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Dynamic Analysis of Planar Mechanism in Numerical Methods

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ABSTRACT

In kinematic synthesis and analysis of planar mechanisms the designer must define a geometry and set of motions for a design task. Then it is logical to determine the forces or torques in order to create such motion in the system, which means finding out a convenient approach to solving for the forces and torques that result from our kinematic system in such a way as to provide the designed accelerations. This task is called dynamic force analysis (or kinetic analysis). Many numerical methods to determine the position, velocity, and acceleration of a planar mechanism are introduced and applied successfully, for example, using the transformation matrix. To solve fully kinetic problems for a planar mechanism, it is necessary to establish a simple and easy procedure that determines the forces or torques maintaining the motion in the system with the help of a computer. This paper will focus on how to determine the forces or torques acting on the planar mechanism by numerical method when acceleration and dynamic properties of the linkage are known. Some of the chosen examples shown here aim to demonstrate in detail the numerical solution for dynamic force analysis. The paper's purpose is not to demonstrate novelty, but to propose a new approach that can be applied to machinery design projects for students. University students can consult the content of this paper to carry out kinematic and kinetic analysis of planar mechanisms using this numerical method, as opposed to the traditional vector drawing method that is widely used.

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1. Introduction

In planar mechanisms, kinetic analysis can be conducted using several methods, such as force or energy analysis. These methods have great practical value, for instance, in selecting the appropriate electric motor and calculating bearings in kinematic pairs [1]. Force analysis is to determine forces and torques act on the links in a mechanism that keep on moving with the designed velocity and acceleration. This approach has the advantage of providing complete information about all interior forces at pin joints as well as about the external forces and torques on the system. Energy methods of solution are established on the law of conservation of energy that do not compute the internal joint forces in a system [2].

There are many papers concern to this topic which research deeply and vastly dynamics of planar mechanism by matrix methods, finite methods and software applications under different aspects [3], [4], [5], [6], [7]. However, it is very difficult to read, understand and apply at a basic level for university students to solve kinematic and kinetic problems of planar mechanism for project of machine design. Indeed, through investigation of how to work machinery design projects in Vietnam universities, students have still based vector equations to compute. They use graphic software to draw position diagrams, velocity diagrams, acceleration diagrams, and force diagrams according to a given number of positions of the drive link in the mechanism. Then, the magnitude of the velocity and acceleration of the links, the force acting at the joints as well as the balance moment will be measured through the length of the corresponding vectors [8]. Obviously, this drawing method has the advantage of being easy to understand and intuitive, but comes with the disadvantages of being cumbersome, inaccurate, and difficult to automate.

This article will build on the problem of calculating the kinematics of planar mechanisms using the transformation matrix method published by the author [9], [10], by showing how to create calculation matrices from the dynamic equations applicable to links in planar motion mechanisms, and formulate the numerical method to solve fully dynamic problems for planar mechanisms. This article does not aim to introduce new theories and new calculation methods, but to suggest another approach to assist students in universities to solve dynamic problems simply and easily automatically, instead of the traditional vector diagram method. The essential content is to determine the forces or torques acting on the planar mechanism when the acceleration and dynamic properties of the linkage are known. A chosen example shown out here directs to demonstrate detailly the numerical solution for dynamic force analysis.

2. Theoretical Basic

A mechanism (or linkage) is considered as a collection of the links that are interconnected by kinematic joints forming a single or multiple degree-of-freedom chain. One link is designated the frame (the ground link) because it served as the frame of reference for the motion of all other links [8]. Links are the individual parts which are considered rigid bodies. Theoretically, a true rigid body does not change shape during motion. A joint is a movable connection and allows relative motion between the links. The two primary joints are revolute and sliding joint. Linkage can be either open or closed chains (Fig. 1a, 1b) [8].

