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Trajectory Tracking and Stabilization Control of Rotary Inverted Pendulum based on LQR and LQT Techniques: Simulation and Experiment

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Tracking the trajectory.

ABSTRACT

Rotary Inverted Pendulum (RIP) plays a vital role in control engineering. Rotary Inverted Pendulum is a complex, nonlinear, non-minimum-phase and under-actuated system which has various applications in the field of Robotics. The main contribution of this paper is to design and control RIP by using Linear Quadratic Regulator (LQR) controller for stabilization at vertically upright position - the unstable equilibrium point, and Linear Quadratic Tracker (LQT) controller for tracking the desired trajectory. Besides, stability of the closed-loop system is analyzed for ensuring the reliability of the developed controller. The simulation is carried out in MATLAB/Simulink environment, and the proposed controllers have been tested on Rotary Inverted Pendulum hardware that is designed by authors. The analysis and results conducted on the system demonstrate the performance of the control schemes, including stabilization of unstable equilibrium point, tracking the desired trajectory, and system response showing the robustness and effectiveness of methods.

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1. Introduction

The classical rotary inverted pendulum (RIP) system is well known in control area to demonstrate the desired performance of control algorithm, which is a highly nonlinear, multi-variables, unstable, strong-coupled system. It makes the control more difficult and complicated. The pendulum rod can intuitively show many abstract concepts such as stability, rapidity, and robustness in the control field, and that's why it is the ideal model for robot control systems, aircraft landing systems, and so on. As a top subject in the control area, there are two problems with studying the inverted pendulum system: one is reposefully controlling the pendulum at the upright position, and the other is controlling the arm link to track a random trajectory when still stabilizing pendulum on inverted position. There are lots of works of literature that focus on stability control, and it is relatively mature. Many algorithms have been conducted on this system, including linear quadratic regulator (LQR) [1], [2], adaptive dynamic programming based linear quadratic regulator design [3], LQR-based ANFIS [4], LQR-based Sliding Mode Control [5], LQR neural network control approach [6], robust adaptive super-twisting sliding mode stability control [7], backstepping control [8], [9], input-output feedback linearization control [10], [11], Lyapunov-based continuous-time nonlinear control using deep neural network [12], robust control for RIP system with unmatched uncertainty [13].

LQR method, known as linear optimal state feedback control and is used in adaptive control mechanisms, aims to minimize the errors that occur in the state output values [14]. Linear quadratic tracking (LQT) - a closed-loop control scheme is designed with purpose that the output response of the system optimally tracks desired trajectory. In this paper, the LQR method, LQT control method are designed and applied for the RIP system. Therefore, the ability of this real-time RIP is proved to be suitable for both simulation and experiment. The application of mathematics in balancing, and tracking the trajectory proves that real-time RIP satisfies the requirement of a standard model for laboratories.

The organization of this paper has 5 parts. Part 1 is introduction. Part 2 describes the RIP system's mathematical model, constraints imposed on the model, and the state transformation for the stabilization task. In Part 3, the analyzed control schemes, and hardware design. Part 4 describes the simulation results and experimental validation of analyzed algorithms, while Part 5 gives conclusion and final remarks.

2. Mathematical model of RIP

2.1. System model of RIP

The structure of RIP system is shown in Figure 1. It contains two links. The first link is pendulum, and the second link is arm. Angle of pendulum and arm are α and β, respectively. The model's parameters are shown in Table 1. For the convenience of adjusting the motor as well as applying the controller to the real model, we transform the control signal from torque of DC motor to voltage that is applied to DC servo motor by formula (5). According to [9], the equations of the RIP in matrix is as follows

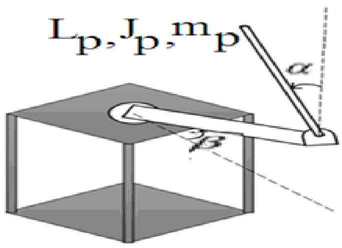

Figure 1. *Structure of RIP*

$$D(\alpha,\beta).(\ddot{\alpha},\ddot{\beta})+C(\alpha,\dot{\alpha},\beta,\dot{\beta}).(\dot{\alpha},\dot{\beta})+G(\alpha,\beta)=v \tag{1}$$

