.

According to Fig 4, under control law (35), the output signal is sine wave because the back-stepping controller guarantees z_1 in (19) to converge to zero. The z_1 function is a combination of x_1, x_3. Then, It does not guarantee x_1, x_3 to move to zero. Therefore, these variables can still vibrate. However, due to the stability of z_1, these variables tend not to be unstable.

4.1.2 Swing-up and Stabilization using back-stepping

The initial vale of the system is set as

$$\left[\alpha(0), \dot{\alpha}(0)\right]^T = \left[\pi, 0\right]^T \qquad (37)$$

Block diagram and simulation result using swing-up controller combination with back-stepping control are shown from Figure 6 to Figure 8.

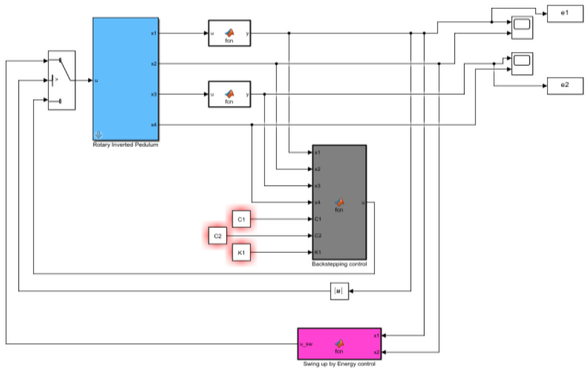

Figure 6. *Block diagram of controllers using swing-up control and Back-stepping*

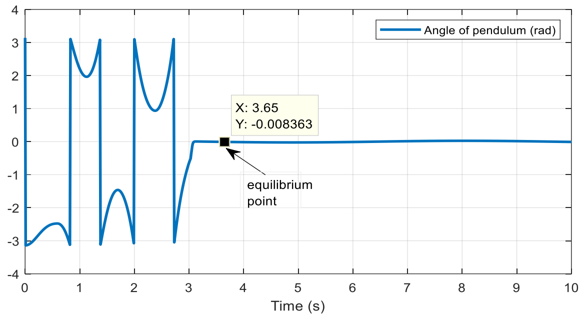

Figure 7. *Simulation of Swing-up by energy control & Stabilization using Back-stepping*

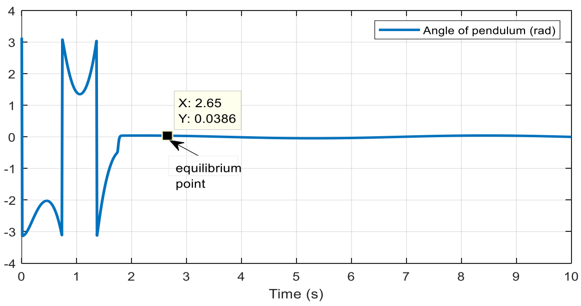

Figure 8. *Simulation of Swing-Up by exponentiation pf the pendulum position & Stabilization using Back-stepping*

The corresponding of system when applied swing-up by energy control and back-stepping control. As we can see in Fig 7. shows that the transition time of the system using back-stepping control is only 3.6 seconds.

The corresponding of system when applying swing-up by exponentiation of the pendulum position and back-stepping control. In Fig 8, we see that the transition time of the system using back-stepping control is only 2.95 seconds.

4.2 Experimental result

The real-time model is shown in Fig 9.

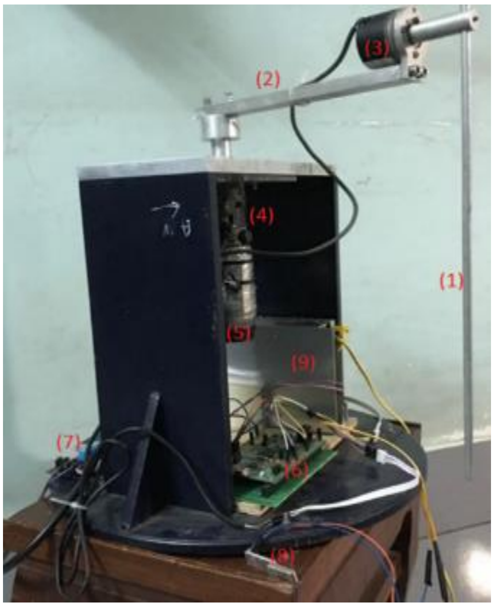

Figure 9. *The real time RIP model*

Components are:

