

Kinematic Modeling and Stable Control Law Designing for Four Mecanum Wheeled Mobile Robot Platform Based on Lyapunov Stability Criterion

Tri Dung Nguyen*, Duy Phuong Dinh, Trung Hao Tran

Industrial Maintenance Training Center, Ho Chi Minh City University of Technology (VNU-HCM), Vietnam

* Corresponding author. Email: tridung.nguyen92@gmail.com

ARTICLE INFO

Received: 28/07/2023
Revised: 18/10/2023
Accepted: 24/10/2023
Published: 28/10/2023

KEYWORDS

Mecanum Wheels;
Omni – Directional Mobile Platform;
Lyapunov Stability;
Line Tracking Robots;
AGV.

ABSTRACT

Transportation in warehouses and production workshops is a matter of urgency today. Most warehouses arrange routes for circulation along the shelves, transportation vehicles will move on this road to perform the task of exporting or importing goods. Routes will be arranged to move in one direction because vehicles do not have enough space to turn around in cramped warehouses. This causes many difficulties in planning the trajectory for transportation vehicles, especially self-propelled vehicles. In order to have an appropriate transportation plan, it is necessary to solve many problems, including: reasonable transport equipment, sufficient number of devices, optimal route layout, algorithm of operation center for Positioning and Navigation of transportation equipment,... This study proposes a method for transportation using an omnidirectional automated guided vehicle (AGV). The AGV's omnidirectional mobility is supported by the mecanum wheels. This study consists of two parts, the first part focuses on kinematic modeling for mecanum wheels and extends to robot's platform using four mecanum wheels. Part two proposes a diagram to calculate the errors of the robot compared to a reference tracking line, design a control law based on the Lyapunov stability criterion. The stability of the control law is verified and confirmed by simulation on MATLAB environment.

Doi: <https://doi.org/10.54644/jte.79.2023.1437>

Copyright © JTE. This is an open access article distributed under the terms and conditions of the [Creative Commons Attribution-NonCommercial 4.0 International License](https://creativecommons.org/licenses/by-nc/4.0/) which permits unrestricted use, distribution, and reproduction in any medium for non-commercial purpose, provided the original work is properly cited.

1. Introduction

Mecanum wheel, sometimes called the Swedish wheel or Ilon wheel, was invented by Bengt Erland Ilon in 1973. In this design, there are many of free rollers attached to the hub. These rollers typically each have an axis of rotation at 45° to the wheel plane and at 45° to the axle line [1]. Each Mecanum wheel is driven by a separate motor for an omnidirectional movement on a plane [2]. The advantage of AGV (Automated Guided Vehicle) using Mecanum wheels is that there is no steering mechanism. Figure 1 [3] present the picture and the drawing model of a Mecanum wheel.

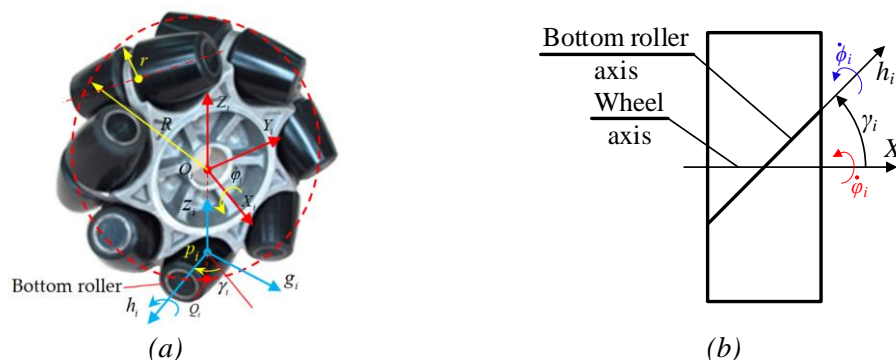


Figure 1. Dimensions for (a): Mecanum wheel and (b): Wheel basic parameter

A robot moving in a plane will have a maximum of 3 degrees of freedom (DoFs). So that, the AGV must have at least three wheels [4]-[10]. In the field of industrial, four Mecanum wheels AGV is widely used. There are many kinds of 4-wheeled Mecanum configuration for a mobile platform. However, in order to implement an omnidirectional AGV, it is necessary to select a properly configuration. To ensure the omni-directional movement ability, the axes of bottom roller of any three wheels have to intersect at two points [11]. There are two configurations that are selected to archive omnidirectional motion, they are illustrated in Figure 2.