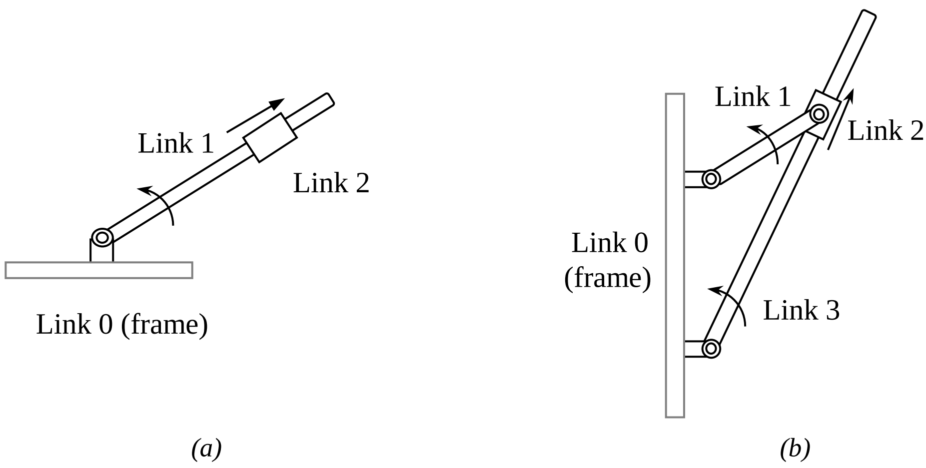

Figure 1. *Mechanism: (a) The opened; (b) The closed*

Dynamic equations describe motion of any link in a mechanism can be written as summation of all forces and moments. Here, bold characters express vector quantities.

$$\sum_{k=1}^{n} \boldsymbol{F}_i = m\boldsymbol{a}\ ;\ \sum_{k=1}^{n} M_G(\boldsymbol{F}_i) = I_G\alpha \quad [11] \tag{1.a}$$

For a planar mechanism, it is convenient to separately sum force components in *X* and *Y* directions, with the coordinate system chosen for convenience. The moments in two dimensional systems are all in the *Z* direction and take about the point *G*, that is center of gravity of links.

$$\sum_{k=1}^{n} F_{ix} = ma_x\ ;\ \sum_{k=1}^{n} F_{iy} = ma_y\ ;\ \sum_{k=1}^{n} M_G(\boldsymbol{F}_i) = I_G\alpha \quad [11] \tag{1.b}$$

where, m is the mass of link; I_G is the mass moment of inertia about the center of gravity of links; a_x and a_y are the acceleration of links at the center of gravity along the *X* and *Y* axes; α is angle acceleration of links.

One more nonrotating local coordinate system (called *Gxy*) is attached each moving member and located at its *G*, which has *Gx* and *Gy* parallel to *Ox* and *Oy*. This coordinate system is used to determine sum moment of forces applied to members about their centers of gravity (Fig. 2).

$$\sum M_G = \sum_{k=1}^{n} \boldsymbol{R}_i \times \boldsymbol{F}_i = \sum_{k=1}^{n} (R_{ix}F_{iy} - R_{iy}F_{ix}) \quad [12] \tag{2}$$

These three equations must be written for each moving body in the system which will lead to a set of linear simultaneous equations for any system. The set of simultaneous equations should be written in the matrix in order to solve conveniently by software. In order to establish set of simultaneous equations easily, we should label for all member in a linkage consistently. Link 0 is named for the frame (grounded link). The remaining moving links are labeled 1, 2, 3... joint to frame in the order.

As usual, these equations do not account for the gravitational force (weight) on a link because the kinematic accelerations of any link in the system are usually large compared to gravity, the weight forces can be ignored in the dynamic analysis.

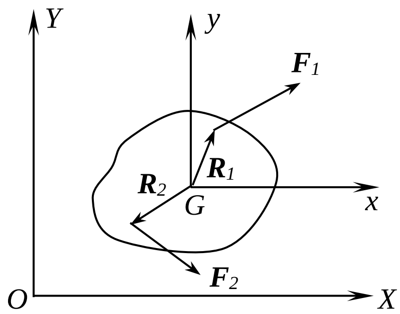

Figure 2. *Free-body diagram of one link*

3. Dynamic force analysis of some typical planar mechanisms

3.1. The single link in pure rotation

To begin the procedure of dynamic force analysis, it should consider a simple mechanism, the single link in pure rotation. Fig 3a. shows the kinematic diagram of a rotating link. Here, the input kinematic parameters must first be fully defined that mean linear acceleration of the moving member center ($\boldsymbol{a}_{G1}$) and angle acceleration of rotation link (α_1) must be found for all position of interest (the readers can refer [9], [10]). The mass (m_1) and the mass moment of inertia respect to the center (I_{G1}) of the moving link must also be known.

