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Synchronizing Newton-Leipnik Fractional Order Chaotic Systems by Sliding Mode Control and Applying in Secure Color Image Communication

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ABSTRACT

In this paper, we present a comprehensive investigation of a sliding mode control (SMC) method, with a focus on its application to synchronize a three-dimensional (3-D) chaotic Newton-Leipnik system with fractional order. This study aims to demonstrate the robustness and efficiency of the SMC technique in addressing the challenges of synchronization in chaotic systems. The proposed control method is rigorously analyzed using Lyapunov stability theory, ensuring that the system’s behavior converges smoothly towards synchronization. The reliability of this approach is validated through both theoretical and practical evaluations.To further assess the performance of the sliding mode control method, MATLAB simulations were conducted, comparing the SMC technique to a conventional PID controller as well as other advanced methods in the context of secure color image communication. The results showed that SMC outperformed these techniques, achieving a lower Root Mean Square Error (RMSE) and a higher Signal-to-Noise Ratio (SNR), which are key indicators of communication security and effectiveness. Additionally, static analyses such as histogram, Mean Square Error (MSE), and SNR measurements highlighted the system's enhanced security and potential for robust image communication applications. These findings emphasize the practical significance of SMC in secure and reliable communication systems.

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1. Introduction

The phenomenon of chaos inherent in nonlinear dynamical systems has attracted much attention in the last two decades and has been extensively studied [1]-[3]. Characterized by unique properties such as broadband noise-like waveforms and sensitive dependence on initial conditions, chaos offers remarkable advantages in the field of secure communication systems [2]. The inherent unpredictability and complexity of chaotic systems make them ideal for secure communication, which has led to extensive research on the control and synchronization [2], [3], [4].

Synchronization of chaotic circuits has become a focus of research in secure communication due to its potential to improve security measures [1]-[4]. Various control methods have been proposed and extensively studied for controlling chaotic systems, including adaptive control, Takagi-Sugeno (T-S) fuzzy control, and sliding mode control (SMC).

In recent years, with the growing demand for secure image transmission, researchers have increasingly focused on applying chaotic systems to image encryption, particularly for color images. Color image encryption presents greater challenges due to its multi-channel nature (e.g., RGB) and the need for robust synchronization during transmission [5], [6]. Traditional methods have proven effective for grayscale images, but new approaches are required to handle the complexity of color image encryption in secure communication systems.

Sliding mode control (SMC) is a particularly powerful and specialized control strategy in the field of variable structure systems [5]-[7]. Over the past two decades, SMC has been recognized for its robustness and effectiveness in coping with system uncertainties and reducing dynamics through proposed switching functions. The robust nature of SMC and its ability to deal with uncertainties make it an ideal technique for synchronizing chaotic systems. The control law based on SMC ensures the occurrence of sliding motion, providing a reliable approach for secure communication applications.

Managing fractional-order control systems is more challenging than conventional integer-order systems due to the complex mathematics of fractional calculus [8]-[10], which involves non-integer integrals and derivatives. This complexity complicates the development and analysis of controllers. Accurate modeling requires advanced techniques to determine the parameters. Implementing these controllers is difficult because fractional derivatives and integrals often require numerical approximations, leading to high computational costs and potential inaccuracies. Stability analysis is complicated and requires new methods as standard tools are not applicable. The frequency behavior differs from whole-order systems, making it difficult to predict system behavior and control over a wide frequency range.

Therefore, a sliding mode control method is presented for chaotic synchronization in this paper. This method is designed to handle the synchronization challenges associated with fractional-order chaotic systems, particularly when applied to secure color image communication, where the synchronization of RGB channels requires high precision.

Several studies have focused on secure communication using chaotic systems. Huynh et al. presented secure audio communication based on a 4-D memristive chaotic system [11], while Lin et al. [12] proposed a hyper-chaotic system for secure image transmission. Giap et al. [13], [14] explored fractional-order chaotic synchronization for secure communication, highlighting its effectiveness in areas such as EEG signal security and wireless communication using a perturbation observer. However, secure color image communication remains a relatively underexplored area, presenting opportunities for further development in synchronization techniques [15], [16], [17]. In this work, we implemented the synchronization of a fractional-order chaotic system for secure communication of color images using sliding mode control.