Where,

- Inertial matrix:

$$D(\alpha,\beta)=\begin{bmatrix} -\frac{1}{2}m_pL_pL_r\cos\alpha & m_pL_r^2+\frac{1}{4}m_pL_p^2-\frac{1}{4}m_pL_p^2\cos^2\alpha+J_r \\ \left(J_p+\frac{1}{4}m_pL_p^2\right) & -\frac{1}{2}m_pL_pL_r\cos\alpha \end{bmatrix} \tag{2}$$

- Radial vector:

$$C(\alpha,\dot{\alpha},\beta,\dot{\beta})=\begin{bmatrix} \left(\frac{1}{2}m_pL_pL_r\sin\alpha\right)\dot{\alpha} & \left(\frac{1}{2}m_pL_p^2\sin\alpha\cos\alpha\right)\dot{\alpha}+B_r+k_2 \\ B_p & -\frac{1}{4}m_pL_p^2\cos\alpha\sin\alpha\dot{\beta}^2 \end{bmatrix} \tag{3}$$

- Gravity vector and output vector:

$$G(\alpha,\beta)=\begin{bmatrix} 0 \\ -\frac{1}{2}m_PL_Pg\sin\alpha \end{bmatrix};\ v=\begin{bmatrix} k_1e \\ 0 \end{bmatrix} \tag{4}$$

- Control input is transformed from torque to voltage:

$$\tau=-k_2\dot{\beta}+k_1e \tag{5}$$

$$k_1=\frac{K_t}{R_m};k_2=\frac{K_t}{R_m}K_b$$

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The system's parameters list in Table 1 based on the real system and variables.

Table 1. *Parameters of system*

Parameter	Description	Unit	Value
m_p	Mass of pendulum	kg	0.062
L_p	Length of pendulum	m	0.2
l_p	The center of mass	m	$L_p/2$
J_p	Inertial moment of pendulum	kgm	0.0046617
L_r	Length of arm	m	0.205
J_r	Inertial moment of arm	kgm	0.0019
g	Gravitation acceleration	m/s^2	9.80665
B_r	Friction of arm	N m s/rad	≈0
B_p	Friction of pendulum	N m s/rad	0.0017

According to [15], DC motor's parameters are listed in Table 2.

Table 2.. *Motor parameters*

K_b (V / (rad / sec))	0.064943
K_t (V / (rad / sec))	0.064943
R_m (Ω)	6.835271

2.2. Linearization

To linear the model in state-space form, we assume that $\sin\alpha \approx \alpha$, $\sin\beta \approx \beta$, $\cos\alpha = 1$, $\cos\beta = 1$. Defining the state and output as given below

$$x_1 = \alpha; x_2 = \dot{\alpha}; x_3 = \beta; x_4 = \dot{\beta}; x_5 = \ddot{\alpha}; x_6 = \ddot{\beta} \tag{6}$$

where: $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T; y = [x_1 \quad x_3]^T$

Hence, the state-space representation of the completed system is obtained as follows

$$\dot{x} = Ax + Be; y = Cx \tag{7}$$

Where, $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$;

$a_{21} = \frac{\partial\ddot{\alpha}}{\partial x_1}$; $a_{22} = \frac{\partial\ddot{\alpha}}{\partial x_2}$; $a_{23} = \frac{\partial\ddot{\alpha}}{\partial x_3}$; $a_{24} = \frac{\partial\ddot{\alpha}}{\partial x_4}$; $a_{41} = \frac{\partial\ddot{\beta}}{\partial x_1}$; $a_{42} = \frac{\partial\ddot{\beta}}{\partial x_2}$; $a_{43} = \frac{\partial\ddot{\beta}_4}{\partial x_3}$; $a_{44} = \frac{\partial\ddot{\beta}_4}{\partial x_4}$; $b_2 = \frac{\partial\ddot{\alpha}}{\partial e}$; $b_4 = \frac{\partial\ddot{\beta}}{\partial e}$

2.3. Stability analysis of the linear model

The stability of system can be determined by examining the location of the eigenvalues values of the transfer function. This characteristic equation can be expressed by det(**λI−A**) = 0 where **λi** (i = 1, . . . , n) are the eigenvalues of **A** ∈ Rn×n, which are also the open-loop poles, and $\mathbf{I_n}$ is the unit matrix. By substituting the values of the parameters from Table 1 and Table 2 into (7), we obtain