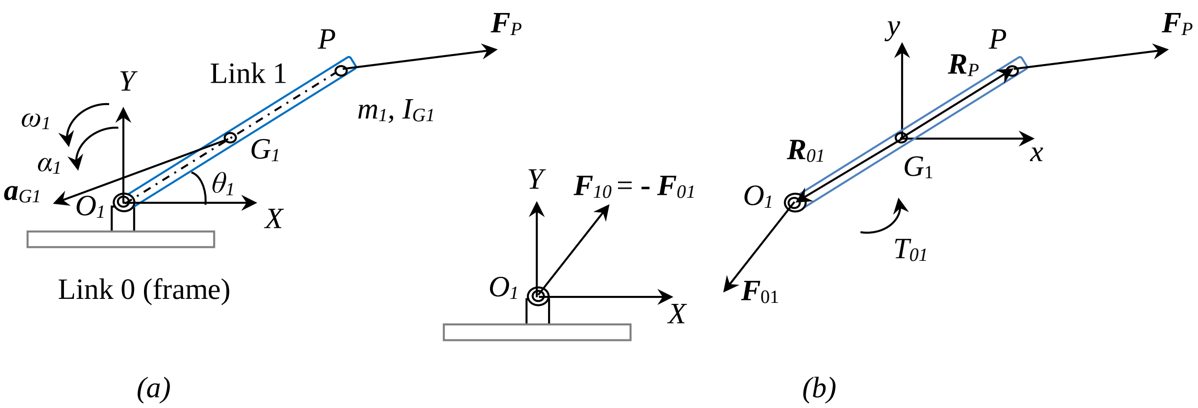

Figure 3. *Analytical diagrams of a single link: (a) Kinematic diagram; (b) Force diagrams*

Fig 3b. displays a free-body diagram of the moving link 1. The pin joint at pin O_1 on link 1 has a force $\boldsymbol{F}_{01}$ due to the mating link 0 acts on link 1, and the x and y components of which denote F_{01x} and F_{01y}. In opposite, the force from link 1 exsert on link 0 is named $\boldsymbol{F}_{10}$. The load applied on the moving link at point P are $\boldsymbol{F}_P$ with components of F_{Px} and F_{Py}. Vectors define positions of application points of forces act on link 1 denote $\boldsymbol{R}_{01}$ and $\boldsymbol{R}_P$ measured from the point G_1 of the member. We will need to resolve them into the x and y components. Finally, there will have to be a source torque from the ground apply on the moving link to drive it at the kinematically defined accelerations which is labeled T_{01}.

We have three unknowns F_{01x}, F_{01y}, T_{01} and three equations as below:

$$\begin{cases} F_{Px} + F_{01x} = m_1 a_{G1x} \\ F_{Py} + F_{01y} = m_1 a_{G1y} \\ T_{01} + R_{01x}F_{01y} - R_{01y}F_{01x} + R_{Px}F_{Py} - R_{Py}F_{Px} = I_{G1}\alpha_1 \end{cases} \quad (3)$$

The above equation system should be written in a matrix formation:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{01y} & R_{01x} & 1 \end{bmatrix} . \begin{bmatrix} F_{01x} \\ F_{01y} \\ T_{01} \end{bmatrix} = \begin{bmatrix} m_1 a_{G1x} - F_{Px} \\ m_1 a_{G1y} - F_{Py} \\ I_{G1}\alpha_1 - (R_{Px}F_{Py} - R_{Py}F_{Px}) \end{bmatrix} \quad (4)$$

Solving this system at every position of the moving link we can receive values of reaction forces at pin O_1 as well as the drive torque.

3.2. The four-bar linkage

Fig 4a. shows a four-bar linkage, in which, *OC* is the ground link, labeled link 0, the drive link is a crank *OA* rotating about a pin *O* which is named link 1, the connecting rod *AB* is labeled link 2, and the pendulum rod is link 3. All links are interconnected by revolute joints. All link lengths, link positions, link masses, locations of links' gravity centers, mass moments of inertia respect to the center, linear accelerations of links' gravity centers, and link angular accelerations and velocities have been previously determined from a kinematic analysis. Note that an external force $\boldsymbol{F}_{P2}$ is shown acting on link 2 and T_3 is an external torque applying on link 3, which are also known directions and magnitudes. Because four-bar linkage has three moving links, we should expect to have nine equations in nine unknowns for this problem.

