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# Propose Theoretical Foundations for Analyzing the Dynamic Control and Optimizing the Structure of Multifunctional Carbon Fiber-Reinforced Material Plate Integrated Piezoelectric

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**ABSTRACT**

In the fields of aviation, energy, construction, biomedicine, chemical technology, and some other fields, the use of functionally graded materials (FGMs) plates, multifunctional carbon nanofibers reinforced material plates, piezoelectric plates, and several others are increasingly popular. The behavior of material plates can be analyzed through partial differential equations (PDEs). However, the PDEs are for complex problems such as solid-liquid interactions, thermoelectric mechanical environments, functional material plates in multi-physics environments, and others that are very difficult or impossible to find a solution. In many numerical methods have been researched and developed, the finite element method (FEM) is a widely used and effective method to find approximate solutions of the PDEs. But the FEM has certain limitations in element techniques, discretizing the weak form for the plate structure problems with many degrees of freedom significantly, and affects the accuracy and efficiency of calculation. Proposing improvements to traditional FEM combines dynamic control analysis in the presence of piezoelectric crystals and optimization method algorithms, meeting the increasing requirements in analyzing the behavior of carbon nanofiber material plates used in many fields. On this basis, the article presents some theoretical foundations to solve the problem of dynamic control and optimizing the structure of multifunctional carbon nanofiber-reinforced material plates with integrated piezoelectric crystal.

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## 1. Introduction

Nowadays, in the problem of dynamics and optimization of plate structures, the analysis can be based on two groups of developed theoretical foundations: the analytical methods and the equivalent single-layer theories.

The analytical methods: Bailey et al. [1] & Shen et al. [2] used in the study of composite beams integrating piezoelectric layers, and Tzou & Tseng [3] studied using 3D elements to control piezoelectric plate and shell vibrations.

As for the equivalent single-layer theories, there are three popular theories: the Classical Lamination Theory (CLT), the First order Shear Deformation Theory (FSDT), and the High order Shear Deformation Theory (HSDT). In CLT, based on the hypothesis of Kirchhoff plate theory, interlayer shear deformation in the composite plate is ignored. In the FSDT theory, the transverse shear deformation is assumed to be constant in the plate thickness direction, and shear stress conditions at the top and bottom of the plate are ignored. Using this method, Liew et al. [4] used the element-free Galerkin method to analyze multilayer beam structures with piezoelectric patches. Milazzo and Orlando [5] studied the free vibrations of composite piezoelectric plates. The mesh-free method based on FSDT was combined and demonstrated by Liew et al. [4] to control the vibration pattern of piezoelectric composite plates with different boundary conditions. Phung-Van et al. [6] have developed the cell-based smoothed discrete shear gap (CS-DSG3) method to control the response of piezoelectric patch-integrated composite plates. Several FEM models have also been developed to analyze the response of piezoelectric beam and slab structures and are reported in [7]-[20]. In both CLT and FSDT, a shear correction factor is required to ensure the stability of the solution. To improve the shear stress accuracy and eliminate the use of a shear correction factor, a FEM-based HSDT has been proposed for layer analysis of piezoelectric patch integration composite problems [21], [22]. This model approximates the displacement field using a quadratic, third-order, or higher-order function. The layerwise model combined with the bidet [23] has also been studied and applied to the piezoelectric composite problem. In the bid, the displacement field is required C1-continuity because in the formula for calculating the stiffness matrix appears, the second derivative of the approximation field. This is the difficulty of applying HSDT to traditional FEM because most traditional EM only C0-continuity. Among the bids, the unconstrained third-order shear deformation theory (UTSDT) [24] demonstrates some effectiveness for multilayer plate structural analysis. The UTSDT satisfies the relaxed traction-free boundary condition at the top and bottom faces of the plate. The arrival of the UTSDT opened up a new application for the flow field. This method gives more accurate results than TSDT [25]. Static and free vibration analysis of composite plates using UTSDT combined with RPIM has been studied by Dinis et al. [26]. In this study, the displacement field is approximately composed of 7 components.