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 68.4244 & 0 & 0 & -0.4053 \\ 0 & 0 & 0 & 1 \\ 10.6368 & 0 & 0 & -0.3464 \end{bmatrix}; B=\begin{bmatrix} 0 \\ 1.6622 \\ 0 \\ 1.4204 \end{bmatrix} \tag{8}$$

We find the open-loop poles of the system, which are λ1 = 0, λ2 = 8.2415, λ3 = -8.3046 and λ4 = -0.2833 show the system unstable. Considering the stability in the vertical position of the system, we have commandability matrix of the system

$$Mc=\begin{bmatrix} B & AB & A^2B & \ldots \mathrm{A}^{n-1}B \end{bmatrix} \tag{9}$$

$$rank(Mc)=4=n \tag{10}$$

The system is unstable due to the poles with positive real part. All closed-loop poles with negative real part, stability can be guaranteed. The state-variable feedback (SVFB) control is effectiveness approach to resolve this problem. Given that the system is completely commandability, the next step is to locate the desired closed-loop poles.

The characteristic equation as follows

$$s^2+2\xi\omega_n s+\omega_n^2=0 \tag{11}$$

They are expressed as

$$s_{1,2}=-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \tag{12}$$

Where, ω_n is natural frequency, ξ is damping ratio

The condition of controller and time response are as follows:

- The maximum overshoot in unit step response is 5%.
- The settling time within 3% acceptable tolerance in the unit step response is 2 secs.

The maximum overshoot POT is given by

$$POT=e^{-\left(\xi/\sqrt{1-\xi^2}\right)\pi}=5\% \tag{13}$$

Thus,

$$5\%=e^{-\left(\xi/\sqrt{1-\xi^2}\right)\pi} \tag{14}$$

Which yields $\xi=0.69$. The settling time t_{xl} for 3% allowable tolerance is given by

$$t_{xl}=\frac{4}{\xi\omega_n} \tag{15}$$

is specified as 2 secs. Thus, $\omega_n=2.89$ rad/sec

We obtain the dominant pairs as $s_{1,2}=-1.9941\pm 2.0918j$. According to [16], the poles can be randomly chosen as long as the complex pairs dominate the response. Hence, we choose $s_3=-10$, and $s_4=-20$.

SVFB pole placement with Ackermann's Formula [16]

$$K_{svfb} = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix} R^{-1} \phi_d(A) \tag{16}$$

Where, Mc commandability matrix, $\phi_d(s)$ is the characteristic equation for the closed-loop poles, which we evaluate for $s = A$.

The controller gain for this system with the desired poles $s_{1,2} = -1.9941 \pm 2.0918j$, and $s_3 = -10$, and $s_4 = -20$

$$K_{svfb} = \begin{bmatrix} 2.5644 & 31.713 & -21 & -13.4269 \end{bmatrix} \tag{17}$$

Hence, using the state feedback control law as follows

$$u = -K_{svfb} x \tag{18}$$

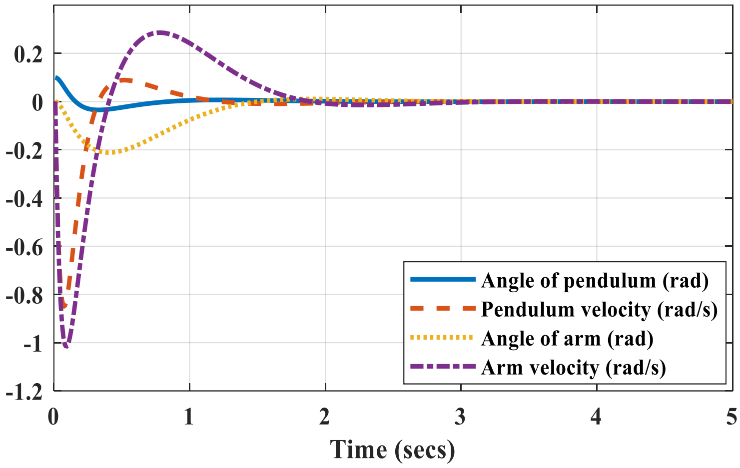

Figure 2. *The output response of the RIP system*

Figure 2 indicates output response for closed-loop system with initial condition $x = [0.1 \quad 0 \quad 0 \quad 0]^T$, according to original position of pendulum position afar from vertically upright position. The result in Figure 2 explains that SVFB controller stabilizes the RIP system at unstable equilibrium point with certain conditions. In conclusion, stability of the closed-loop system has already been analyzed for ensuring the reliability of the developed controller.

