COMPARISON BETWEEN TWO SWING-UP ALGORITHMS: PARTIAL FEEDBACK LINEAR AND ENERGY BASED METHOD

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ABSTRACT

Beside the balancing algorithms, swing-up is also an important problem in controlling under-actuated system. Swing-up is necessary to move automatically system to a suitable position for others balancing controls to operate. However, a limited number of researches has been focused on this problem. In this paper, the authors compare two swing-up algorithms from other researches: partial feedback linear and energy-based methods in both simulation and experiment. In previous studies, these methods were presented in only a mathematical description and simulation. Moreover, no comparison was concerned between these methods. Thence, experimental results from this paper implement more descriptions in a real-time system. Our both simulation and experiment results prove that recent energy-based method gives better controlling response than classical partial feedback linear method under external force. In this research, pendubot, a popular model in control engineering, is an under-actuated object.

Keywords: *pendubot; swing-up; partial feedback linear; energy-based; balancing control; under-actuated system.*

1. INTRODUCTION

Under-actuated systems are model types in which a number of control inputs are less than the number of degrees of freedom. During the past decades, research into these systems and their problems are strongly developed. Among under-actuated system categories, pendubot (shown in Fig. 1) is popularly used because of its simple mechanical structure and highly nonlinear model (see [1-12]). Three basic problems of pendubot are balancing on TOP and MID positions, trajectory tracking and swinging up. Although many types of research on balancing and tracking the desired trajectory have been introduced in [3-12], fewer researches on swing-up controlling has been presented. J. Block [3] introduced a partial feedback linear method (PFLM) to swing up pendubot. This controller was proved to work successfully in simulation. Anyway, in that research, Block also stated that when testing on an experiment, that swing up controller did not work well in all cases. Especially, from real-time controlling pendubot in the study [15], when the DC motor warmed up after the long-time operation, its torque constant was decreased slightly. This uncertainty of system parameters causes difficulty in controlling. Also, this controller was not guaranteed by mathematics. In [1, 12], another swing-up controller so-called sliding mode control (SMC) was presented. Its mathematical background was guaranteed by Lyapunov's criteria and proved by simulation and no experimental result was shown. In [2], the authors used the energy-based method (EBM) for swing up control. The controller was designed from calculating potential energy at upright position and control law was obtained by defining suitable Lyapunov function. Thence, in all cases, the controller could mathematically guarantee to bring both links to the nearby upright position. But, controller in [2] was not tried under external force and results were only verified through simulation and experimental results were not shown.

In presented researches, simulation results are shown but the actual results are not sufficiently introduced. Thence, in this paper, we focus mostly on an experiment of two swings up controllers, from partial PFLM (in early period) and from EBM (in recent period). The comparison of the two methods is concerned in both simulation and experiment. Surveys of pendubot under external force and under no external force are also inferred. From those investigations, controller from EBM gives better responses than from PFLM in all cases. This paper comprises four sections. Firstly, the introduction of paper is shown in this chapter. Then, mathematical equations of pendubot are described in section 2. In section 3, authors design controllers in order to swing up. Results in simulation and real-time system are presented in section 4. Finally, section 5 shows the conclusion.

2. DYNAMIC EQUATION

From [3], mathematical model is described as

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=B\tau_1 \quad (1)$$

where $q=[q_1 \quad q_2]^T$ are angles of link1 and link 2; τ_1 is the moment applied to link one. Matrix $D(q), C(q,\dot{q}), G(q)$ are show in *Appendix*.

In the experiment, a control input of the system is the voltage u that is supplied to the DC motor. Thence, the relation of τ_1 and u has to be considered. According to [13], this relation is shown in Fig. 2 and (2). System parameters are described in Table 1.

$$u=\frac{\tau_1+a_2\dot{q}_1+a_3\ddot{q}_1}{a_1} \quad (2)$$

where $a_1=\frac{K_t}{R_m}; a_2=\frac{K_tK_b}{R_m}+C_m; a_3=J_m$.