This article deals with the complicated dynamics of chaotic systems and the effectiveness of sliding motion in synchronizing these systems for secure image communication. The novel aspect of this research lies in the application of fractional-order chaotic synchronization to color image encryption, which requires handling multiple data channels with precision to maintain both security and synchronization.

While sliding mode control (SMC) has been previously utilized in various studies for the synchronization of fractional-order chaotic systems, this paper emphasizes its specific application in the context of secure color image communication. This work is significant as it addresses the unique challenges associated with multi-channel (RGB) image encryption, necessitating precise synchronization across multiple data channels to achieve both robust security and high-quality image transmission can be summarized as follows:

1. Application of Sliding Mode Control for Synchronization of Chaotic Fractional Order Systems: While Sliding Mode Control (SMC) has been used in various fractional order systems in the past, this research explores its novel application in the field of secure color image communication. The use of SMC in this context helps synchronize chaotic fractional-order systems that are complex and dynamic, and ensures that color image communication remains secure and resistant to external disturbances or attacks.
2. Stability of the proposed method: The stability of the control method presented in this study is rigorously demonstrated using Lyapunov theory. This mathematical framework ensures that the system remains stable over time and provides a solid foundation for its reliable performance in synchronizing chaotic systems for secure communication.
3. Introduction of a new secure communication method for color images: A new method specifically designed for secure communication of color images is introduced. This approach utilizes the advantages of SMC and fractional-order chaotic systems to protect the integrity and confidentiality of transmitted images, making it ideal for secure data transmission.

4. Security and effectiveness analysis: Several analysis methods are used to evaluate the security and effectiveness of the proposed communication method, such as: histogram, mean square error (MSE) and signal to noise ratio (SNR)

This paper is organized as follows: Section 2 shows the problem formulation, Section 3 shows simulation results for chaotic synchronization between master and slave with fractional order. Section 4 shows the method for secure color image communication and the results, and Section 5 contains the conclusion.

2. Problem Formulation

This section examines the synchronization between master system and slave chaotic system of Newton-Leipnik fractional-order chaotic system by using sliding mode control. The master system is shown in following equation.

$$\begin{cases} D^{q_1}x_1 = -a_1x_1 + y_1 + 10y_1z_1 \\ D^{q_2}y_1 = -x_1 - 0.4y_1 + 5x_1z_1 \\ D^{q_3}z_1 = b_1z_1 - 5x_1y_1 \end{cases} \tag{1}$$

Parameters can be chosen as [1]: constants are $a_1 = 0.4, b_1 = 0.175$, $x_1 = 0.19$, $y_1 = 0$, $z_1 = -0.18$ and $q_i = 0.95$ $(i = 1,2,3)$. The slave equation for Newton–Leipnik chaotic system can be defined in following equation.

$$\begin{cases} D^{q_1}x_2 = a_2\left(y_2 - x_2\right) + u_1 \\ D^{q_2}y_2 = c_2y_2 - x_2z_2 + u_2 \\ D^{q_3}z_2 = x_2y_2 - b_2z_2 + u_3 \end{cases} \tag{2}$$

Parameters can be chosen as [1], constants are $a_2 = 36$, $b_2 = 3$, $c_2 = 20$, initial conditions are $\left(x_2, y_2, z_2\right) = \left(0.2, 0.5, -1\right)$, and $q_i = 0.95$ is the order of the derivative. The unknown terms u_1, u_2, u_3 in (2) are the control functions to be determined by sliding mode control method as below. Finally, the error system modeling can be defined as follows

$$e_3 = z_2 - z_1, e_2 = y_2 - y_1, e_1 = x_2 - x_1 \tag{3}$$

From equations (1) and (2), deriving error system is presented in following equation.

$$\begin{cases} D^{q_1}e_1 = -a_2e_1 + a_2e_2 + a_2\left(y_1 - x_1\right) + a_1x_1 - y_1 - 10y_1z_1 + u_1 \\ D^{q_2}e_2 = c_2e_2 + \left(c_2 + 0.4\right)y_1 - x_2z_2 + x_1 + 0.4y_1 - 5x_1z_1 + u_2 \\ D^{q_3}e_3 = -b_2e_3 - \left(b_2 + b_1\right)z_1 + 5x_1y_1 + u_3 \end{cases} \tag{4}$$