3. Methodology

3.1. Linear Quadratic Regulator controller (LQR) [17]

The A and B matrices are introduced in 2.3 of this paper. *Q* and *R* matrices are selected as identity matrix like (19)

$$Q = I_{4\times 4}; R = 1 \tag{19}$$

Implementing MATLAB command *K= lqr(A, B, Q, R)*. The feedback control matrix is

$$K_{lqr} = \begin{bmatrix} 103.4602 & 12.8266 & -1 & -2.1580 \end{bmatrix} \tag{20}$$

3.2. Linear Quadratic Tracking controller (LQT) [17]

Consider a linear, time-invariant (LTI) system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{21}$$

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With cost function as given below

$$\lim_{t_f \to \infty} J = \lim_{t_f \to \infty} \frac{1}{2} \int_0^{\infty} [e'(t)Qe(t) + u'(t)Ru(t)]dt \quad (22)$$

Where $x \in \mathbb{R}^n$ is system state, $y \in \mathbb{R}^p$ is the output of system, *z(t)* is desired output trajectory, and tracking error *e(t)* = *z(t)*-*y(t)*. As $t_f \to \infty$, the $P(t)$ tends to the steady-state value $\bar{P}$ as the solution

$$-\bar{P}A - A'\bar{P} + \bar{P}BR^{-1}B'\bar{P} - C'QC = 0 \quad (23)$$

The vector function $g(t)$ tends to a finite function $\bar{g}(t)$ as the solution of

$$\dot{\bar{g}}(t) = \left[\bar{P}E - A'\right]\bar{g}(t) - Wz(t); \text{ where, } E = BR^{-1}B' \text{ and } W = C'Q \quad (24)$$

The optimal control law as follows

$$u(t) = -R^{-1}B'\left[\bar{P}x(t) - \bar{g}(t)\right] \quad (25)$$

4. Simulation and experimental results

4.1. Simulation results

4.1.1. Stabilization by using LQR

Stabilization of RIP is shown from Figure 3 to Figure 5. The parameters for this simulation follow Table 1, Table 2. Initial values of system are chosen as follows

$$x = [0.2 \quad 0.01 \quad 0.1 \quad 0.03]^T \quad (26)$$

Selecting the penalty matrices *Q* and *R* as is (19) and initial condition as is (26), the output responses of system show in Figures 3, 4, respectively. According to Figure 3, LQR controller brings pendulum angle (α) back to vertical upright position "0 rad" in 1 second and the arm is stable after 4 seconds. The control effort is indicated in Figure 4 and performance index is shown in Figure 5. With matrix *K* (20), Figure 4 shows the performance index is minimized. On the one hand, the maximum overshoot range of arm position and pendulum position is quite large. We can reduce the overshoot range by eliminating the matrices *Q* and *R*. On the other hand, the rotary inverted pendulum could achieve the optimal control.