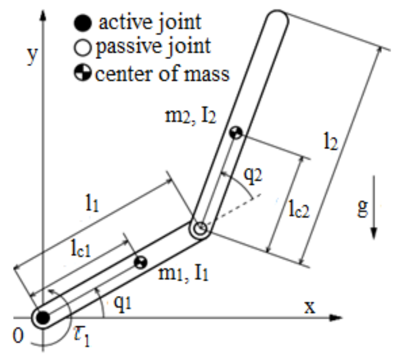

***Fig. 1** Schematic of pendubot [3]*

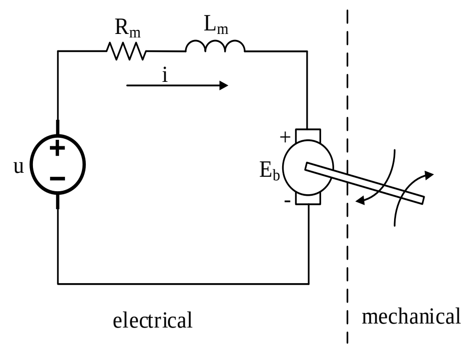

***Fig. 2** Structure of DC Motor*

In Fig. 1, pendubot on Oxy-coordinate axis is shown. Fig. 2 shows the relation between electrical and mechanical parts of DC motor.

***Table 1.** System parameters*

Symbol	Unit	Value	Description
m_1	Kg	0.15	Mass of link 1
l_1	m	0.2	Length of link 1
l_{c1}	m	0.1	Distance from active joint to center of mass of link 1
I_1	kg.m^2	0.002	Moment of inertia of link 1
m_2	kg	0.055	Mass of link 2
l_2	m	0.22	Length of link 2
l_{c2}	m	0.11	Distance from passive joint to center of mass of link 2
I_2	kg.m^2	2.5x10^{-4}	Moment of inertia of link 2
g	m/s^2	9.81	Gravitional acceleration
K_t	Nm/A	0.0644	Torque constant

Symbol	Unit	Value	Description
K_b	V/(rad/s)	0.0644	Back EFM constant (Force constant of motor)
R_m	Ohm	4.6	Resistor of rotor
J_m	kg.m_2	$5x10^{-3}$	Moment of inertia of rotor
C_m	Nm/(rad/s)	$8x10^{-5}$	Viscous friction constant

3. SWING-UP CONTROLLER

3.1 Partial feedback linear method

PFLM was introduced by J. Block [3]. In that study, (3)-(9) are listed as below. From (1), it yields

$$D_{11}\ddot{q}_1 + D_{12}\ddot{q}_2 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + G_1 = \tau_1 \quad (3)$$

$$D_{21}\ddot{q}_1 + D_{22}\ddot{q}_2 + C_{11}\dot{q}_1 + G_2 = 0 \quad (4)$$

From (4), acceleration of link 2 is

$$\ddot{q}_2 = \frac{-D_{21}\ddot{q}_1 - C_{21}\dot{q}_1 - G_2}{D_{22}} \quad (5)$$

Substituting (5) into (3), it yields

$$\bar{D}_{11}\ddot{q}_1 + \bar{C}_{11}\dot{q}_1 + \bar{C}_{12}\dot{q}_2 + \bar{G}_1 = \tau_1 \quad (6)$$

where $\bar{D}_{11} = D_{11} - \frac{D_{12}D_{21}}{D_{22}}$; $\bar{C}_{11} = C_{11} - \frac{D_{12}C_{21}}{D_{22}}$; $\bar{C}_{12} = C_{12}$; $\bar{G}_1 = G_1 - \frac{D_{12}G_2}{D_{22}}$

Moment that is applied to control link 1 after linearizing q_1 is proposed as

$$\tau_1 = \bar{D}_{11}v + \bar{C}_{11}\dot{q}_1 + \bar{C}_{12}\dot{q}_2 + \bar{G}_1 \quad (7)$$

The dynamic equations of system are

$$\ddot{q}_1 = v \quad (8)$$

$$D_{22}\ddot{q}_2 + C_{21}\dot{q}_1 + G_2 = -D_{21}v \quad (9)$$

Therefore, link 1 will track the desired trajectory according to (8) and response of link 2 will track the desired trajectory according to (9). At upright position, set-point is $q_{1d} = \frac{\pi}{2}, \dot{q}_{1d} = 0$. To achieve the object, we use the PD controller acceleration feedback as shown in Fig. 3 as

$$v = -K_D\dot{q}_1 + K_P\left(\frac{\pi}{2} - q_1\right) \quad (10)$$

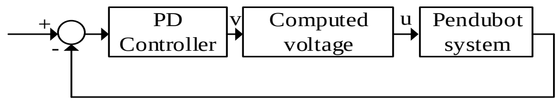

***Fig. 3** Controller structure for PFLM*

In Fig. 3, the state variables of pendubot are feed-backed to "PD controller" block to obtain v (based on (10)). Then, from (2)-(7), voltage u is obtained as control signal in "computed voltage" block.