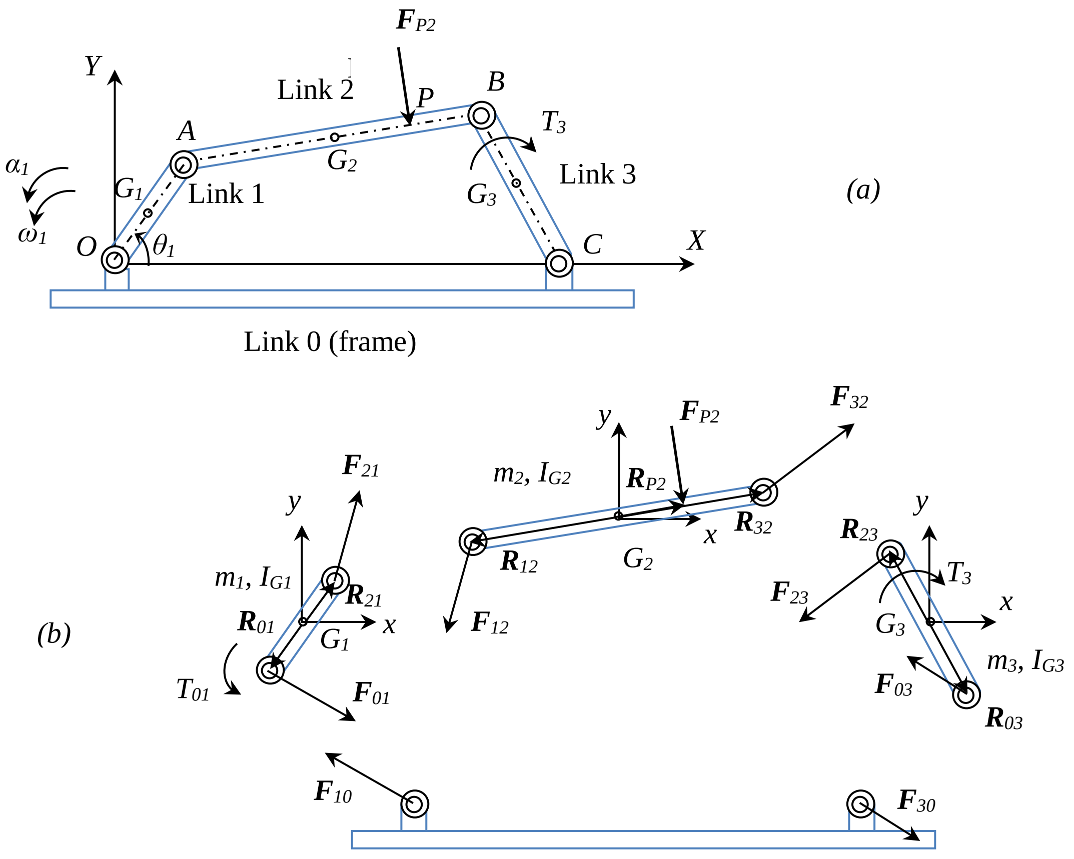

Figure 4. *Analytical diagrams of four-bar linkage: (a) Kinematic diagram; (b) Force diagrams*

Figure 4b. separates mechanism into discrete links, in which, the reactions at the joints satisfy Newton's 3rd law: $\boldsymbol{F}_{ij} = -\boldsymbol{F}_{ji}$. Vectors defining positions of application points of forces act on links measured from their centers are also labeled by $\boldsymbol{R}$ with indices respectively. Similar to equations (3), every moving link has three algebraic equations written as follow.