1. Pendulum link
2. Arm link
3. Encoder for pendulum link
4. DC motor
5. Encoder for arm link
6. STM32F407 Discovery
7. H Bridge
8. UART CP2102
9. Voltage power supply 24VDC-10A

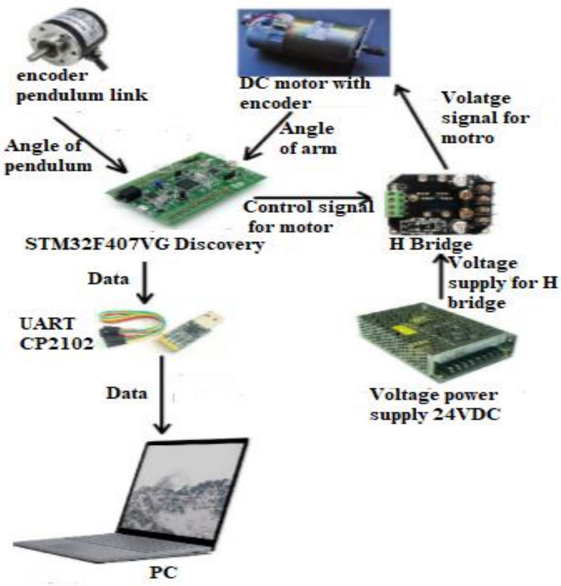

Figure 10. *Electrical diagram*

4.2.1 Stabilization using backstepping

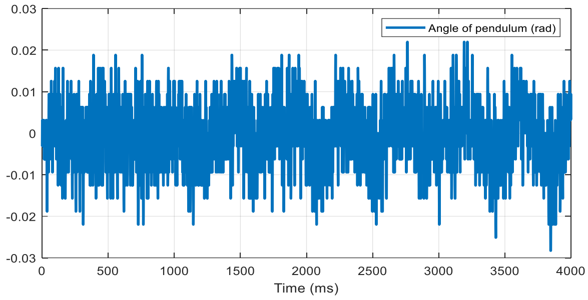

Figure 11. *Angle of pendulum (rad)*

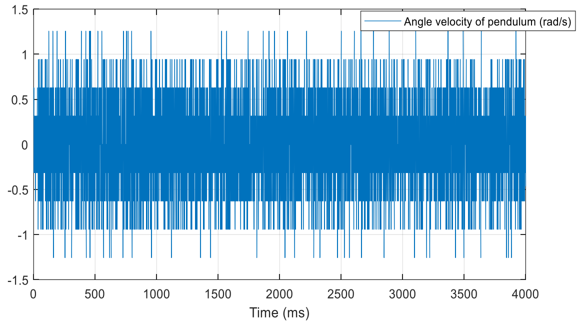

Figure 12. *Angle velocity of pendulum (rad/s)*

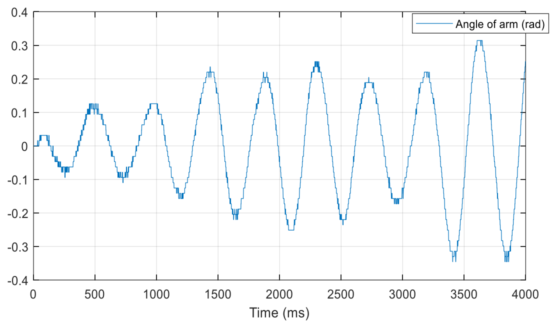

Figure 13. *Angle of arm (rad)*

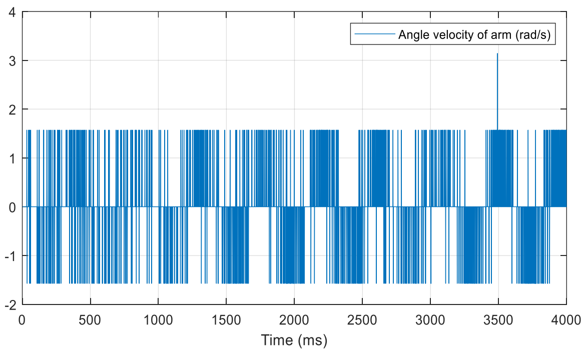

Figure 14. *Angle velocity of arm (rad/s)*

As a experimental result, around the equilibrium point of the rotary inverted pendulum system, the linear backstepping control can keep the pendulum link at the vertical upward position.