For the problem of optimal control of piezoelectric plates, Kumar et al. [27] and Rao et al. [28] used a genetic algorithm (GA) to find the optimal position for the piezoelectric patch on the plate and array. Chang-Qing et al. [29] used the theory of independent modal space control (IMSC) to control piezoelectric structures optimally. Using GA, Artificial neural networks (ANN) to find piezoelectric pad positions to control structural vibrations were also studied by Bruant et al. [30]. In this study, two variables for each piezoelectric pad are used for the optimization problem, including its position and orientation. A semi-analytic solution based on linear quadratic regulators (LQR) for optimal control of the piezoelectric plate is also reported in [31].

For multifunctional material and piezoelectric plate structures, a FEM model based on the differential principle and linear electrical theory has been developed by Liew et al. [32] to control the functionally graded material (FGM) plate actively. ) integrates sensors and actuators. Reddy and Cheng [33] have developed a semi-analytic solution for the intelligent 3D FGM plate model. Huang and Shen [34] used HSDT to analyze piezoelectric FGM plates in a thermal environment. Nonlinear analysis of piezoelectric FGM plates subjected to thermomechanical loads has also been reported [35]. Butz et al. [36] studied the material nonlinear and geometric nonlinear effects on 3D piezoelectric beams. Panda and Ray [37] have also studied nonlinear analysis of piezoelectric layer-reinforced FGM plates using FSDT. Based on the bidet, the nonlinear analysis of the piezoelectric FGM plate structure has also been presented in [38]-[40].

With the above studies, most of the authors in the world use numerical methods (FEM, BEM, kp-Ritz, Meshfree, ...) to analyze the plate problems. This shows the superiority of the numerical method over the analytical method in computing complex problems. In this study, we will develop the recently proposed Smooth Finite Element Method - SFEM [41] to analyze the behavior of piezoelectric patch-integrated multifunctional materials. In these methods, the strain smoothing technique [42] has been incorporated into FEM to produce a series of smooth finite elements [41], such as cell/element-based smoothed FEM (CS-FEM) [43], node-based smoothed FEM (NS-FEM) [44], edge-based smoothed FEM (ES-FEM) [45], and face-based smoothed FEM (FS-FEM) [46]. Each element type of S-FEM has properties and advantages because of its differences compared with FEM; it has been widely used for benchmarking problems and classes of problems in mechanics such as analysis of plate and shell structures [47], [48], piezoelectric materials [49], fracture mechanics [50], the visco-elastoplastic problem [51], [52], limit and shakedown analysis for solids [53], etc.

For the group of piezoelectric plate problems, some domestic authors have also begun to study in recent years. Nguyen Xuan Hung and colleagues [49], [54] typically analyzed and calculated two-dimensional planar piezo structures using ES-FEM and NS-FEM. Tran Ich Thinh and Ngo Nhu Khoa [55] have developed bids based on quadrilaterals for composite plates. Tran Ich Thinh and Le Kim Ngoc [56]-[58] used a 9-node quadrilateral element for static analysis, oscillation control, and optimizing composite plates containing a piezoelectric layer. These studies show that the works all use high-order

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quadrilaterals and require a high computational cost. This will make it difficult for problems with complex geometric boundaries such as circles, triangles, polygons, etc.

Meanwhile, Nguyen Xuan Hung has only reused the application of smooth finite elements to 2D planar piezo textures but has not extended through the analysis of piezo plate textures. Phung Van Phuc [59] and his colleagues recently developed the CS-DSG3 method based on FSDT for piezoelectric composite plate structures. Phung Van Phuc [60] used isogeometric analysis (IGA) to analyze static and free vibrations for the carbon nanofiber-reinforced composite plate structure. Through some of the above studies, there are still relatively few behavioral studies of carbon nanofiber-reinforced plates and carbon nanofiber-reinforced plates of piezoelectric materials. Therefore, this can be considered a relatively new and promising research direction.