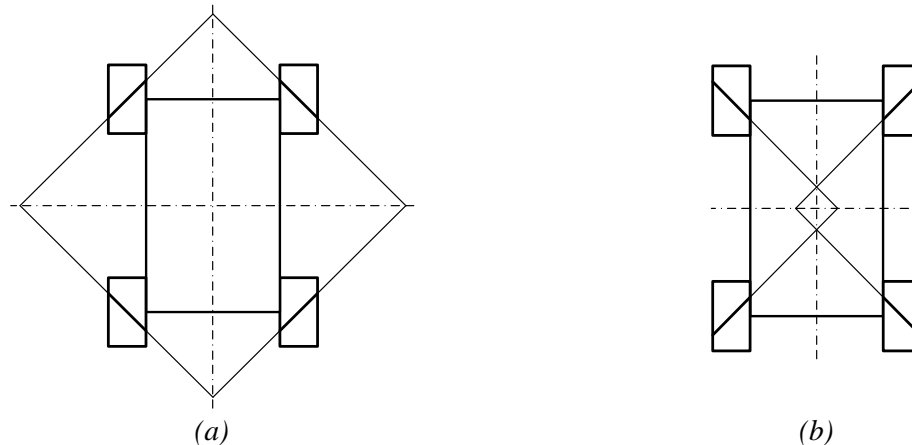


Figure 2. Two optimal configurations (a): Rectangular configuration and (b): Cross configuration

In Figure 2, the axes of bottom roller of any three wheels intersect with other at two distinguishing points. But especially in right hand side of Figure 2, if the center of four wheels forms a square, the axes of bottom roller of any three wheels intersect at only one point. In this case, the Jacobi matrix [3] will be singular and the platform is no longer an omnidirectional system. Jingyang et al. [13] designed a prototype of omnidirectional mobile robot, named Savvy, and a software system framework for this platform based on ROS environment, SLAM. Taheri et al. [14] introduced omnidirectional Mecanum wheels and its kinematic relation of a mobile platform using four Mecanum wheels. The study also obtained the experimental of 8 different motions without the change of its orientation. Doreftei et al. [15] presented a literature review of practical application for mobile robot using special wheels. According to the study of Doreftei, mobile equipment with omnidirectional property can be moved in widely environment, such as: factory workshops, warehouses, hospitals, etc. Li et al. [16] proposed a method for modeling the Mecanum wheel platform. In other to verify the modeling results, a virtual model is established in SolidWorks and a virtual prototype used for simulation is conducted in RecurDyn software. Daniil et Vasily [17] discussed about the design feature of a Mecanum wheel platform. This study also mentioned to the forward and revert kinematics problem for this kind of wheel. The results of mathematics model are proved by using Mobile Robotics Simulation Toolbox package, an additional package of the MATLAB Simulink.

This study will focus on the kinematic analysis for the Mecanum wheel and extend to the robot model using four Mecanum wheels. Then, using the modeling results for the four Mecanum wheeled platform combined with the Lyapunov stability criterion to design the control law for the robot to follow the given reference trajectory.

2. System Modeling

2.1. Modeling of four Mecanum wheeled platform

The Mecanum wheel is modeled as shown in Figure 1. Coordinate system $O_i X_i Y_i Z_i$ is attached to i -th wheel, where x_i is the axis of rotation of this wheel, O_i is coincided with the center of i -th wheel. Z_i is the vertical axis passing through O_i in the upward direction. \vec{X}_i , \vec{Y}_i and \vec{Z}_i follow the right-hand rule. In addition, \vec{h}_i is the axis of rotation of the roller. γ_i denotes the angle from \vec{X}_i to \vec{h}_i . R and r are wheel and roller radius, respectively. $\dot{\phi}_i$ and $\dot{\phi}_i$ are angular rotational speed about \vec{X}_i of i -th wheel and \vec{h}_i of its roller, respectively. Where $i = 1..4$ is the wheel's index. In addition, $P_i g_i h_i z_i$ is the frame that

coincides with the roller in contact with the ground, g_i , h_i and z_i also follow the right-hand rule. Let P_i be the center of the roller in contact with the ground, Q_i be the point of contact of the roller and the ground. According to the above assumption, we have $\overrightarrow{P_i Q_i}$ parallel to $\overrightarrow{Z_i}$, but in the opposite direction.

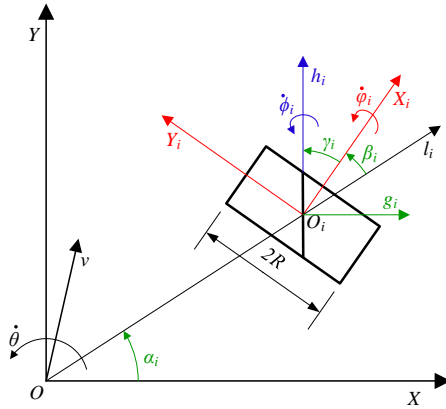


Figure 3. A Mecanum wheel's model

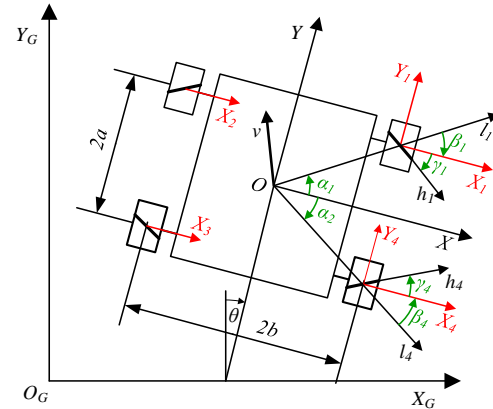


Figure 4. Four-wheeled mobile platform with Mecanum wheels modeling seen from stationary coordinate