According to Fig 13, the output signal of arm angle in experimental (in Fig 13) oscillates same as the output signal of arm angle in simulation. In both simulation and experiment, the arm follows the sine function due to explanation in Section 4.1. In experiment, after 2.5s, the angle of arm follows a sine function which has amplitude 0.3 rad and works in indefinite period. It cannot move to zero due to the selection of z_1 in (19). This is a disadvantage of the back-stepping method for the SIMO system. However, the better selection of function for z_1 can give the better results.

4.2.2 Swing-up and Stabilization using backstepping

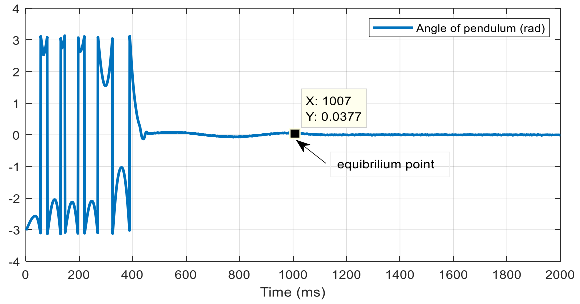

Figure 15. *Experimental Swing-Up by energy control & Stabilization using Back-stepping control*

In Fig 16., as we see, that when transiting from swing-up to balancing. By using exponentiation of the pendulum position to swing the pendulum and back-stepping control to balance the pendulum, the transition time is nearly 660 milliseconds, while using the energy method, the transition time is 1000 milliseconds.

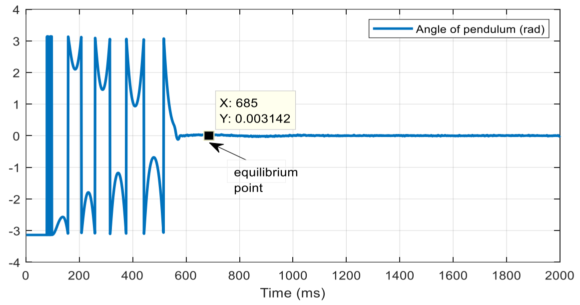

Figure 16. *Experimental Swing-Up by exponentiation of pendulum position & Stabilization using Back-stepping control*

As an experimental result, the control input from combination of a swing-up control a balancing control is able to keep and stabilize the pendulum link from stable equilibrium point $x=[\pi \quad 0 \quad 0 \quad 0]^T$ to unstable equilibrium point $x=[0 \quad 0 \quad 0 \quad 0]^T$.

5 CONCLUSION

The problem of swing up the RIP combination with stabilization of the pendulum by back-stepping was analyzed. The block of back-stepping designed can keep the system at an unstable zone. Two proposed swing-up methods are able to be applied and tested on real laboratory set up. The first one method is traditional control based on energy of the system, the second one is based on exponentiation function over the pendulum position for appreciating the energy to be delivered to the system. After the comparison between energy control and exponentiation of the pendulum position, as we see, the second method is better. Both simulation and experimental results are mostly similar. Our future research is control design nonlinear back-stepping control for this model.

APPENDIX A

Derivative (21), it yields

$$\dot{V}_1 = z_1\dot{z}_1 \Leftrightarrow \dot{V}_1 = z_1(x_2 - k_1x_4) \qquad \text{(A.1)}$$

We choose sub-function to guarantee (22). Then, by equalize these formulas, we obtain:

$$z_1(x_2 - k_1x_4) = -c_1z_1^2 \Leftrightarrow x_2 - k_1x_4 = -c_1z_1 \qquad \text{(A.2)}$$

A suitable stabilizing function has been selected to find out the desired value of the virtual input for the first subsystem. We need to choose the stabilizing function (23):

$$\alpha_1 = x_2 = k_1x_4 - c_1z_1 \qquad \text{(A.3)}$$

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