## 2. Research orientation

This study aims to analyze the dynamics and optimize the structure of multifunctional carbon nanofiber-reinforced material plates incorporating piezoelectric crystals, including three main stages:

- ✓ First, analyze the linear and nonlinear behavior of multifunctional carbon nanofiber-reinforced material plates integrated with piezoelectric stickers subjected to simultaneous mechanical-thermal-electrical loads.
- ✓ Second, analyze the dynamics & input control (voltage, impact force, etc.) to find the smallest deformation energy level during the process and analyze the optimal position of the piezoelectric crystal stickers for the behavior of multifunctional carbon nanofiber-reinforced material plates.
- ✓ Then, optimization analysis determines the best position and thickness of the piezoelectric crystal stickers in the structure.

The research approach and method includes three main steps:

- ➢ Research from general to detailed theoretical foundations. Each issue will be analyzed, evaluated, and compared with previous publications.
- ➢ Write algorithms and program numerical simulations using Matlab for common problems with exact solutions or many practical applications in life.
- ➢ Analyze and compare numerical simulation results with results of analytical solutions, with results published in international journals, or with results of other software. From these comparison results, this study will present each method's advantages and disadvantages and suggest improvement directions. *Specifically, improving the traditional finite element method, solving optimization methods and developing multifunctional material plate structures and piezoelectric integrated material plates.*

## 3. Proposed theoretical foundations

### *3.1. Materials*

#### *3.1.1. Composite materials*

The materials combine the advantages of two or more different materials to create materials with outstanding properties such as high durability, good corrosion resistance, etc. Composite materials with reinforcement are commonly in fibers and are divided into three common types as follows [61]:

- The fibrous composites: fibers and matrix materials mixed
- The particulate composites: micro-sized fibers in the base material
- The laminated composites: each layer is a different type of material or fiber direction

In this study, the material chosen is laminated composites to analyze the mechanical behavior of the structure (shown in Figure 1). Layers I, II, III, and IV represent different materials or fiber orientations.

For anisotropic materials, Hooke's law formula is written as:

$$\sigma_i = Q_{ij}\varepsilon_j \tag{1}$$

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where $\sigma_i$ is the stress, $\varepsilon_j$ is the strain, and $Q_{ij}$ is the material coefficient matrix, with *i* and *j* being the Cartesian coordinates in the *x* and *y* directions.

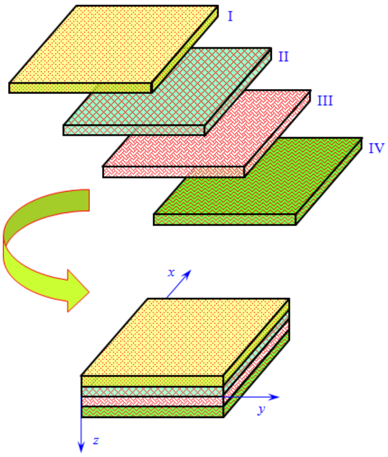


**Figure 1.** *A structure of common multi-layer composite materials*

The material constant is defined as follows:

$$E_1 = E_f \upsilon_f + E_m \upsilon_m \quad ; \quad \nu_{12} = \nu_f \upsilon_f + \nu_m \upsilon_m$$
$$E_2 = \frac{E_f E_m}{E_f \upsilon_m + E_m \upsilon_f} \quad ; \quad G_{12} = \frac{G_f G_m}{G_f \upsilon_m + G_m \upsilon_f} \qquad (2)$$

where $E_f$ & $E_m$ are Young's modulus, $\nu_f$ & $\nu_m$ are Poisson's ratio, $\upsilon_f$ is the volume ratio, *f* & *m* represent the fiber and matrix of the multi-layer composite, *1* & *2* represent the axial direction *x* & *y*.

The sliding modulus $G_f$ & $G_m$ are determined by:

$$G_f = \frac{E_f}{2(1+\nu_f)} \quad ; \quad G_m = \frac{E_m}{2(1+\nu_m)} \qquad (3)$$

*3.1.2. Functional carbon nanofiber-reinforced materials*

Four common distributions of carbon nanofibers in this material, including UD, FG-V, FG-O, and FG-X, are shown in Figure 2.