Defining the active control functions as follows.

$$\begin{cases} u_1 = V_1 - a_2\left(y_1 - x_1\right) - a_1x_1 + y_1 + 10y_1z_1 \\ u_2 = V_2 - \left(c_2 + 0.4\right)y_1 + x_2z_2 - x_1 - 0.4y_1 + 5x_1z_1 \\ u_3 = V_3 + \left(b_2 + b_1\right)z_1 - 5x_1y_1 \end{cases}. \tag{5}$$

With V_1, V_2, V_3 are respectively linear functions of e_1, e_2, and e_3. When u_i $(i = 1,2,3)$ are chosen as in equation (5), the derivate error systems in equation (4) become:

$$\begin{cases} D^{q_1}e_1 = -a_2e_1 + a_2e_2 + V_1 \\ D^{q_2}e_2 = c_2e_2 + V_2 \\ D^{q_3}e_3 = -b_2e_3 + V_3 \end{cases} \quad (6)$$

The control conditions $V_i \ (i=1,2,3)$ are chosen to ensure the stability of the error system (3). And $V_i \ (i=1,2,3)$ can be chosen as follows.

$$V_i = -Ce_i, \ (i=1,2,3) \quad (7)$$

where C is a constant matrix that ensures stability. To ensure that the error system converges to zero and the desired synchronization is achieved, the matrix C can be chosen as follows.

$$C = \begin{pmatrix} -a_2+1 & 0 & 0 \\ 0 & c_2+1 & 0 \\ 0 & 0 & -b_2+1 \end{pmatrix} \quad (8)$$

The sliding surface is designed to ensure that the error values converge to zero. The control functions are:

$$\begin{aligned} u_1 &= V_1 - a_2y_1 - x_1 + a_1x_1 + y_1 - 10y_1z_1 \\ u_2 &= V_2 - c_2y_1 + x_2z_2 - x_1 - 0.4y_1 + 5x_1z_1 \\ u_3 &= V_3 + b_2 + b_1z_1 - 5x_1y_1 \end{aligned} \quad (9)$$

The Lyapunov theory is proposed to point out the stability of the SMC method. Choosing Lyapunov function in following equation.

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \quad (10)$$

Devirative equation (10) gets

$$\dot{V} = e_3\dot{e}_3 + e_2\dot{e}_2 + +e_1\dot{e}_1 \quad (11)$$

Substituting the error equations, equation (11) equavalent to:

$$\dot{V} = e_1\left(-a_2e_1 + a_2e_2 + u_1\right) + e_2\left(c_2e_2 + u_2\right) + e_3\left(-b_2e_3 + u_3\right) \quad (12)$$

Substituting $u_i (i=1,2,3)$ from (9) to (12), equation (12) equivalent to:

$$\dot{V}(e) = -C\left(e_1^2 + e_2^2 + e_3^2\right) \le 0 \quad (13)$$

This shows that the stability of the proposed control system, meaning the error states e_i will come to zero when time $t \to \infty$.

3. Simulation Results

With the initial 3D chaotic fractional order synchronization and using the SMC method, using Matlab Software in personal computer to simulate the synchronization method. Figure 1 shows the 3D trajectories of the chaotic system with Newton-Leipnik fractional parts, Figure 2 shows (a) the tracking trajectory of each channel in the chaotic system with Newton-Leipnik fractional parts and (b) its zoom in 5 seconds. Figure 3 shows the tracking error of each channel in the chaotic system with fractional

Newton-Leipnik parts, and Figure 4 shows the control signal of each channel in the chaotic system with fractional Newton-Leipnik parts.