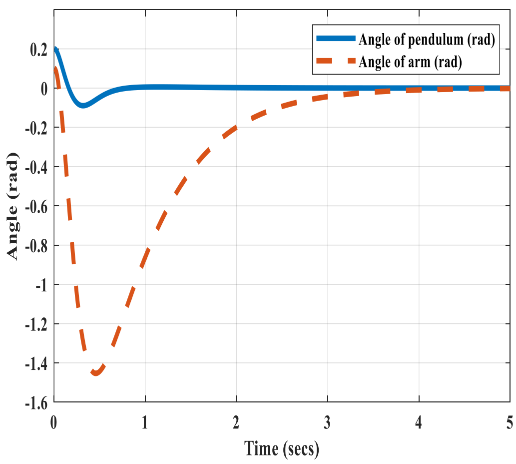

Figure 3. *The output response of the RIP system*

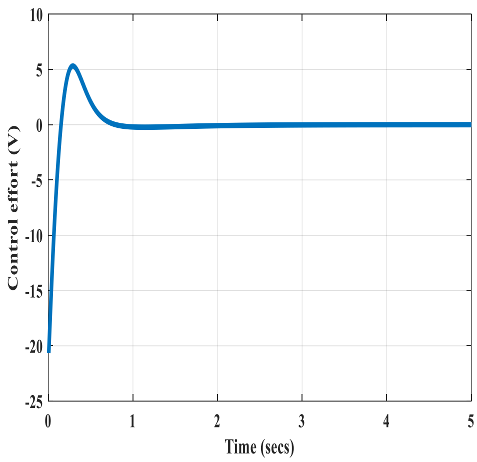

Figure 4. *The control input (V) with LQR control*

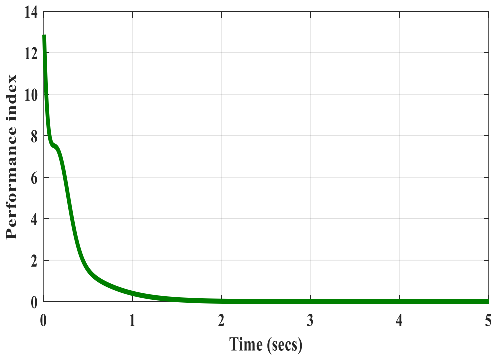

Figure 5. *The performance index of the RIP system with LQR control*

4.1.2. LQT simulation results

We design LQT controller for RIP system with sample time T = 0.001 second. Know the reference status $z_1(t) = 0$, and $z_3(t)$ is function like: $z_3(t)$ = sine function with amplitude ±1 rad, period 12 seconds.

$$Q = diag([10;1;100;1]); R = 1 \quad (27)$$

$$K_{lqt} = [5.6709 \quad 92.5461 \quad -10 \quad -14.1913] \quad (28)$$

The Q and R matrices are selected as is (23), we put the value Q_{33} = 100, aim to increase the control effort and focus on the angle of arm. The state feedback gain K_{lqt} is presented as is (28) and initial condition for this testing as is (26).

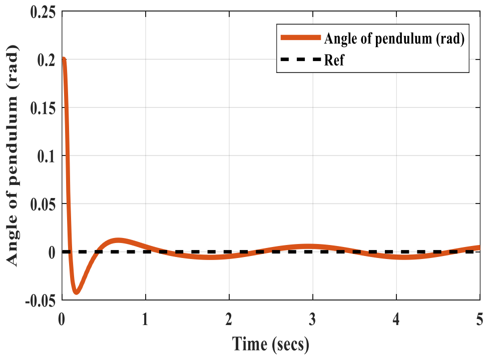

Figure 6. *The output response of the RIP system with LQT control*

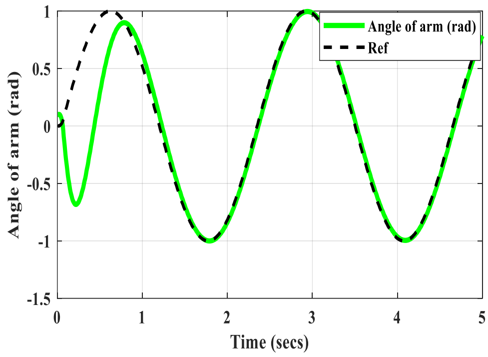

Figure 7. *The output response of the RIP system with LQT control*

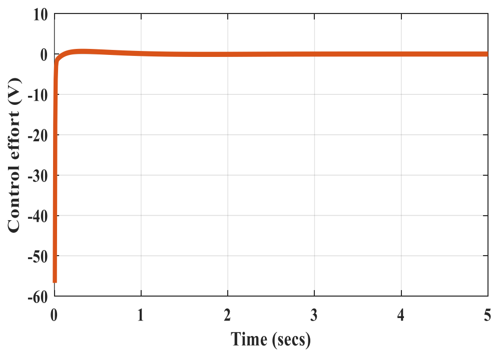

Figure 8. *The control effort (V) of the RIP system with LQT control*

Figures 6, and 7 illustrate the simulation results using LQT controller developed in subsection 3.3. Figure 6 shows the time response of pendulum angle has ripple compared with the reference (black) because the arm oscillates back and forth to track desired trajectory $z_3(t)$. Figure 7 shows the time