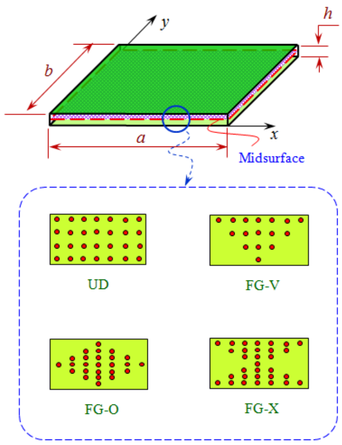


**Figure 2.** *Distributions of carbon nanofibers in functional carbon nanofiber-reinforced materials*

The properties of functional carbon nanofiber-reinforced (CNT) materials are determined as follows [62]:

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m \qquad (4)$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^m} \tag{5}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^m} \tag{6}$$

where $E_{11}^{CNT}$, $E_{22}^{CNT}$, and $G_{12}^{CNT}$ are Young's modulus in the *x* & *y* axis and sliding modulus of CNT, respectively; $E^m$ and $G^m$ are the matrix's elastic and sliding modulus.

The relationship between the volume fraction and matrix components in this material is determined by:

$$V_{CNT} + V_m = 1 \tag{7}$$

Poisson's coefficient and density are also determined by:

$$\nu_{12} = V_{CNT}^{*}\nu_{12}^{CNT} + V_m \nu^m \tag{8}$$

$$\rho = V_{CNT}\rho^{CNT} + V_m \rho^m \tag{9}$$

where $\nu_{12}^{CNT}$, $\rho^{CNT}$ và $\nu^m$, $\rho^m$ are Poisson's coefficient and density of CNT and matrix.

CNT distribution along the thickness of the plate is determined by [63]:

$$V_{CNT} = \begin{cases} V_{CNT}^{*} & \text{(UD)} \\ (1+\frac{2z}{h})V_{CNT}^{*} & \text{(FG-V)} \\ 2(1-\frac{2|z|}{h})V_{CNT}^{*} & \text{(FG-O)} \\ 2(\frac{2|z|}{h})V_{CNT}^{*} & \text{(FG-X)} \end{cases} \tag{10}$$

$$V_{CNT}^{*} = \frac{w_{CNT}}{w_{CNT} + (\rho_{CNT}/\rho_m) - (\rho_{CNT}/\rho_m)w_{CNT}} \tag{11}$$

where $w_{CNT}$ is the mass ratio of CNTs in the plate.

### *3.1.3. Piezoelectric materials*

This material is considered by the laws of electricity and mechanics, and the electrical effect is considered linear. Therefore, in the piezoelectric problem, two fields of mechanical and electrical variables will be approximated. This material includes elastic modulus *E*, Poisson's coefficient $\nu$, density $\rho$, piezoelectric coefficient *d*, and electric permittivity *p*.

The behavior of a piezoelectric plate is defined by:

$$\begin{bmatrix} \boldsymbol{\sigma} \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{c} & -\mathbf{e}^T \\ \mathbf{e} & \mathbf{g} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{\varepsilon}} \\ \mathbf{E} \end{bmatrix} \tag{12}$$

where $\boldsymbol{\sigma}$ and $\bar{\boldsymbol{\varepsilon}}$ are the stress and strain of the mechanical field, **D** is the dielectric displacement, **E** is the electric field; **g** and **e** are the electrical material matrix and are determined as follows [64]:

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \; ; \; \mathbf{g} = \begin{bmatrix} p_{11} & 0 & 0 \\ 0 & p_{22} & 0 \\ 0 & 0 & p_{33} \end{bmatrix} \tag{13}$$

## ***3.2. Theoretical foundations of plates***

The behavior of composite plates is a problem solved by many theories that have been researched and developed, which can be divided into three main groups of theories:

- The equivalent single layer (ESL) theories: the classical laminated plate (CLP) theory and the shear deformation laminated plate (SDLP) theories

- The three-dimensional elasticity theory: the traditional three-dimensional elasticity formulations and the layerwise theory.
- The model combines two-dimensional and three-dimensional.