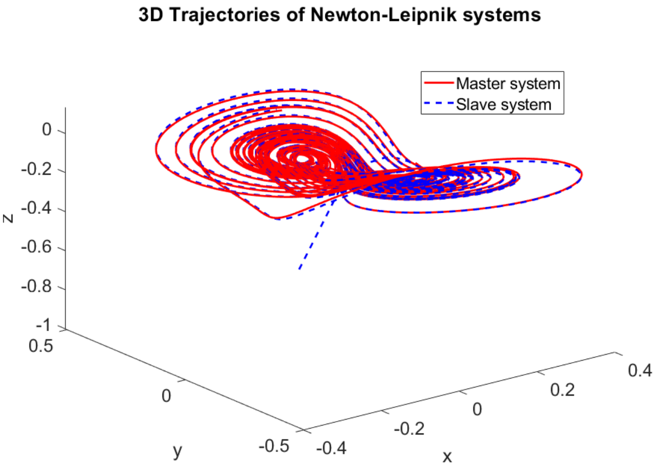
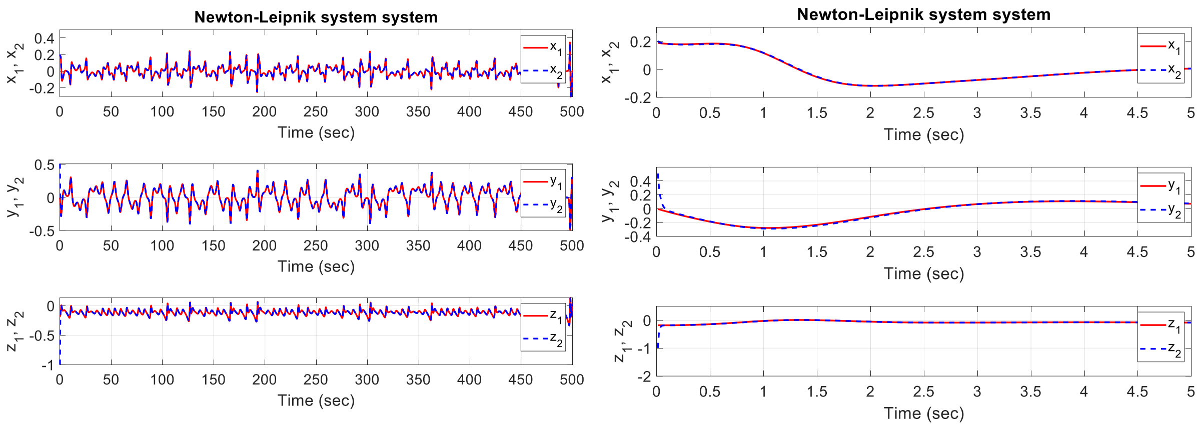
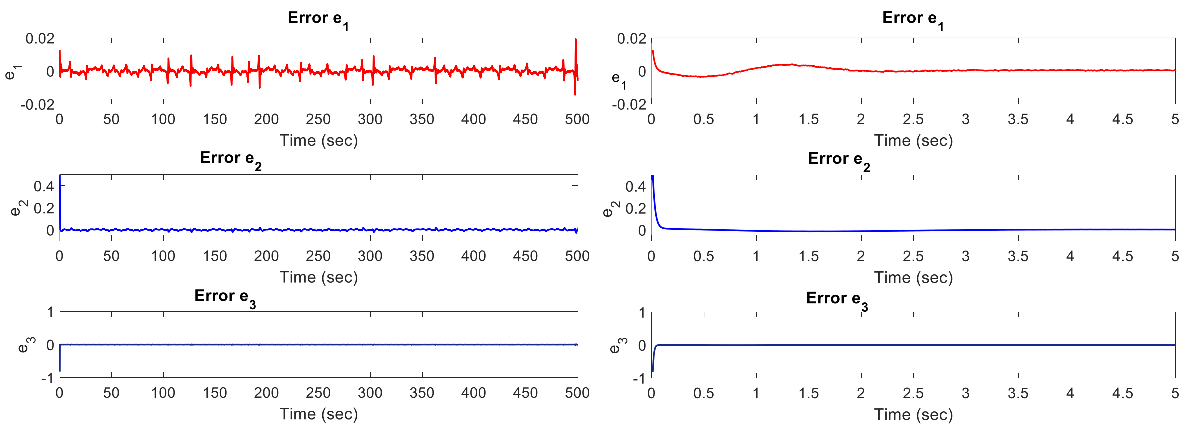


Figure 1. *3D trajectories of Newton- Leipnik fraction order chaotic system*



(a) (b)