In [3], PD controller for this method worked well in simulation, but Block also stated that this controller did not work in all cases in a real-time system. Otherwise, when the DC motor is hot after a long-time operation, system parameters are changed (this problem is proved in [14]). Therefore, another method [2] is suggested in the next section.

3.2 Energy based method

EBM was present in [2]. In this method, control law τ_1 which satisfied conditions are presented as

$$\lim_{t\to\infty} E(q,\dot{q}) = E_p \ , \lim_{t\to\infty}(q_1) = \frac{\pi}{2} \ , \lim_{t\to\infty}\dot{q}_1 = 0 \quad (11)$$

where:

$E(q,\dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q} + P(q)$ is total energy of pendubot; $P(q) = \beta_4 g\sin(q_1) + \beta_5 g\sin(q_1+q_2)$ is potential of pendubot; $E_p = (\beta_4+\beta_5)g$ is energy of pendubot at upright position.

Equations (12) to (14) are obtained from [[2]]. Authors define Lyapunov function

$$V = \frac{1}{2}K_E\left(E-E_p\right)^2 + \frac{1}{2}K_P\left(q_1 - \frac{\pi}{2}\right)^2 + \frac{1}{2}K_D\dot{q}_1^2 \quad (12)$$

where K_E , K_P , K_V is positive contant parameters.

Derivative V respect to time t

$$\dot{V} = K_E\dot{E}\left(E-E_p\right) + K_P\dot{q}_1\left(q_1 - \frac{\pi}{2}\right) + K_D\ddot{q}_1\dot{q}_1 \quad (13)$$

$$\begin{aligned}\dot{E} &= \dot{q}^T D(q)\ddot{q} + \frac{1}{2}\dot{q}^T\dot{D}(q)\dot{q} + \dot{P}(q) \\ &= \dot{q}^T\left(B\tau_1 - C(q,\dot{q})\dot{q} - G(q)\right) + \frac{1}{2}\dot{q}^T\dot{D}(q)\dot{q} + \dot{q}^T G(q) \quad (14) \\ &= \dot{q}^T B\tau_1 + \frac{1}{2}\dot{q}^T\left(\dot{D}(q) - 2C(q,\dot{q})\right)\dot{q}\end{aligned}$$

where $\dot{D}(q)-2C(q,\dot{q})=\beta_3\sin(q_2)(2\dot{q}_1+\dot{q}_2)\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ is a skew-symmetric matrix.

According to properties of skew-symmetric matrix, it yields $z^T\left(\dot{D}(q)-2C(q,\dot{q})\right)z=0 \quad \forall z\in R^{2\times1}$. Substituting this result to (14), we obtain

$$\dot{E}=\dot{q}^T B\tau_1=\dot{q}_1\tau_1 \tag{15}$$

Thence, substitute (15) into (13), then, we obtain

$$\dot{V}=\dot{q}_1\left(K_E\left(E-E_p\right)\tau_1+K_P\left(q_1-\frac{\pi}{2}\right)+K_D\ddot{q}_1\right) \tag{16}$$

τ_1 is selected to satisfy

$$K_E\left(E-E_p\right)\tau_1+K_P\left(q_1-\frac{\pi}{2}\right)+K_D\ddot{q}_1=-K_V\dot{q}_1 \tag{17}$$

where $K_V>0$

Thence, this inequality is obtained

$$\dot{V}=-K_V\dot{q}_1^2<0 \tag{18}$$

and Lyapunov's stability condition is satisfied.

From (1), after some calculations, it yields

$$\ddot{q}_1=B^T\ddot{q}=B^T D^{-1}\left(B\tau_1-C\dot{q}-G\right) \tag{19}$$

Substituting (19) into (17), it yields

$$\Psi\tau_1=-K_P\left(q_1-\frac{\pi}{2}\right)+K_D B^T D^{-1}\left(C\dot{q}+G\right)-K_V\dot{q}_1 \tag{20}$$

where $\Psi=K_E\left(E-E_p\right)+K_D B^T D^{-1}B\neq 0$.