In this study, the used material plate is thin, and the equivalent single layer (ESL) theories give quite reasonable results.

*3.2.1. The classical laminated plate (CLP) theory*

The CLP theory is the most straightforward of the ESL theories. In the CLP theory, some assumptions based on Love-Kirchhoff theory are expressed as follows [65]:

- The generating line along the thickness is still perpendicular to the neutral line before and after deformation.
- The transverse strain and the transverse shear strain are ignored.

Consider the plate as shown in Figure 3, and the displacement field is determined as follows:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\frac{\partial w_0}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) + z\frac{\partial w_0}{\partial y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{14}$$

where $u_0$, $v_0$, and $w_0$ are displacements along $x$, $y$, and $z$ axes.

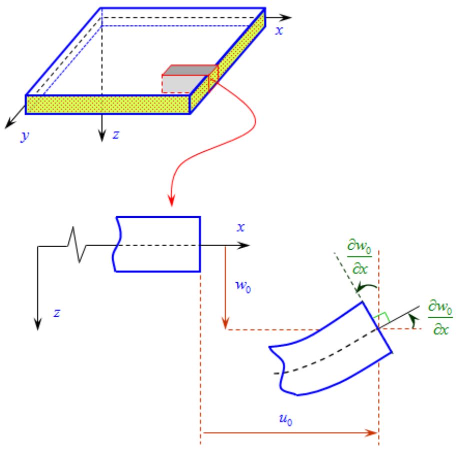

**Figure 3.** *Deformation and non-deformation plates in the CLP theory*

In the CLP theory, the deflection of the plate is caused by the bending process.

*3.2.2. The first order shear deformation theory (FSDT)*

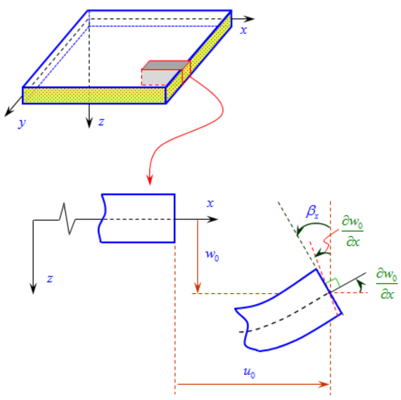

**Figure 4.** *Deformation and non-deformation plates in the FSDT*

The displacement field in the FSDT is described as follows [66], [67]:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\beta_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\beta_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{15}$$

where $u_0$, $v_0$, and $w_0$ are displacements in the neutral plane and $\beta_x$, $\beta_y$ are rotation angles along x and y axes.

In the FSDT, the shear strain is non-zero (shown as Figure 4), and the shear correction factor is proposed, which is discussed in detail in [68], [69].

*3.2.3. The high order shear deformation theory (HSDT)*

Reddy [70] developed this theory, and the displacement field is approximated as a high-order function described as follows:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\beta_x(x,y,t) + cz^3\left(\beta_x + \frac{\partial w_0}{\partial x}\right) \\ v(x,y,z,t) &= v_0(x,y,t) + z\beta_y(x,y,t) + cz^3\left(\beta_y + \frac{\partial w_0}{\partial x}\right) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{16}$$

with $c = 4/3h^2$

### 3.3. The finite element method for coupling problem

*3.3.1. The mathematical model of electro-mechanical problem:*

The dynamic equation**:**

$$\sigma_{ij,j} + f_i = \rho\ddot{u}_i;\ i,j = 1,2,3 \Leftrightarrow \begin{aligned} &\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} + f_x = \rho\ddot{u} \\ &\frac{\partial\sigma_{yx}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{yz}}{\partial z} + f_y = \rho\ddot{v} \\ &\frac{\partial\sigma_{zx}}{\partial x} + \frac{\partial\sigma_{zy}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} + f_z = \rho\ddot{w} \end{aligned} \tag{17}$$