Figure 2. *(a) Tracking trajectory of each channel in Newton-Leipnik fraction order chaotic system and (b) its zoom in 5 second.*



(a) (b)

Figure 3. (a) *Tracking error of each channel in Newton-Leipnik fraction order chaotic system and (b) its zoom in 5 second.*

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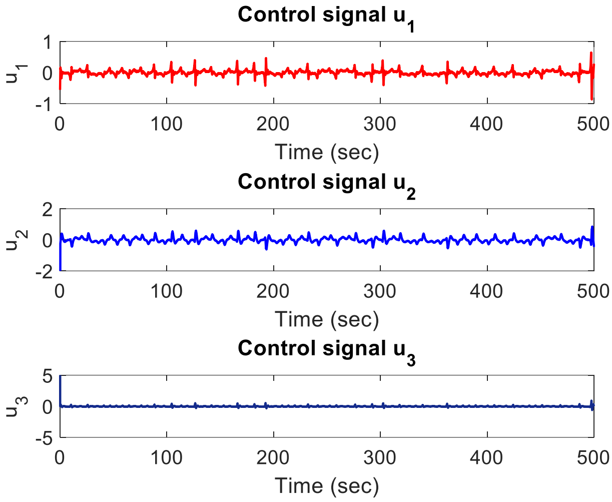

Figure 4. *Control signal of each channel in Newton- Leipnik fraction order chaotic system*

The root mean square error (RMSE) values are $RMSE_1$=0.0056, $RMSE_2$= 0.0135, $RMSE_3$ = 1.7626e-04 for each channel. These results are so low that we can summarize that the SMC method is effective. Furthermore, comparing with the PID method in [18] in Table I as below. The significantly lower RMSE values for SMC, particularly the average of 0.0064 compared to 0.0183 for PID, indicate that the SMC method provides much more accurate synchronization across all channels. This demonstrates that the proposed SMC approach is not only effective but also outperforms traditional PID control methods in terms of precision.

Table 1. *RMSE values for synchronization*

Method	$RMSE_1$	$RMSE_2$	$RMSE_3$	Average RMSE
Proposed SMC	0.0056	0.0135	1.7626e-04	0.0064
PID [18]	0.016	0.028	0.011	0.0183

4. Applying chaotic synchronization to image secure communication

After synchronization chaotic system by SMC method, the proposed algorithm is used for color image secure communication. This method combines 5 steps as follows.

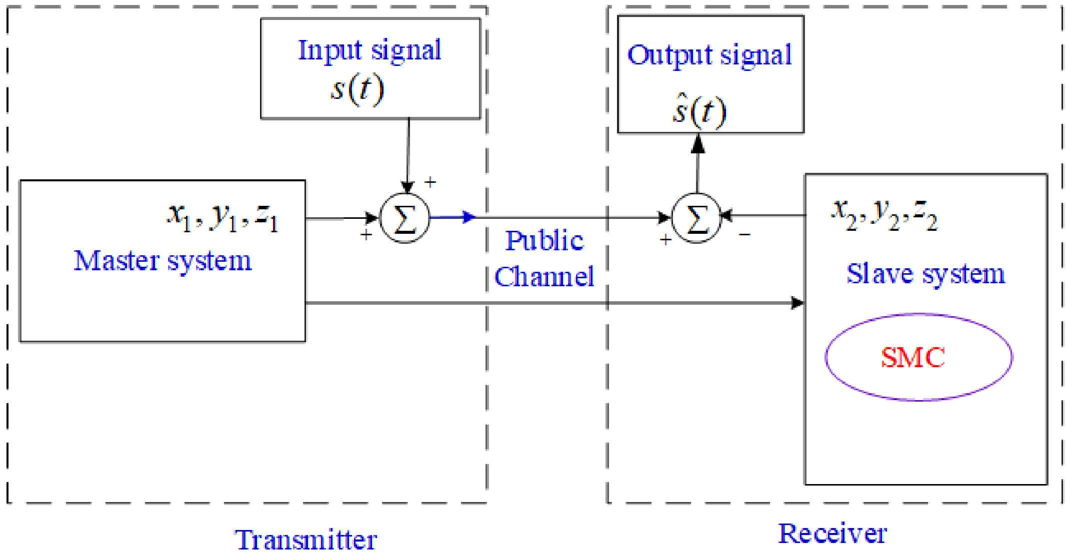

Figure 5. *Secure color image communication using Master and slave chaotic synchronization and sliding mode control method.*

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Step 1: Discretize the continuous chaotic with the sampling time h=0.001 (s).