Let $\overrightarrow{v_{oi}}$ be the absolute velocity of the point O_i , $\overrightarrow{v_{pi}}$ be the absolute velocity of the point P_i , $\overrightarrow{v_{OP}}$ be the relative velocity of O_i seen from P_i , we have:

$$\overrightarrow{v_{oi}} = \overrightarrow{v_{pi}} + \overrightarrow{v_{OP}} \quad (1)$$

Denoted $\overrightarrow{\omega_{oi}}$ and $\overrightarrow{\omega_{pi}}$ are the angular rotational velocity of the i -th wheel and its roller respectively in the fixed coordinate system, they can be calculated as follows:

$$\overrightarrow{\omega_{oi}} = \dot{\theta} \overrightarrow{Z_i} + \dot{\phi}_i \overrightarrow{X_i} \quad (2)$$

And

$$\overrightarrow{\omega_{pi}} = \overrightarrow{\omega_{oi}} + \dot{\phi}_i \overrightarrow{h_i} \quad (3)$$

The values of linear velocities $\overrightarrow{v_{pi}}$ is determined by multiplying the equations (2) with the level arm $\overrightarrow{Q_i P_i}$ as follow:

$$\overrightarrow{v_{pi}} = \overrightarrow{\omega_{pi}} \times \overrightarrow{Q_i P_i} = (\dot{\theta} \overrightarrow{Z_i} + \dot{\phi}_i \overrightarrow{X_i} + \dot{\phi}_i \overrightarrow{h_i}) \times r \overrightarrow{Z_i} = -r(\dot{\phi}_i \overrightarrow{Y_i} - \dot{\phi}_i \overrightarrow{g_i}) \quad (4)$$

Similarly, $\overrightarrow{v_{OP}}$ is obtained:

$$\overrightarrow{v_{OP}} = \overrightarrow{\omega_{oi}} \times \overrightarrow{P_i O_i} = (R - r)(\dot{\theta} \overrightarrow{Z_i} + \dot{\phi}_i \overrightarrow{X_i}) \times \overrightarrow{Z_i} = (r - R)\dot{\phi}_i \overrightarrow{Y_i} \quad (5)$$

Where R and r are wheel and roller radius respectively. Then, $\overrightarrow{v_{oi}}$ is calculated by adding the two above equations together:

$$\overrightarrow{v_{oi}} = -r(\dot{\phi}_i \overrightarrow{Y_i} - \dot{\phi}_i \overrightarrow{g_i}) + (r - R)\dot{\phi}_i \overrightarrow{Y_i} = r\dot{\phi}_i \overrightarrow{g_i} - R\dot{\phi}_i \overrightarrow{Y_i} \quad (6)$$

On the other hand, let \overrightarrow{v} be velocity of centre point O seen from the ground, we have the relationship between $\overrightarrow{v_{oi}}$, \overrightarrow{v} and $\dot{\theta}$ as following equation:

$$\overrightarrow{v_{oi}} = \overrightarrow{v} + \dot{\theta}(\overrightarrow{Z} \times \overrightarrow{l_i}) \quad (7)$$

So that:

$$r\dot{\phi}_i \overrightarrow{g_i} - R\dot{\phi}_i \overrightarrow{Y_i} = \overrightarrow{v} + \dot{\theta}(\overrightarrow{Z} \times \overrightarrow{l_i}) \quad (8)$$

Multiply the scalar by $\overrightarrow{h_i}$ on both sides of the above equation, we get:

$$r\dot{\phi}_i \overrightarrow{h_i} \cdot \overrightarrow{g_i} - R\dot{\phi}_i \overrightarrow{h_i} \cdot \overrightarrow{Y_i} = \overrightarrow{h_i} \cdot \overrightarrow{v} + \dot{\theta} \overrightarrow{h_i} \cdot (\overrightarrow{Z} \times \overrightarrow{l_i}) \quad (9)$$

In fact:

$$\overrightarrow{g_i} \cdot \overrightarrow{h_i} = 0; \text{ and } \overrightarrow{Y_i} \cdot \overrightarrow{h_i} = \sin \gamma_i \quad (10)$$

Therefore:

$$-R\dot{\phi}_i \sin \gamma_i = \overrightarrow{h_i} \cdot \overrightarrow{v} + \dot{\theta} \overrightarrow{h_i} \cdot (\overrightarrow{Z} \times \overrightarrow{l_i}) \quad (11)$$

In the configuration of the mobile robot shown in Figure 4, because \vec{X}_i is always set to parallel with \vec{X} , so:

$$\alpha_i + \beta_i = 0 \quad (12)$$

Apply equation (12), vector \vec{h}_i seen from stationary coordinate system is described as follows:

$$\vec{h}_i = [\cos(\alpha_i + \beta_i + \gamma_i); \sin(\alpha_i + \beta_i + \gamma_i); 0] = [\cos \gamma_i; \sin \gamma_i; 0] \quad (13)$$