The Gauss's Law (Electricity)

$$D_{i,i} - q_e = 0;\ i,j = 1,2,3 \Leftrightarrow \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} - q_e = 0 \tag{18}$$

The kinetic relationships (Mechanics)

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \Leftrightarrow S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{19}$$

The Maxwell's Law (Electricity)

$$E_i = -\phi_{,i} \Leftrightarrow E_i = -\frac{\partial\phi}{\partial x_i} \tag{20}$$

The behavioral relationships in the electro-mechanical problem:

$$\begin{matrix} F^{(m)} \to u^{(m)} \to \varepsilon^{(m)} \to \sigma^{(m)} \\ \updownarrow \\ F^{(e)} \to \phi^{(e)} \to E^{(e)} \to D^{(e)} \end{matrix} \Rightarrow \begin{aligned} \sigma_{ij} &= c_{ijkl}S_{kl} - e_{kij}E_k \\ D_i &= e_{ikl}S_{kl} + \varepsilon_{ik}E_k \end{aligned} \tag{21}$$

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where $c_{ijkl}$ is the material elastic constant with constant electric field, $\varepsilon_{ik}$ is the dielectric constant with constant strain field, $e_{kij}$ is the piezoelectric constant

*3.3.2. The dynamic control*

Consider a base plate with a piezoelectric sensor on the bottom and a piezoelectric actuator on top, as shown in Figure 5:

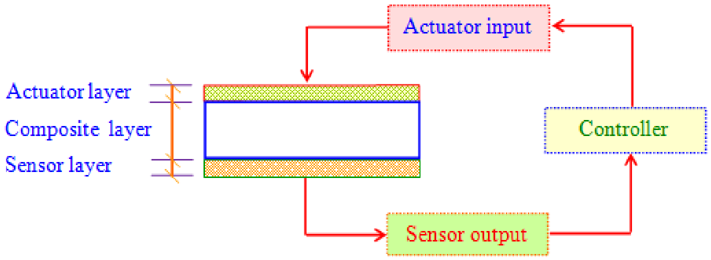


**Figure 5.** *The active control for plate*

The equations of motion is presented in matrix form:

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \ddot{d} \\ \ddot{\phi} \end{bmatrix}+\begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix}\begin{bmatrix} d \\ \phi \end{bmatrix}=\begin{bmatrix} F \\ Q \end{bmatrix} \tag{22}$$

$$M\ddot{d}+(K_{uu}+K_{u\phi}K_{\phi\phi}^{-1}K_{\phi u})d=F+K_{u\phi}K_{\phi\phi}^{-1}Q \tag{23}$$

The control constants are used to link the actuator's input potential $\varphi_a$ and the sensor's output potential $\varphi_s$.

The control gain:

$$\phi_a=G_d\phi_s+G_v\dot{\phi}_s \tag{24}$$

$G_d$ and $G_v$ are the feedback gain during displacement and velocity control.

Because the sensor has no initial action potential, Q is zero. Therefore, apply the potential equation generated by the sensor:

$$\varphi_s=\left[K_{\phi\phi}\right]_s^{-1}\left[K_{\phi u}\right]_s \tag{25}$$

Substitute Equations 24 and 25 into Equation 22:

$$Q_a=\left[K_{uu}\right]_a\{d_a\}-G_d\left[K_{\phi\phi}\right]_a\left[K_{\phi\phi}\right]_s^{-1}\left[K_{\phi u}\right]_s\{d_s\}-G_v\left[K_{\phi\phi}\right]_a\left[K_{\phi\phi}\right]_s^{-1}\left[K_{\phi u}\right]_s\left\{\dot{d}_s\right\} \tag{26}$$

Substitute Equation 26 into Equation 23:

$$[M]\{\ddot{q}_k\}+\left(G_v\left[K_{q\varphi}\right]_a\left[K_{\varphi\varphi}\right]_s^{-1}\left[K_{\varphi q}\right]\right)_s\{\dot{q}\}+\left(\left[K_{qq}\right]+G_d\left[K_{q\varphi}\right]_s\left[K_{\varphi\varphi}\right]_s^{-1}\left[K_{\varphi q}\right]_s\right)\{q\}=\left\{\overline{f}\right\} \tag{27}$$