Step 2: Encrypt the original image with RGB (red-green-blue) method and convert the RGB image into normalization signal, called input signal $s(t)$.

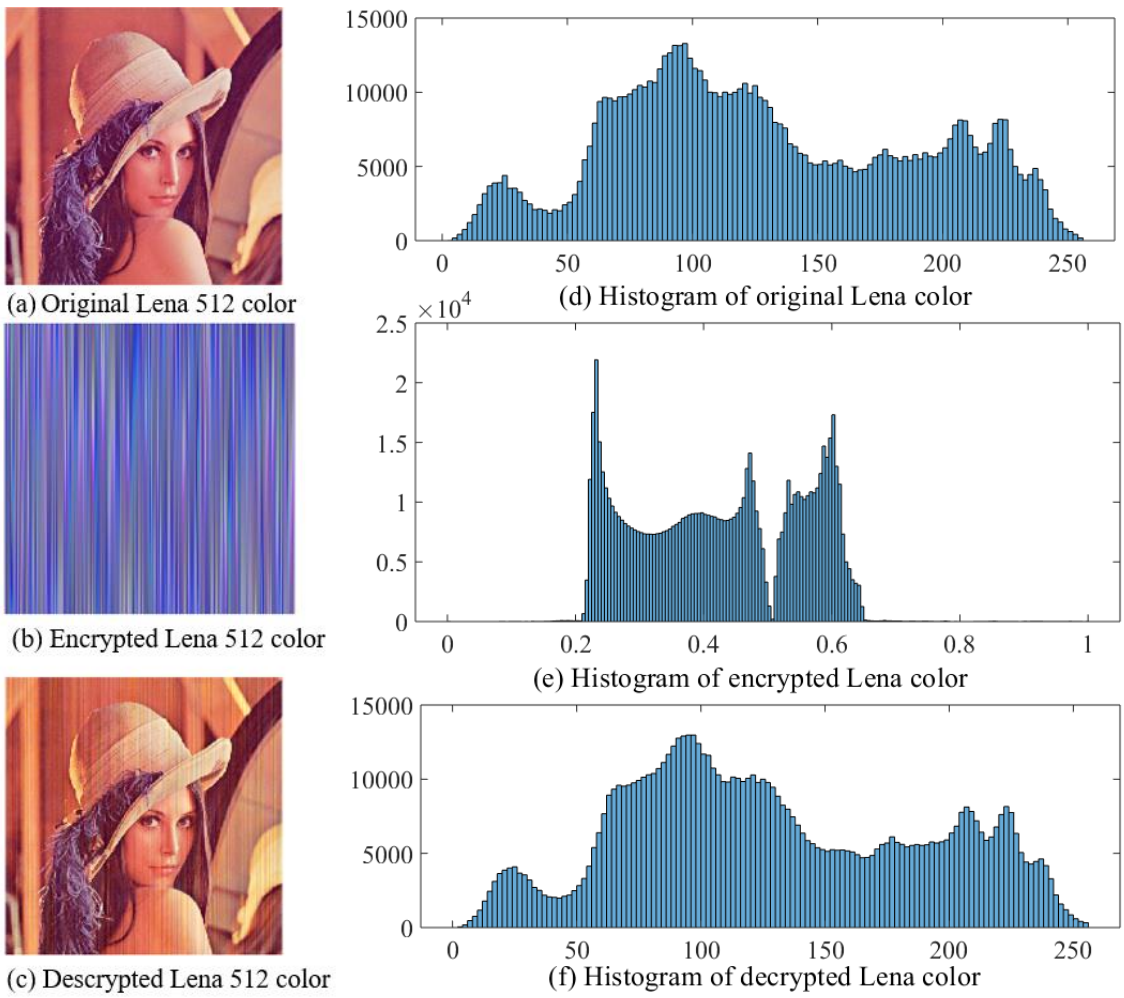
Step 3: Send the input signal $s(t)$ to the public channel.