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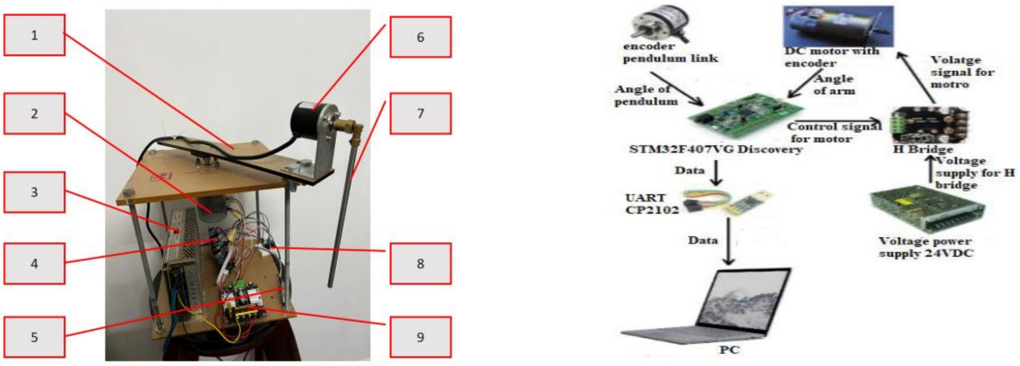
evolution of the horizontal arm position denoted by x_3 during tracking of desired trajectory (black) with the amplitude ± 1 rad. Initially, LQT controller needs 1 second to bring the arm position back to trajectory. The time-evolution of the horizontal arm position still have a delay compared with reference (black). Likewise, Figure 8 depicts simulation results for the applied voltage (V).

4.2. Experimental results

4.2.1. System setup

In this subsection, we show the RIP system in 10 which used in this experiment. It consists of the following components: 1. Arm link; 2. Nisca DC servo motor with encoder (200 rpm); 3. Power supply; 4. STM32F407VG Discovery; 5. USB-TTL UART CP2102 module; 6. Encoder pendulum (600 rpm); 7. Pendulum link; 8. Pull-up resistor board; 9. Driver IR2184



(a) *(b)*

Figure 9. *(a) RIP system; (b) Diagram of devices connection*

4.2.2. Stabilization by using LQR approach

In this subsection, the state feedback gain (20) is used for the experimental validation. Figures 10, and 11 show the angular position of the pendulum and the arm during the balancing progress. According to the results, the balancing controller does not have any overshoot and it has no steady-state error as desired. Figure 12 shows the control input during the balancing motions.