If we consider a system with dampers:

$$\begin{gathered}[M]\{\ddot{q}_k\}+\left(G_v\left[K_{q\varphi}\right]_a\left[K_{\varphi\varphi}\right]_s^{-1}\left[K_{\varphi q}\right]+\alpha[M]+\beta\left[K_{qq}\right]\right)_s\{\dot{q}\}+ \\ \left(\left[K_{qq}\right]+G_d\left[K_{q\varphi}\right]_s\left[K_{\varphi\varphi}\right]_s^{-1}\left[K_{\varphi q}\right]_s\right)\{q\}=\left\{\overline{f}\right\}\end{gathered} \tag{28}$$

where $\alpha$ and $\beta$ are the Rayleigh damping coefficients.

### *3.4. Optimization using the combination of Artificial Neural Network (ANN) and differential evolution (DE) algorithms*

The Multi-Layer Perceptron (MLP) is one of the most useful ANNs in function approximation used in this study. The MLP consists of an input layer, several hidden layers, and an output layer, as shown in Figure 6.

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Optimization using the combination of ANN and DE algorithms: calculating the objective function values of the individuals in the population in each generation and evaluating the test vectors in steps performed using finite element analysis as in the original DE algorithm is replaced by the ANN approximation.

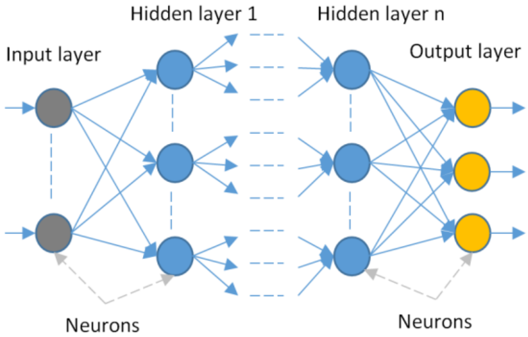


**Figure 6.** *The MLP model*

The difference between the traditional ANN model and the improved ANN model is shown in Figures 7 and 8.