Thence, swing up controller is

$$\tau_1=\frac{-K_P\left(q_1-\frac{\pi}{2}\right)+K_D B^T D^{-1}\left(C\dot{q}+G\right)-K_V\dot{q}_1}{\Psi}\xi \tag{21}$$

After swinging up controllers, when two links are near the upright position, balancing controller, such as LQR, PID…, will replace swing up controllers to keep pendubot balanced. Switching condition is chosen as

$$\left(\frac{\pi}{2}-q_1\right)+0.1\dot{q}_1+q_2+0.1\dot{q}_2<\xi \tag{22}$$

where ξ is small constant to achieve better performance. If it is too small, the controller cannot switch to the LQR controller, and the system is un-stabilized. Vice versa, the swing-up controller switches to balancing controller too soon, when angular velocities of links are still quite big or both links is not adequately near the upright position. Thence, the balancing controller cannot work well. In this paper, in simulation and experiment, we choose $\zeta=0.7$ through trial and error test.

4. RESULTS

4.1 Simulation results

Under PFLM, $K_P=82$, $K_D=28$ are obtained through a genetic algorithm (GA). From simulation results in Fig. 4, PFL controller swings pendubot from down-ward position to TOP position in about 1.5 second. However, from 5th to 6th second, if we add external force, both links are un-stabilized, and PFLM cannot swing up links again. Link 1 oscillates around its balancing upright position, but link 2 drops down and cannot swing up to its set point again.

Under EBM controller, $K_P=200$, $K_D=8$, $K_E=141$ and $K_V=62$ are obtained through GA. From simulation results in Fig. 5, EBM controller swing pendubot to TOP position in about 1.5 seconds. This settling time is the same as in PFLM. However, when both links are un-stabilized under external force, this controller can swing up links again to an upright position where the balancing controller can continue keeping model at that position.

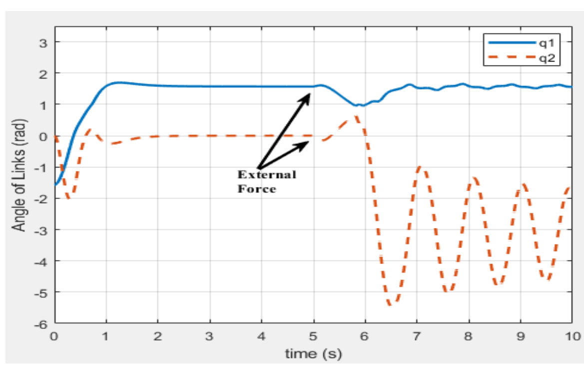

Fig. 4 *Angles of links under PFLM control*

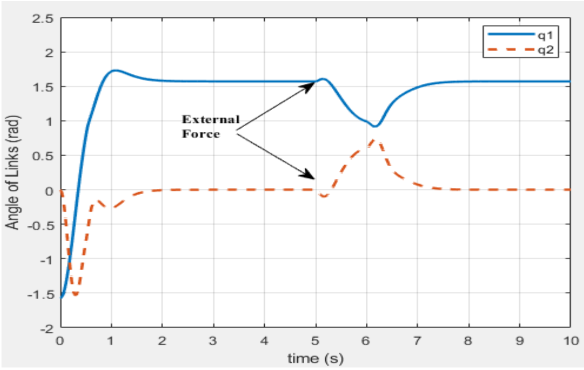

***Fig. 5** Angles of links under EBM control*

4.2 Experimental results

The real time model of pendubot for this research is shown in Fig. 6. Experimental results are shown in Fig. 7, 8, 9.