And vector \vec{l}_i is also given:

$$\vec{l}_i = [l_i \cos \alpha_i; l_i \sin \alpha_i; 0] \quad (14)$$

The results of scalar product and cross product between \vec{h}_i , \vec{v} , \vec{Z} and \vec{l}_i are recognized as follows:

$$\vec{h}_i \cdot \vec{v} = v_x \cos \gamma_i + v_y \sin \gamma_i \quad (15)$$

$$\vec{Z} \times \vec{l}_i = [-l_i \sin \alpha_i; l_i \cos \alpha_i; 0] \quad (16)$$

Substitute equation (15) and (16) into formula (11), we get:

$$-R\dot{\phi}_i \sin \gamma_i = v_x \cos \gamma_i + v_y \sin \gamma_i - \dot{\theta} l_i \sin \alpha_i \cos \gamma_i + \dot{\theta} l_i \cos \alpha_i \sin \gamma_i \quad (17)$$

Because $R \sin \gamma_i$ is never equal to zero, divide both sides of the equation (17) by $R \sin \gamma_i$, the i -th wheel angular rotational speed $\dot{\phi}_i$ is determined through the robot's linear velocity and angular velocity as in the below equation:

$$\dot{\phi}_i = -\frac{1}{R_i} J_i \begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} \quad (18)$$

In which:

$$J_i = [\cot \gamma_i \quad 1 \quad -l_i \sin \alpha_i \cot \gamma_i + l_i \cos \alpha_i] \quad (19)$$

Is the i -th row of Jacobian matrix J .

Applying the equation (18) to the wheels $i = 1..4$, the values of the angular constants α_i and γ_i are described as equation (20) and (21), we get the fully kinematic equation for the four Mecanum wheeled robot as given in equation (22):

$$\gamma_2 = \gamma_4 = -\gamma_1 = -\gamma_3 = 45^\circ \quad (20)$$

$$\alpha_1 \in (0; 90^\circ); \alpha_2 \in (90^\circ; 180^\circ); \alpha_3 \in (-90^\circ; -180^\circ); \alpha_4 \in (0; -90^\circ) \quad (21)$$

Fully kinematic equation for the four Mecanum wheeled robot is described as below:

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix} = -\frac{1}{R} \begin{bmatrix} -1 & 1 & a+b \\ 1 & 1 & -a-b \\ -1 & 1 & -a-b \\ 1 & 1 & a+b \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} \quad (22)$$

Equation given above is the inverse kinematic equation of the four-wheeled robot, in which the angular rotational speed of each wheel is calculated according to the linear and angular velocity of the robot in a fixed coordinate system.

2.2. Line tracking's deviations modeling

In this section, we will propose a mathematical model for a mobile robot to follow a given reference line. Tracking line is assumed as a curve in $O_G X_G Y_G$ coordinate system. The setting reference point $R(x_R, y_R)$ is assumed to be moving along the reference orbit with linear velocity \vec{v}_r , the absolute value $|\vec{v}_r|$ is set to be constant. Mobile platform with parameters as mentioned in the previous section, the control point $C(x_C, y_C)$ will track to the reference point R with constraint that the \vec{Y} axis is always tangent to the reference curve.

In most cases, point C is chosen to coincide with the robot's centre of gravity. In this study, the control point C is chosen to coincide with the geometric centre O of the Mecanum platform. We have the kinematics equation of point C in a fixed coordinate system as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\tau}_C \end{bmatrix} = \begin{bmatrix} \cos \tau_C & 0 \\ \sin \tau_C & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_C \\ \omega_C \end{bmatrix} \quad (23)$$

Where v_C and ω_C are the linear and angular velocity of the control point C. On the other hand, we also have the kinematic equation of the reference point R in the fixed coordinate system as follows:

$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\tau}_R \end{bmatrix} = \begin{bmatrix} \cos \tau_R & 0 \\ \sin \tau_R & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_R \\ \omega_R \end{bmatrix} \quad (24)$$

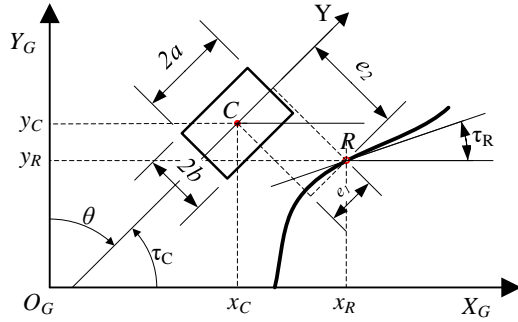


Figure 5. Diagram of deviations calculation for line tracking

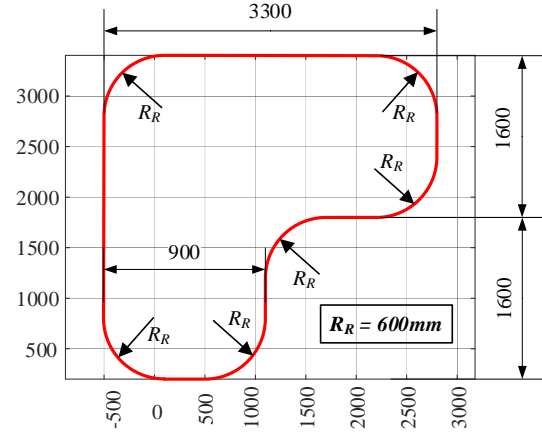


Figure 6. Reference line's parameters used in simulation

The position deviation between C and R can be considered as three components as shown in Figure 5, including:

- Longitudinal deviation e_1 :

$$e_1 = (x_R - x_C) \cos \tau_C + (y_R - y_C) \sin \tau_C \quad (25)$$

- Transverse deviation e_2 :

$$e_2 = (x_R - x_C) \sin \tau_C - (y_R - y_C) \cos \tau_C \quad (26)$$

- And angular deviation e_3 :

$$e_3 = \tau_R - \tau_C \quad (27)$$

These errors can be represented in matrix form as follows:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos \tau_C & \sin \tau_C & 0 \\ \sin \tau_C & -\cos \tau_C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R - x_C \\ y_R - y_C \\ \tau_R - \tau_C \end{bmatrix} \quad (28)$$

Taking the derivative of both sides of the above equation, we can get:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -1 & -e_2 \\ 0 & e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_C \\ \omega_C \end{bmatrix} + \begin{bmatrix} v_R \cos e_3 \\ -v_R \sin e_3 \\ \omega_R \end{bmatrix} \quad (29)$$

3. Control law Designing

The objective of this section is to design a stable control law to ensure that the robot moves along the reference line, satisfying the moving conditions in section 2.2. The control object here is a robot with four Mecanum wheels. Here we only consider the kinematic relationship without regard to the dynamical parameters.

As analysed above, linear velocity v_C , angular velocity ω_C , moving direction θ of platform and the angular rotational speed of each Mecanum wheels are presented in equation (22). From Figure 5 we can recognize that:

$$\tau_C = 90^\circ + \theta \quad (30)$$

And

$$\dot{\theta} = \dot{\tau}_C \quad (31)$$

Let choose Lyapunov function as follows:

$$L = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + k_1(1 - \cos e_3) \quad (32)$$

Noted that $L \geq 0 \forall (e_1, e_2, e_3)$ and $L = 0$ only when $(e_1, e_2, e_3) = (0,0,0)$.

Taking the derivative of the equation (32) versus time, we get:

$$\dot{L} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + k_1 \dot{e}_3 \sin e_3 \quad (33)$$

Combine equations (32), (28) and (29), we get:

$$\dot{L} = e_1(-v_C + v_R \cos e_3) + k_1 \sin e_3 \left(\omega_R - \omega_C - \frac{e_2 v_R}{k_1} \right) \quad (34)$$

If we choose v_C and ω_C as below equations:

$$\begin{cases} v_C = v_R \cos e_3 + k_2 e_1 \\ \omega_C = \omega_R - \frac{1}{k_1} (e_2 v_R - k_3 \sin e_3) \end{cases} \quad (35)$$

Then the equation (34) is simplified as following equation:

$$\dot{L} = -k_2 e_1^2 - k_3 \sin^2 e_3 \leq 0, \forall k_2, k_3 > 0 \quad (36)$$

According to Lyapunov stability criterion [12], robot platform will be stable with this control law.

4. Simulation Results

The simulation is done in MATLAB environment in two cases. First case is the simulation with all knows the tracking line parameters and second case is the simulation in the situation of some parameters are unknown.

4.1. Simulate results in case the tracking curve's parameters are known

The control parameters and kinematics parameters of the robot are shown in Table 1 and Table 2, the layout of tracking line used in this simulation is shown in Figure 6. Assume that robot's initial position has a deviation relative to reference point R as shown in Table 3.

Table 1. Control parameters

Parameters	Value (No unit)
k_1	1
k_2	10
k_3	2

Table 2. Robot's kinematics parameters

Parameters	Symbols	Value	Unit
Platform's half-width	b	120	mm
Platform's half-length	a	100	mm
Wheel radius	R	45	mm
Roller radius	r	10	mm
Reference curve radius	R_R	600	mm
Reference linear velocity	v_r	10	mm/s
Sampling time	t_s	0.1	s

Table 3. Initial position error relative to reference point R

Parameters	Symbols	Value	Unit
Longitudinal deviation	e_1	0	mm
Transverse deviation	e_2	10	mm
Angular deviation	e_3	$\pi/18$	rad

Figure 7 to Figure 9 show deviations of simulation result with sampling time $t_s = 0.1$ second. The vertical axis of each chart represents the value of the error (mm for e_1 and e_2 , and rad for e_3), the horizontal axis represents the simulation time in second. The time it takes for the robot to complete this orbit is about 572.6 seconds. From the charts it can be seen that, due to the initial error of e_2 and e_3 , the errors are somewhat fluctuating at the beginning, after about 15 seconds the steady-state value is approximately 0 mm for e_1 and e_2 or 0 rad for e_3 . Figure 10 to Figure 12 show the simulation results of deviations for period from 0 to 15 seconds only.