For link 1:

$$\begin{cases} F_{01x} + F_{21x} = m_1 a_{G1x} \\ F_{01y} + F_{21y} = m_1 a_{G1y} \\ T_{01} + R_{01x}F_{01y} - R_{01y}F_{01x} + R_{21x}F_{21y} - R_{21y}F_{21x} = I_{G1}\alpha_1 \end{cases} \tag{5}$$

For link 2:

$$\begin{cases} F_{12x} + F_{32x} + F_{P2x} = m_2 a_{G2x} \\ F_{12y} + F_{32y} + F_{P2y} = m_2 a_{G2y} \\ R_{12x}F_{12y} - R_{12y}F_{12x} + R_{32x}F_{32y} - R_{32y}F_{32x} + R_{P2x}F_{P2y} - R_{P2y}F_{P2x} = I_{G2}\alpha_2 \end{cases} \tag{6}$$

For link 3:

$$\begin{cases} F_{23x} + F_{03x} = m_3 a_{G3x} \\ F_{23y} + F_{03y} = m_3 a_{G3y} \\ T_3 + R_{23x}F_{23y} - R_{23y}F_{23x} + R_{03x}F_{03y} - R_{03y}F_{03x} = I_{G3}\alpha_3 \end{cases} \tag{7}$$

Note again that T_{01} considered as the source torque, only appears in the equation for link 1 as that is the drive crank to which the motor is attached. From Newton's 3rd law, we can substitute the x and y components of the reaction forces $\boldsymbol{F}_{12}$ for $-\boldsymbol{F}_{21}$, $\boldsymbol{F}_{23}$ for $-\boldsymbol{F}_{32}$. Finally, we can obtain nine equations with nine unknowns of F_{01x}, F_{01y}, F_{21x}, F_{21y}, F_{32x}, F_{32y}, F_{03x}, F_{03y}, T_{01} in a matrix form as below:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{01y} & R_{01x} & -R_{21y} & R_{21x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{12y} & -R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{23y} & -R_{23x} & -R_{03y} & R_{03x} & 0 \end{bmatrix} . \begin{bmatrix} F_{01x} \\ F_{01y} \\ F_{21x} \\ F_{21y} \\ F_{32x} \\ F_{32y} \\ F_{03x} \\ F_{03y} \\ T_{01} \end{bmatrix} = \begin{bmatrix} m_1 a_{G1x} \\ m_1 a_{G1y} \\ I_{G1}\alpha_1 \\ m_2 a_{G2x} - F_{P2x} \\ m_2 a_{G2y} - F_{P2y} \\ I_{G2}\alpha_2 - R_{P2x}F_{P2y} + R_{P2y}F_{P2x} \\ m_3 a_{G3x} \\ m_3 a_{G3y} \\ I_{G3}\alpha_3 - T_3 \end{bmatrix} \tag{8}$$

Solving this system at every position of the moving link we can obtain reactions at joints as well as the drive torque.

3.3. The slider-crank linkage

The slider-crank linkage can be considered as a transformer of the four-bar linkage that the pendulum rod is replaced by a slider block moving linear translation, so that the approach taken for the pin-joint four-bar mechanism is equally valid for a four-bar slider-crank linkage. The main difference needs attention that the slider block (link 3) will have no angular acceleration and the x and y components of reaction at slide-joint depend on each other according to Coulomb friction relation [12]:

$$F_{03x} = \pm \mu F_{03y} \tag{9}$$

where F_{03x} is friction force, F_{03y} is pressure force and μ is a known coefficient of friction. The plus and minus signs on the coefficient of friction are to recognize the fact that the friction force always opposes motion.

Fig 5a. shows a slider-crank linkage with an external force $\boldsymbol{F}_P$ acting on slider block. Fig 5b. presents free body diagrams (force diagram) of each link. Dynamic equation written for crank is same as equations (5) while dynamic equation for link 2 and link 3 will have a change as following.

For link 2, remove F_{P2x} and F_{P2y} from equations (6), we obtain:

$$\begin{cases} F_{12x} + F_{32x} = m_2 a_{G2x} \\ F_{12y} + F_{32y} = m_2 a_{G2y} \\ R_{12x}F_{12y} - R_{12y}F_{12x} + R_{32x}F_{32y} - R_{32y}F_{32x} = I_{G2}\alpha_2 \end{cases} \tag{10}$$

For link 3 moving translation on a path coincide with x axis, it leads $a_{G2y} = 0$; $\alpha_2 = 0$, so dynamic equations can be written in a form:

$$\begin{cases} F_{23x} + F_{03x} + F_{Px} = m_3 a_{G3x} \\ F_{23y} + F_{03y} + F_{Py} = 0 \end{cases} \tag{11}$$