Step 4: Once the synchronization is completed, the message is the result at the receiver. By subtraction, the synchronized chaotic signal is removed from the message and we get the decrypted image.

Step 5: Test the results.

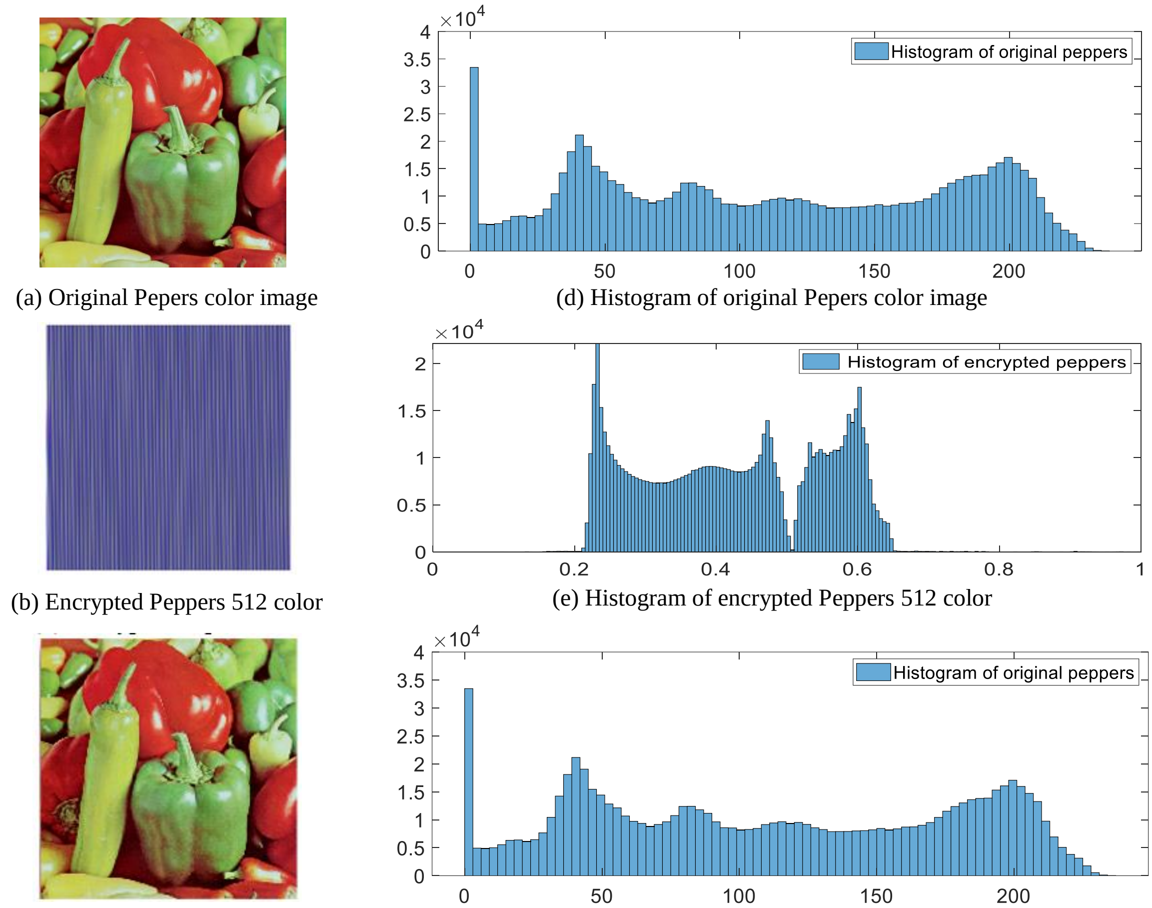
In summary, figure 5 shows the secure color image communication using master and slave chaotic synchronization and sliding mode control method as below.

In this section, we choose two standard color images: Lena with a size of 512x512 pixels and Peppers with a size of 512x512 pixels. And the simulation results of the color image secure communication of the proposed method are presented as below in figure 5 for Lena image, and in figure 6 for Peppers image.