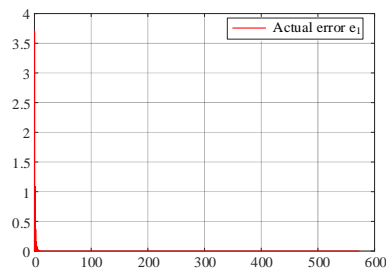


Figure 7. Simulation result of longitudinal deviation

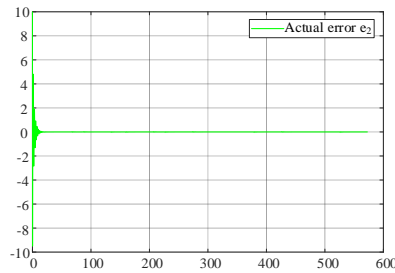


Figure 8. Simulation result of transverse deviation

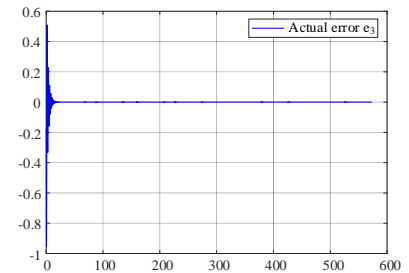


Figure 9. Simulation result of angular deviation

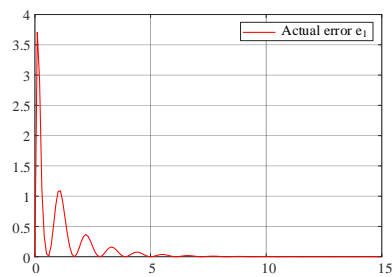


Figure 10. Simulation result of longitudinal deviation

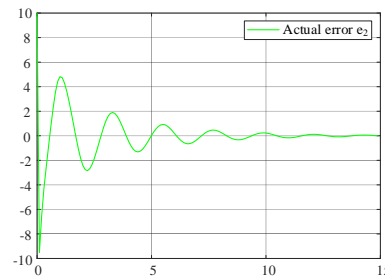


Figure 11. Simulation result of transverse deviation

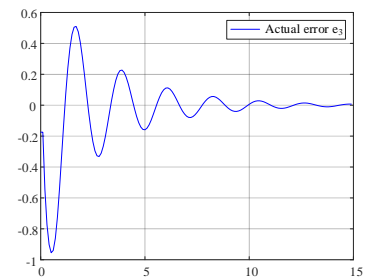


Figure 12. Simulation result of angular deviation

4.2. Simulate results in case the tracking curve's parameters are unknown

In fact, robots moving in warehouses and production workshops will follow some reference curve such as magnetic line, induction line or laser virtual line. When moving along these lines, parameters such as ω_R and e_1 cannot be determined. In addition, the error value e_3 may or may not be determined depending on type and number of sensors used. Here we will investigate in both cases of deviation value e_3 : can and cannot be determined. The kinematics and control parameters are unchanged. Figure 13 and Figure 14 show simulation results in case e_3 can be determined, Figure 15 show simulation result in case the value of e_3 is not available.

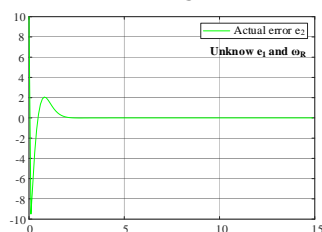


Figure 13. Simulation result of transverse deviation

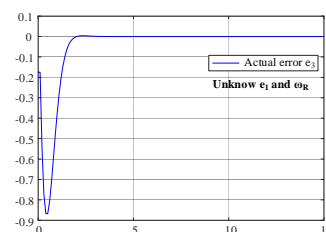


Figure 14. Simulation result of angular deviation

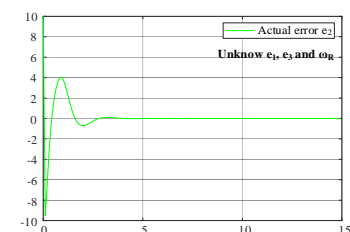


Figure 15. Simulation result of transverse deviation

The angular rotational velocity of each wheel in the case of unknown value of e_3 based on the relationship in equation (22) is shown as Figure 16 to Figure 19. The vertical axis is the value of the wheel angular velocity in rad/s, the horizontal axis is the time of simulation in second.

The most basic goal of the controller is to make the robot follow a given line, in which e_2 represents the transverse deviation between the robot 's center and the reference line, e_1 represents the ability to maintain robot 's velocity during the process of tracking, and e_3 represents the angle deviation. Thus, the error e_2 is decisive in whether the robot can stick to the line or not. From the simulation results, it shows that the line tracking deviations always converges to 0 even in cases where the error components (e_1, e_3) and even the line parameters (ω_R) cannot be determined, the setting time from Figure 15 is about 3s. Therefore, they shows that this control law is stable and able to be applied to actual robot models.