Again, using action-reaction laws at the joints *B* and *C*, we have:

$$F_{21x} = -F_{12x};\ F_{21y} = -F_{12y};\ F_{32x} = -F_{23x};\ F_{32y} = -F_{23y}; \tag{12}$$

Combine equations (5), (9), (10), (11), and (12) we get a system including eight equations with eight unknowns of F_{01x}, F_{01y}, F_{21x}, F_{21y}, F_{32x}, F_{32y}, F_{03y}, T_{01} in a matrix form (13). Solving this system at every position of the crank we can find reactions at joints as well as the drive torque.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{01y} & R_{01x} & -R_{21y} & R_{21x} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{12y} & -R_{12x} & -R_{32y} & R_{32x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \pm\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} F_{01x} \\ F_{01y} \\ F_{21x} \\ F_{21y} \\ F_{32x} \\ F_{32y} \\ F_{03y} \\ T_{01} \end{bmatrix} = \begin{bmatrix} m_1 a_{G1x} \\ m_1 a_{G1y} \\ I_{G1}\alpha_1 \\ m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} - F_{Px} \\ -F_{Py} \end{bmatrix} \tag{13}$$

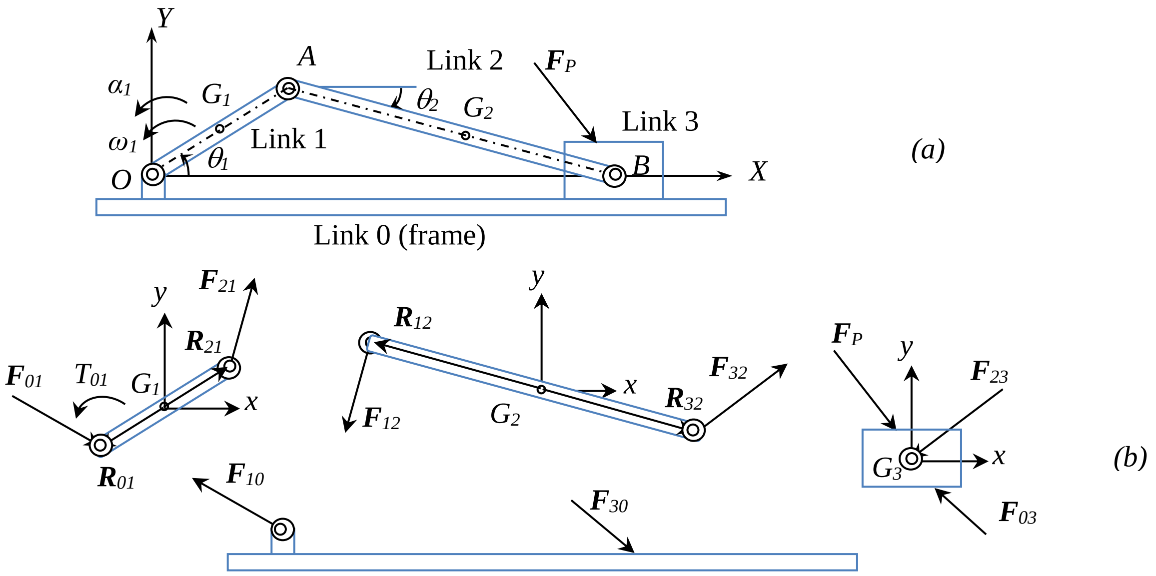

Figure 5. *Analytical diagrams of slider-crank linkage: (a) Kinematic diagram; (b) Force diagrams*

4. Illustration example

One example chosen here is to compute dynamic force for a slider-crank linkage including homogenous links with input data:

- Length and mass of crank (link 1): L_1 = 90 mm; m_1 = 0.9 kg
- Length and mass of connected rod (link 2): L_2 = 180 mm; m_2 = 1.8 kg
- Mass of slider block (link 3): m_3 = 0.4 kg
- Horizontal external force F_P = 500 N acting on slider block directed oppose to slider motion
- Friction coefficient between slider and cylinder: μ = 0.1
- The crank rotates at a constant angular velocity ω_1 = 3.14 rad/s (angular acceleration α_1 = 0)

A code file is written by Matlab software to calculate reactions at joints and source torque. An algorithm is implemented previously by calculating kinematic parameters, such as angular accelerations of links, acceleration of links' gravity centers at every of crank locations considered as input data for calculating mechanism kinetics. Slider-crank linkage kinematic equations were written by own author and be referred in [9], [10]. The valuable table of angular accelerations of links, acceleration of links' gravity centers at each crank angle with rotation angular step of 5 degrees is shown in table 1.