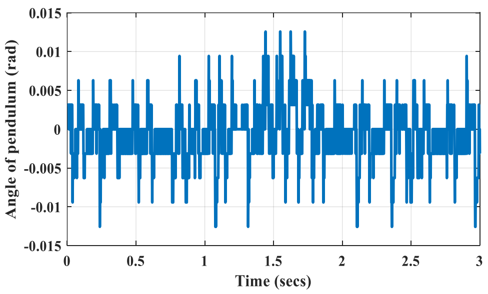


Figure 10. *Angle of pendulum (rad)*

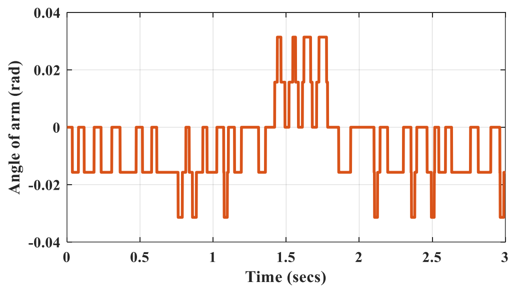


Figure 11. *Angle of arm (rad)*

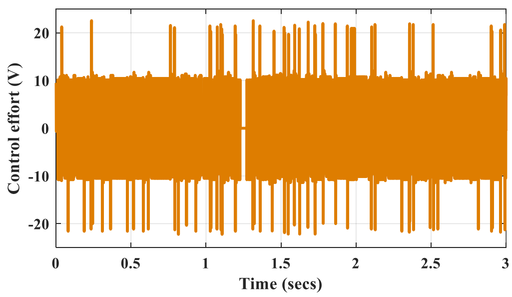


Figure 12. *The control input (V) with LQR control*

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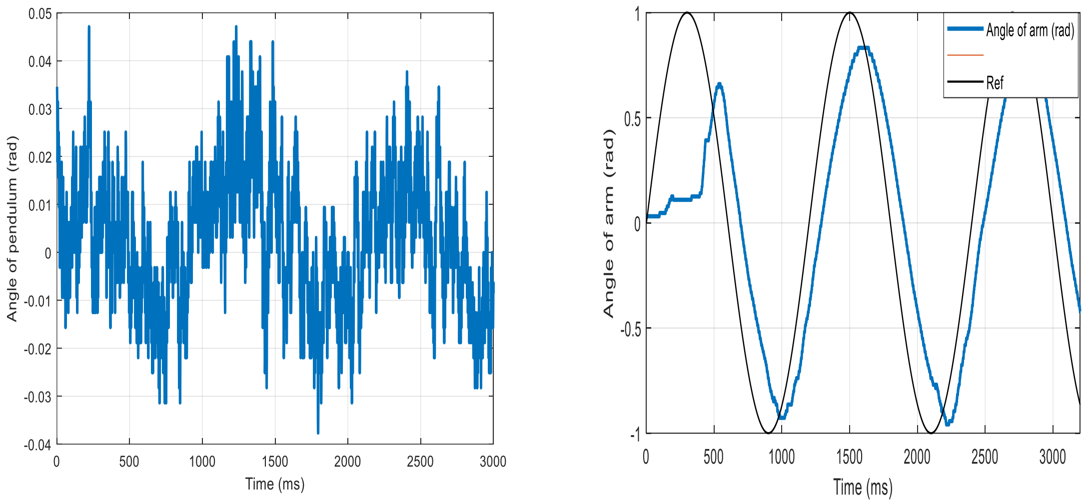
4.2.3. LQT experimental results

In this subsection, the state feedback gain (28) is used for the experiment. The results of experimental evaluation of the proposed controller are depicted in Figures 13 and 14. Specifically, Figure 13b shows the time response of the horizontal arm position during the tracking of the desired trajectory with the amplitude +-1 rad. The arm position still has a delay during the tracking of the desired trajectory. Besides, in Figure 13a, we see the pendulum oscillates around zero point (range +-0.04 rad). Likewise, Figure 14 depicts simulation results for the applied voltage (V).

As observed in Figures 7 and 13b, the experiment and the simulation are similar, which have a delay during the tracking of desired trajectory.



(a) *(b)*

Figure 13. *(a) Angle of pendulum (rad); (b) Angle of arm (rad)*

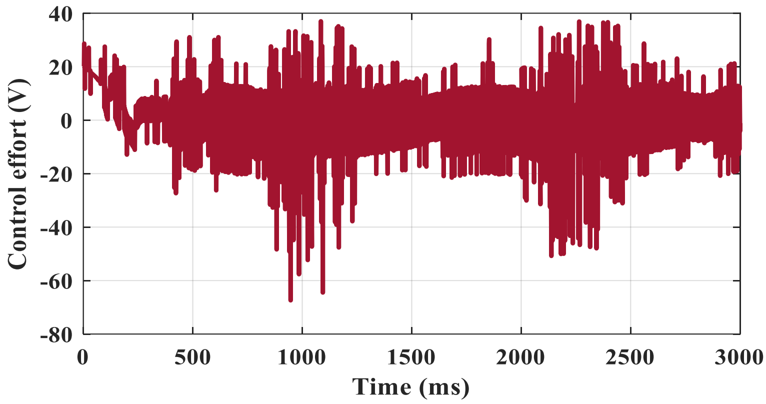


Figure 14. *The control input (V) with LQT control*

5. Conclusions

Identifying the research subjects, controller design, and response analysis on both simulation and experiment acceptable results proves that real-time RIP in this paper is standard and has exactly dynamic equations and system parameters. The authors successfully built a control method for the rotary inverted pendulum, maintained the system's pendulum around the equilibrium region at the upright position by LQR approach and tracked the desired trajectory by LQT controller. We propose some opportunities for future research: Based on the present study, the researchers focus on applying the proposed controller to the RIP system. In addition, we can focus more on other research subjects, such as the Parallel-type Rotary Double Inverted Pendulum, the Series-type Rotary Double Inverted Pendulum and the triple RIP are more challenging than single RIP.

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