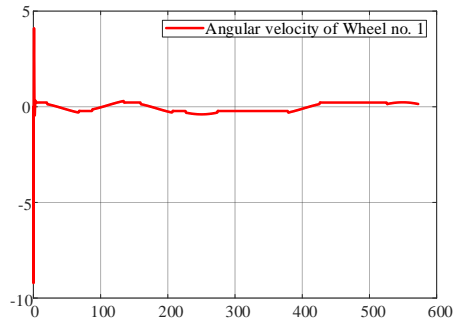


Figure 16. Simulation result for angular velocity of wheel No. 1

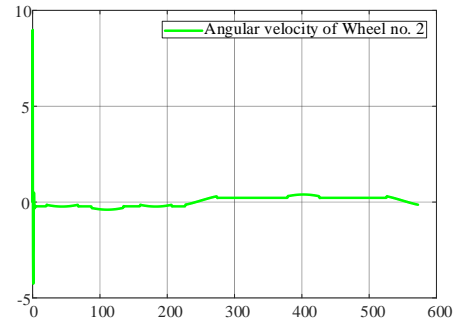


Figure 17. Simulation result for angular velocity of wheel No. 2

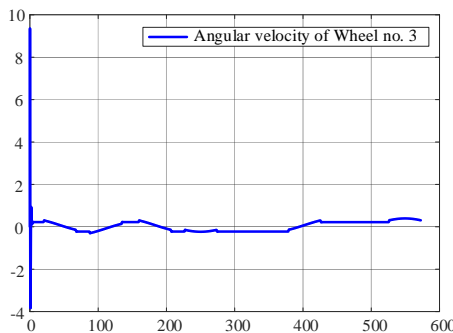


Figure 18. Simulation result for angular velocity of wheel No. 3

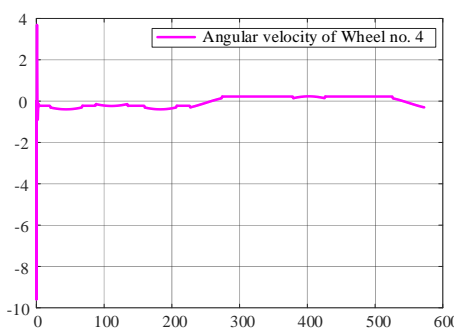


Figure 19. Simulation result for angular velocity of wheel No. 4

5. Conclusions

This study focuses on modeling the kinematics in general for only one Mecanum wheel, then extended to mobile robot using four Mecanum wheels. Based on this result, the research continues with a specific application of controlling a four Mecanum wheeled robot to follow a given reference line. This study does not refer to the type of line or the method of measuring and determining the error by real sensors. This research also provides a diagram to calculate the tracking deviation based on the position of the robot relative to the desired curve, and also proposes a control law to ensure that the robot moves along the line with a constant speed based on the Lyapunov stability criterion. The simulation results on MATLAB environment have verified the correctness of the mathematical model and the stability of the control law.

Acknowledgements

We acknowledge the support of time and facilities from Ho Chi Minh City University of Technology (HCMUT), VNU-HCM for this study.