Table 1. *Values of angular accelerations of links, acceleration of links' gravity centers at every of crank location*

θ_1 (Degree)	θ_2 (Degree)	a_{G1x} (m/s^2)	a_{G1y} (m/s^2)	a_{G2x} (m/s^2)	a_{G2y} (m/s^2)	α_2 (rad/s^2)	a_{G3x} (m/s^2)
0	0	-0.4441	0	-1.1103	0	0	-1.3324
5	-2.4969	-0.4424	-0.0387	-1.1034	-0.0388	0.3233	-1.3219
10	-4.9749	-0.4374	-0.0771	-1.0827	-0.0776	0.6485	-1.2905
15	-7.4153	-0.4290	-0.1150	-1.0487	-0.1164	0.9768	-1.2394
…	…	…	…	…	…	…	…
360	0	-0.4441	0	-1.1103	0	0	-1.3324

Subsequently, a program used a for loop is written from the linear equation system (13) to determine reactions at joints and balance torque at every crank angle value of θ_1, where the values of quantities of a_{G1x}, a_{G1y}, a_{G2x}, a_{G2y}, a_{G3x}, α_2 vary according to crank locations that are taken from table 1. The other parameters concern to mass and geometry of mechanism are determined from input data and are shown in table 2, in which the mass moment of inertia about the center of gravity of each link are determined by formulas:

$$I_{Gi} = \frac{m_i L_i^2}{12} \tag{14}$$

Table 2. *Values of property parameters of Slider-crank linkage*

Links	m (kg)	L (m)	I_{Gi} (10^{-8} kgm^2)
1	0.9	0.09	60,750
2	1.8	0.18	486,000
3	0.4		

Vector components defining positions of joint forces act on links measured from their centers are computed by trigonometric functions of θ_1 and θ_2 as equations (15).

$$\begin{gathered} R_{01x} = -\frac{L_1}{2}\cos\theta_1\,;\; R_{01y} = -\frac{L_1}{2}\sin\theta_1\,;\; R_{21x} = \frac{L_1}{2}\cos\theta_1\,;\; R_{21y} = \frac{L_1}{2}\sin\theta_1\,; \\ R_{12x} = -\frac{L_2}{2}\cos\theta_2\,;\; R_{12y} = -\frac{L_2}{2}\sin\theta_2\,;\; R_{32x} = \frac{L_2}{2}\cos\theta_2\,;\; R_{32y} = -\frac{L_2}{2}\sin\theta_2 \end{gathered} \tag{15}$$

The output graphs of joint reactions and drive torque are displayed in Fig 6. Generally, all forces at joints and source torque in slider-crank linkage have a change of magnitude and direction at each position of drive link. These joint forces reach the maximum values at two positions of crank (at θ_1 = 90^o, 270^o) and down to the minimum values at three positions of crank (at θ_1 = 0^o, 180^o, 360^o) while drive torque acting on crank increases to the maximum value at θ_1 = 70^o, 290^o. The x components of the reaction forces at joints have insignificant varied magnitudes but their directions change at θ_1 = 0^o, 180^o, 360^o. The y components of the reaction forces remain directions and vary magnitudes dramatically. The balance torque supply on drive link also remains directions and changes magnitude continuously. The readers can also refer force analysis for Slider-crank linkage in documents [13], [14].

5. Conclusion

The article presents a numerical method to calculate forces and torques in planar mechanisms. This approach was built from dynamic equations describe translation and rotation motions of all moving links, which assemble a linkage. Reciprocal forces between two links at joints were used to integrate unknowns to reduce to a minimum. Assemble discrete equations into a system of linear equations to be able to write a solution program using specialized software such as Matlab, Maple. Finally, we proceed

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to input data and output the results in the form of tables or graphs. Some systems of linear equations written in matrix form for some typical mechanisms are presented detail in the article. One demo example for slider-crank linkage is also to demonstrate clearly. The paper's purpose is not to demonstrate novelty, but to propose a new approach that can be applied in machinery design projects for students. The author hopes that university students can consult the content of this paper to carry out kinematic and kinetic analysis of planar mechanisms by this numerical method, as opposed to the traditional vector drawing method that is widely used.