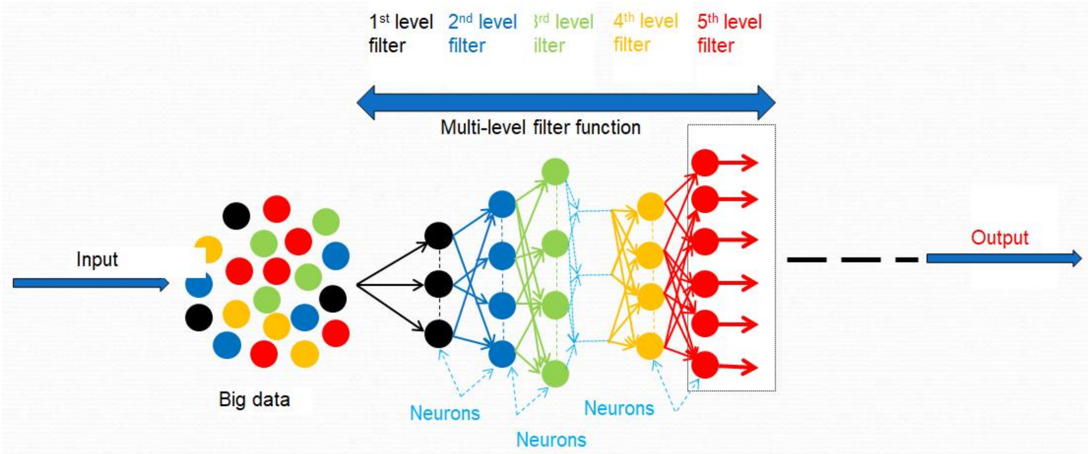


**Figure 7.** *The traditional ANN model*

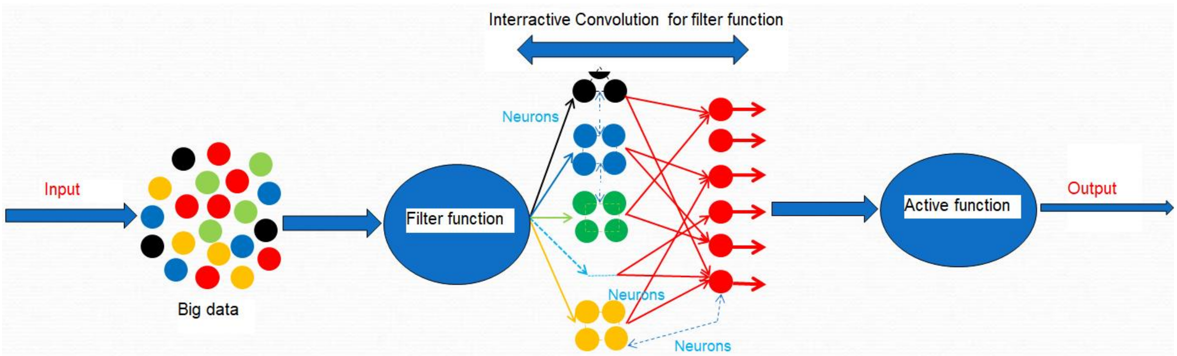


**Figure 8.** *The improved ANN model*

This combination is intended to improve traditional ANN, even using improved DE (iDE) to create a filter to get the necessary data to reduce the number of hidden layers and promote achieving the goal quickly.

The combined algorithm diagram is presented in Figure 9.

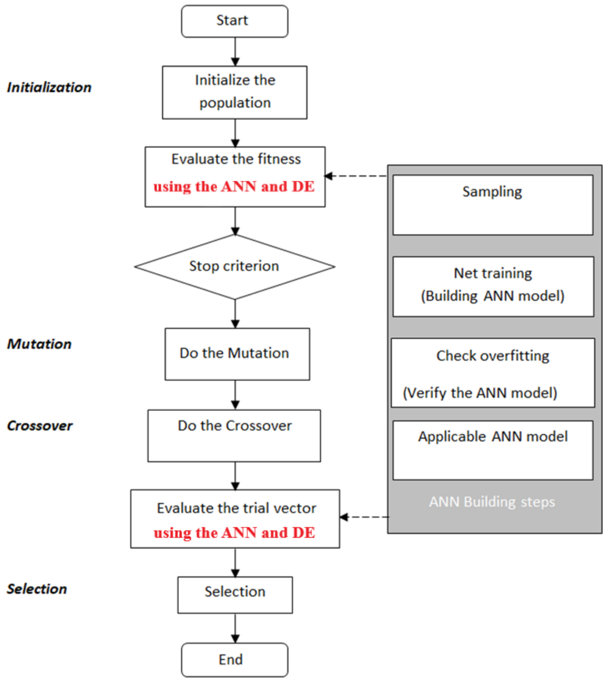


**Figure 9.** *The combination of ANN and DE algorithms diagram*

## 4. Conclusions

Previously improved finite element methods have only been applied to a few problems in simple single-physics, plate problems, or simple two-dimensional (2D) multi-physics environments. Therefore, the new point of this article is the further development of finite element methods and improved control and optimization analysis methods for complex problems. Specifically, the problems of structural plates made of multifunctional materials reinforced with carbon nanofibers integrated with piezoelectric stickers.

Proposing structural behavior solutions in real-time mode through the combination of control laws and optimization method improvements to meet increasing requirements in calculating plate structural problems in diverse multi-physics environments.

The results of this research will provide real-time structural analysis and evaluation and improved optimization methods that can be effectively applied in computer calculations of complex material plate structural problems.

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## Conflict of Interest

The authors declare no conflict of interest.

## Data Availability Statement

The data supporting this study's findings are available from the corresponding author upon reasonable request.

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