REFERENCES

- [1] O. Diegel, A. Badve, J. Potgieter, and S. Tlate, "Improve mecanum wheel design for omni-directional robots", in *Proceedings 2002 Australasian Conference on Robotics Automation*, Auckland, New Zealand, Nov. 2002, pp. 117-121.
- [2] A. Gfrerrer, "Geometry and kinematics of the mecanum wheel", *Trans. Computer Aided Geometric Design*, vol. 25, no. 9, pp. 784-791, 2008.
- [3] Y. Li, S. Ge, S. Dai, L. Zhao, X. Yan, Z. Yuwei, and Y. Shi, "Kinematic Modeling of a Combined System of Multiple Mecanum-Wheeled Robots with Velocity Compensation", *Trans. Sensors*, vol. 20, no. 1, p. 75, 2019.
- [4] I. Doroftei, V. Spinu, and V. Grosu, "Omnidirectional mobile robot – design and implementation", in *Bioinspiration and Robotics Walking and Climbing Robots*, Rijeka, Croatia: IntechOpen, 2007, p. 544.
- [5] P. Alvito, C. Marques, P. Carriço, and J. Freire, "A Robotic Platform for the Social Robot Project." in *Proceedings of the 23rd IEEE International Symposium on Robot and Human Interactive Communication (ROMAN 2014) Workshop on Interactive Robots for Aging and/or Impaired People*, Edinburgh, UK, Aug. 25–29, 2014.
- [6] J. Qian, B. Zi, D. Wang, Y. Ma, and D. Zhang, "The design and development of an omni-directional mobile robot oriented to an intelligent manufacturing system", *Sensors*, vol. 17, no. 9, p. 2073, 2017.
- [7] F. A. Tei and I. Doroftei, "Practical applications for mobile robots based on mecanum wheels-a systematic survey." in *Proceedings of the 3rd International Conference on Innovations, Recent Trends and Challenges in Mechatronics, Mechanical Engineering and New High-Tech Products Development (MECAHITECH'11)*, Bucharest, Romania, Sep. 22–23, 2011, pp. 112–123.
- [8] P. Hryniewicz, A. Gwiazda, W. Banas, A. S'ekala, and K. Foit, "Modelling of a mecanum wheel taking into account the geometry of road rollers." in *Proceedings of the IOP Conference Series: Materials Science and Engineering*, Sibiu, Romania, Jun. 14–17, 2017, p. 012060.
- [9] C. He, D. Wu, K. Chen, F. Liu, and N. Fan, "Analysis of the Mecanum wheel arrangement of an omnidirectional vehicle." in *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.*, 2019, pp. 5329–5340.
- [10] Y.N. Zhang, S. Wang, J. Zhang, and J. Song, "Research on motion characteristic of omnidirectional device based on Mecanum wheel." in *Proceedings of the 2011 International Conference on Electric Information and Control Engineering*, Wuhan, China, Apr. 15–17, 2011, pp. 6094–6097.
- [11] Y. Li, S. Dai, L. Zhao, X. Yan, and Y. Shi, "Topological design methods for mecanum wheel", *Symmetry in Engineering Sciences II*, vol. 11, no. 10, pp. 1-27, 2019.
- [12] K. H. Khalil, "Lyapunov stability", *Control Systems, Robotics and Automation*, vol. 12, pp. 115-126, 2009.
- [13] J. Wu, C. Ly, L. Zhao, R. Li, and G. Wang, "Design and implementation of an omnidirectional mobile robot platform with unified I/O interfaces." in *Proceedings of 2017 IEEE International Conference on Mechatronics and Automation (ICMA)*, Takamatsu, Japan, Aug. 06–09, 2017.
- [14] H. Taheri, B. Qiao, and N. Ghaeminezhad, "Kinematic model of a four mecanum wheeled mobile robot," *International Journal of Computer Applications (0975 – 8887)*, vol. 113, no. 3, pp. 6-9, 2015.
- [15] F. Adăscăliței and I. Doroftei, "Practical Applications for Mobile Robots based on Mecanum Wheels - a Systematic Survey," in *Proceedings of International Conference on Innovations, Recent Trends and Challenges in Mechatronics, Mechanical Engineering and New High-Tech Products Development – MECAHITECH'11*, 2011, p. 21.
- [16] Y. Li, S. Dai, Y. Zheng, F. Tian, and X. Yan, "Modeling and Kinematics Simulation of a Mecanum Wheel Platform in RecurDyn," *Journal of Robotics*, vol. 2018, 2018, Art. no. 9373580.
- [17] S. A. Daniil and I. R. Vasily, "Development of the laboratory work: "Modeling of a mobile robot on mecanum wheels kinematics"", in *Proceedings of Web International Conferences IT-Technologies for Engineering Education: New Trends and Implementing*, 2020, Art. no. 04001.



Nguyen Tri Dung: Lecturer at the Industrial Maintenance Training Center, Ho Chi Minh City University of Technology (HCMUT, VNU-HCM). He graduated with a master's degree in Mechatronics in 2017, Department of Mechatronics, Faculty of Mechanical Engineering, Ho Chi Minh City University of Technology (HCMUT, VNU-HCM). Won a medal at the 2015 International Robot Competition, 30-31 October 2015, Seoul, Korea. His favorite research fields include: Automatic control, Humanoid robots, Automated Guided Vehicle, Industrial Maintenance and Machine Vibration. Email: tridung.nguyen92@gmail.com.



Dinh Duy Phuong: Industrial Maintenance Engineer. He graduated in major of Industrial Maintenance in 2023, Industrial Maintenance Training Center, Ho Chi Minh City University of Technology (HCMUT, VNU-HCM). Winner Team with a Gold medal at the 2022 Mechanical Engineering Robot Contest (MERC 2022), February – April 2022, Hochiminh City, Vietnam. His favorite research fields include: Automatic control, Humanoid robots, Automated Guided Vehicle, Industrial Maintenance and Machine Vibration. Email: phuong.dinh09052001@hcmut.edu.vn.



Tran Trung Hao: Industrial Maintenance Engineer. He graduated in major of Industrial Maintenance in 2023, Industrial Maintenance Training Center, Ho Chi Minh City University of Technology (HCMUT, VNU-HCM). Winner Team with a Gold medal at the 2022 Mechanical Engineering Robot Contest (MERC 2022), February – April 2022, Hochiminh City, Vietnam. His favorite research fields include: Automatic control, Humanoid robots, Automated Guided Vehicle, Industrial Maintenance and Machine Vibration. Email: hao.tran2032001@hcmut.edu.vn.