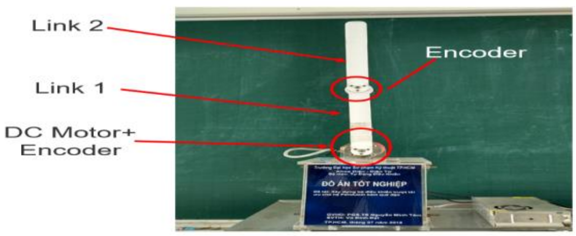

***Fig. 6** Experimental pendubot*

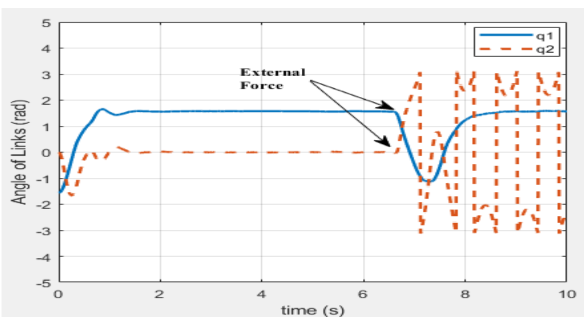

***Fig. 7** Angles of links under PFLM control*

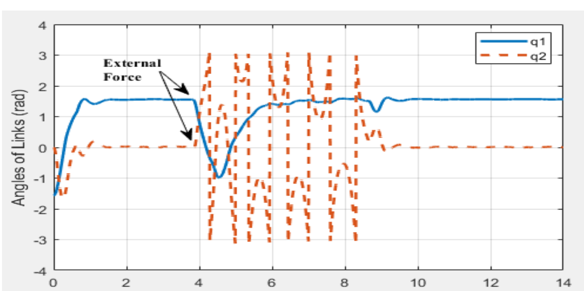

***Fig. 8** Angles of links under EBM control*

From Fig. 7, PFLM controller can swing up both links to TOP position in about 1.5 second. But, after both links are unstable being under external force at 7th second, link 1 can swing up to setpoint while link two cannot. It drops down and oscillates around a down-ward position. This result suits simulation results in Fig. 4 with $q_1, q_2 \in [-\pi, \pi]$.

In Fig. 8, EBM controller works as successfully as PFLM one in Fig. 7

Moreover, when both links drop-down under external force at 4th second, this swing up a controller can swing up them again to set point. At 9th second, both links are stable again at the balancing position same as the simulation result shown in Fig. 5 with $q_1, q_2 \in [-\pi, \pi]$.

Sequence images of pendubot operation (in Fig 6) under EBM are shown in Fig.9.

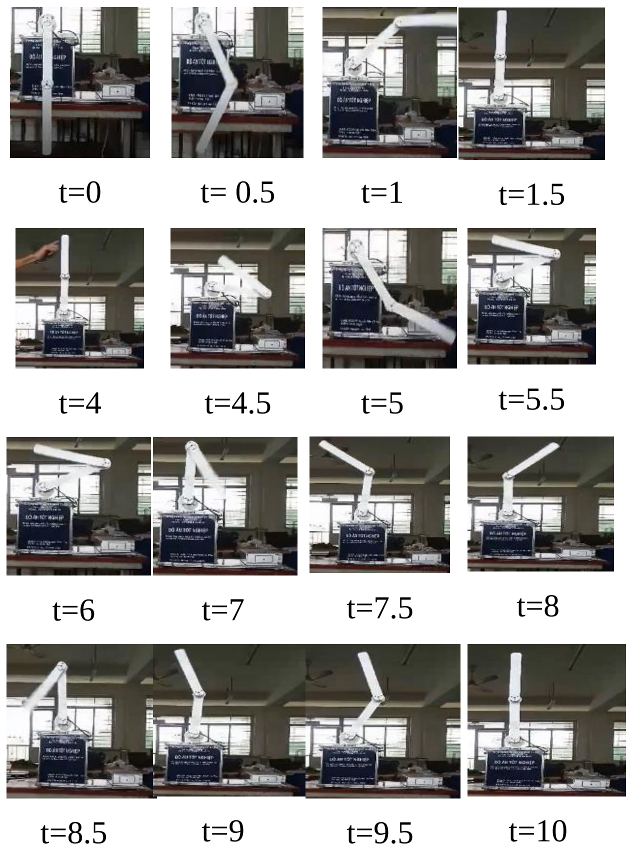

***Fig. 9** Response of actual pendubot system*

5. CONCLUSION

In this paper, two swing-up methods for pendubot are examined: PFLM and EBM. Simulations are reviewed for both methods. Moreover, experiments are implemented for previous researches of swing up controlling to verify their operation in practice. From a survey of this study, two kinds of controllers both work well in free state, condition when there is no external force on model. But, in the case that system is un-stabilized, EBM is more effective than PFLM, which cannot swing up model again to upright position (TOP position).

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