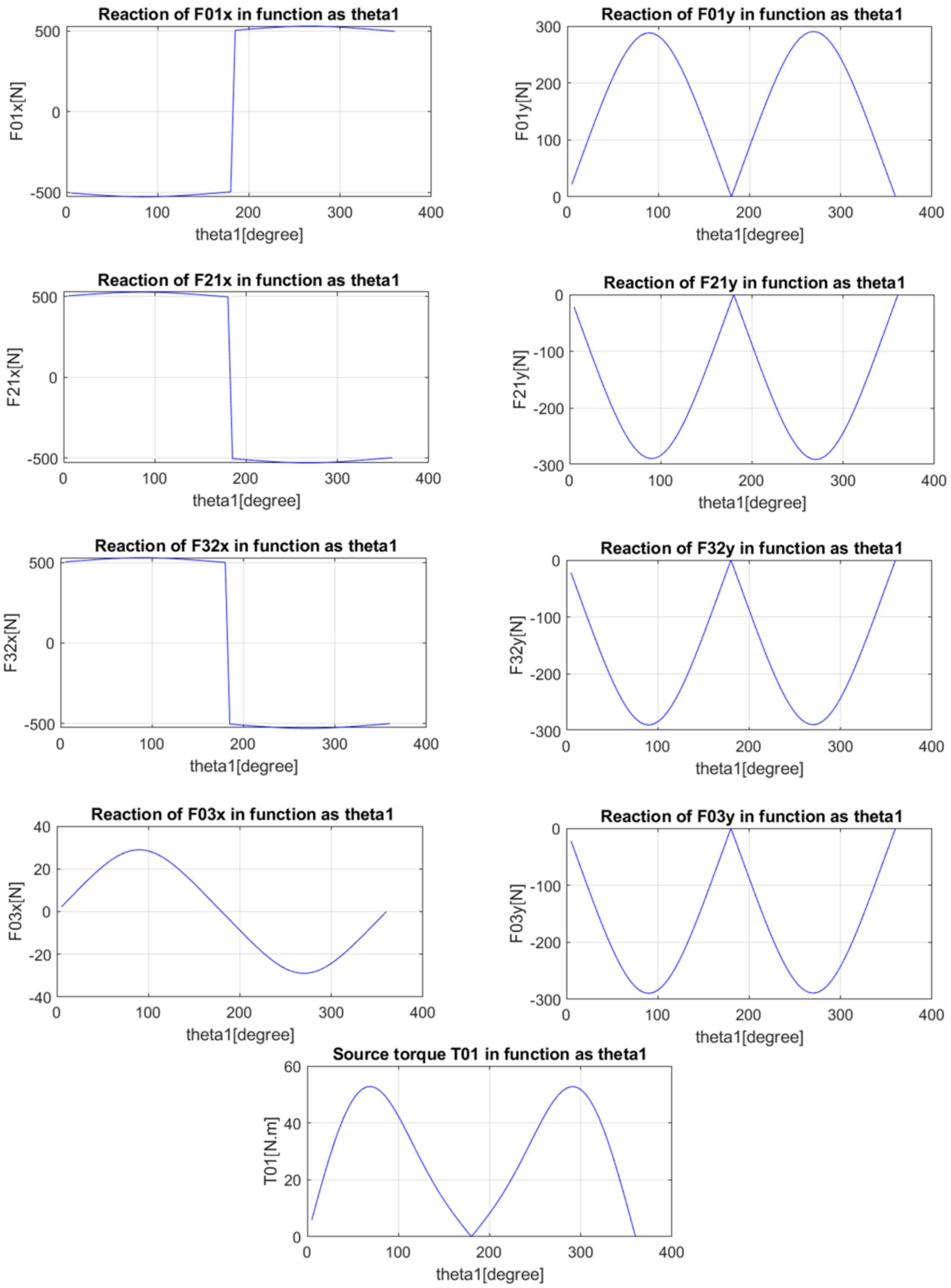

Figure 6. *The graphs of joint reactions and drive torque for slider-crank linkage*

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Conflict of Interest

The authors declare no conflict of interest.

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