Case 1: Lena 512 color picture

Figure 6. *The simulation for Lena image*.



(a) Original Pepers color image

(d) Histogram of original Pepers color image

(b) Encrypted Peppers 512 color

(e) Histogram of encrypted Peppers 512 color

(c) Decrypted Peppers 512 color

Case 2: Peppers 512 color picture

(f) Histogram of decrypted Peppers 512 color

Figure 7. *The simulation for Peppers image*

For the chaotic 3D system, we use all channels x, y and z. The histogram analysis show that there is a large difference in the pixel distribution of the original color image with the encrypted image (see and compare Figure 6 (e) with Figure 6 (d) and see and compare Figure 7 (e) with Figure 7 (d)). And the histogram of the decoded color image is approximately similar to the histogram of the original image (see and compare Figure 6 (f) with Figure 6 (d), and see and compare Figure 7 (f) with Figure 7 (d)). The results point out the proposed secure communication is safe and active.

Mean square error (MSE) and peak signal-to-noise ratio (PSNR)

The histogram analysis shows a clear difference in the pixel distribution of the encrypted image. However, for a more accurate evaluation of the proposed encryption algorithm, the differences between the original image and the encrypted image were calculated. MSE and PSNR are used to measure the distances between the pixels.

$$\text{MSE} = \frac{1}{(N \times M)} \sum_{i=1}^{N} \sum_{j=1}^{M} [I(i,j) - E(i,j)]^2 \tag{14}$$

$$\text{PSNR} = 10 \times log_{10}\left(\frac{I_{Max}^2}{\text{MSE}}\right) \tag{15}$$

where $(N \times M)$ is the number of pixels in the image. $I_{Max} = 255$ is the maximum pixel value of the encrypted image. And $I(i, j)$ and $E(i, j)$ are respectively the pixel gray value of the original and encrypted images at a location (i, j). MSE for the original and encrypted images for Lena and Pepers are respectively 1.2459 x 10^4 and 1.3773 x 10^4. The PSNR between the proposed method and lossless color image encryption method [16] and [17] is presented in Table 2. Analysis results indicate that our method achieves better performance and quality than the encryption method in [19] and [20].

In addition to the histogram analysis, the differences between the original and encrypted images were measured using MSE and PSNR. MSE values for the original and encrypted images of Lena and Peppers were 1.2459 x 10^4 and 1.3773 x 10^4, respectively. PSNR comparisons with methods from [19] and [20] are shown in Table 2, indicating that the proposed method outperforms these in terms of encryption quality. Specifically, our method demonstrated better performance in both MSE and PSNR, leading to higher security and image quality during encryption and decryption

To further enhance the comparison, our method not only excels in PSNR, achieving values such as 8.1080 and 7.6040 for the original-encrypted images of Lena and Peppers, but also shows the infinite PSNR for original-decrypted images, which matches methods [19] and [20]. However, the slight differences in the encrypted image PSNR values between methods [19] and [20] indicate that while the performances are similar, the proposed method demonstrates a more balanced and secure approach to color image encryption.

This comparative analysis highlights that the proposed SMC-based method provides a superior balance between encryption strength and image quality retention, especially when compared to DNA-based and chaotic system approaches in other recent studies.

Table 2. *PSNR of the encrypted images for different methods*

Images	PSNR (original-encrypted images)			PSNR (original-decrypted images)		
	Our method	Image encryption method [19]	Method [20]	Our method	Image encryption method [19]	Method [20]
Lena	8.1080	8.1293	8.7084	∞	∞	∞
Peppers	7.6040	7.6393	8.2003	∞	∞	∞

5. Conclusions

This study proposed a sliding mode control (SMC) method for synchronizing master-slave chaotic systems with fractional order. Lyapunov stability theory confirmed the system's convergence to synchronization. MATLAB simulations showed that SMC outperforms conventional PID controllers, achieving lower RMSE. In secure color image communication, SMC also surpassed recent approaches. Static analysis, including histogram, MSE, and PSNR, validated the system's security and efficiency, making it suitable for practical applications. Future work will focus on developing advanced observers to enhance synchronization accuracy in complex chaotic systems.

Conflict of Interest

The authors declare no conflict of interest.

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