

Probability Distribution of Solar Radiation to Determine the Output Power of the PV System Under Uncertain Conditions

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ABSTRACT

This study introduces a probabilistic modeling framework for solar irradiance that aims to enhance the output power stability of photovoltaic (PV) systems by explicitly accounting for the stochastic nature of solar radiation. Real – world irradiance measurements are first preprocessed and smoothed to remove noise and short – term fluctuations, thereby improving the reliability of statistical estimation. Three probability distributions Weibull, Beta, and Lognormal – are modeled, with their parameters estimated through the Maximum Likelihood Estimation (MLE) method. The suitability of each distribution is assessed using multiple statistical performance indicators, including Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and the coefficient of determination (R^2). Comparative analysis reveals that the Beta distribution provides the highest degree of fit and the lowest prediction error, making it the most appropriate model for representing the temporal variability of solar irradiance in the given dataset. Synthetic irradiance samples generated from the optimal distribution are subsequently integrated into a 250 kW grid – connected PV array model implemented in MATLAB/Simulink to evaluate power output characteristics under uncertain meteorological conditions. Simulation results confirm the method’s ability to capture dynamic fluctuations in solar energy availability and support robust power prediction. The proposed approach can assist in power system analysis, operational planning, and decision-making for PV integration, thereby contributing to improved grid stability and energy management strategies in renewable-rich environments.

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1. Introduction

In the context of increasing global energy demand and the gradual depletion of fossil fuel resources, renewable energy has emerged as a highly important and sustainable solution [1]. With its clean, abundant, readily available [2], and environmentally friendly characteristics, renewable energy – particularly solar energy – is considered one of the most viable alternatives. However, the large-scale exploitation and utilization of solar energy still face significant challenges, especially in ensuring the quality of electricity supplied to the grid. Consequently, accurately forecasting solar irradiance is essential for stabilizing the output power of photovoltaic systems.

To address this issue, recent studies have increasingly focused on modeling the probability distribution of solar irradiance to improve the accuracy of output power forecasting in PV systems. Research in [3], [4] has developed probability distribution curves for different mean values, which are then used to estimate the short – term average intensity of solar irradiance. Furthermore, several studies [5], [6] have shown that the Weibull distribution is highly effective for modeling right – skewed and long – tailed data, making it a useful tool for representing solar irradiance in temperate climate regions. In addition, the Beta distribution [7]–[9], with normalized data in the range (0, 1) and flexible shape parameters, can simulate a wide variety of weather conditions. However, these studies primarily address the uncertainty of solar energy without explicitly considering the stability of the generation output. Other

works [10], [11] have demonstrated that applying probability distributions not only quantifies risks but also facilitates the assessment of potential output power ranges of PV systems. This is particularly valuable for operational planning and ensuring output stability under fluctuating weather conditions. Notably, studies [12]–[15] highlight that the Lognormal distribution offers mathematical simplicity while effectively capturing the variability of solar irradiance influenced by multiple complex meteorological factors. These findings create opportunities to optimize solar energy utilization, enhance system reliability, and assist operators in operational planning.

This study proposes selecting the most suitable probability distribution function (PDF) for modeling solar irradiance. In this work, one year of solar irradiance data from the National Solar Radiation Database (NSRDB) is used as simulation data, which is then applied for forecasting subsequent days. The input parameters are processed using the Maximum Likelihood Estimation (MLE) method [16]. The goodness of fit is evaluated using statistical metrics, including mean squared error (MSE), mean absolute error (MAE), coefficient of determination (R^2), and root mean squared error (RMSE). The output power of the PV system is directly determined by inputting the forecasted solar irradiance data into a PV array model developed in MATLAB.

2. Method

The stochastic and uncertain nature of solar irradiance is typically represented using a probability density function (PDF). To model the uncertainty in solar irradiance, three probability distributions Weibull, Beta, and Lognormal – are employed to determine the best fit. These distributions are used to generate irradiance samples from reference data at a specific location for power estimation through a PV array system model. Figure 1 illustrates the overall sequence of steps in the proposed study.

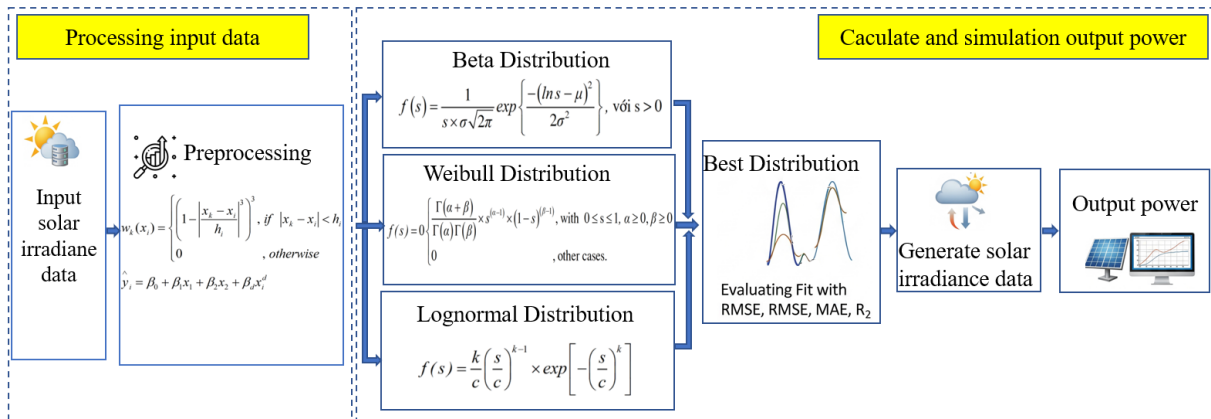


Figure 1. Process of Analyzing Solar Irradiance Probability Distributions and Their Application in PV System Power Simulation.

2.1. Data preprocessing

Prior to simulation, the data must be preprocessed to ensure integrity and accurately reflect physical trends. In this study, solar irradiance data are converted into a one – dimensional vector, with undefined or physically meaningless values removed. The data are then smoothed using the Local Regression (Loess) algorithm [17] with a window size of 15. The Loess method performs weighted low – degree polynomial regression on neighboring data points.

At each point, the algorithm selects the $r = 15$ nearest surrounding points and assigns weights using the Tricube function.

$$w_k(x_i) = \begin{cases} \left(1 - \left|\frac{x_k - x_i}{h_i}\right|^3\right)^3, & \text{if } |x_k - x_i| < h_i \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Subsequently, a linear regression function is fitted by minimizing the weighted sum of squared errors. The smoothed value at data point i is estimated as:

$$\hat{y}_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_d x_i^d \quad (2)$$

Where β_j are the regression coefficients internally determined from the data within the window. This approach reduces random noise in the irradiance data while preserving the actual trend.

2.2. Probability distributions

In this study, three probability distributions Weibull, Beta, and Lognormal – are employed to describe the stochastic characteristics of solar irradiance. These are applied after the input data are processed and are expressed from [18]–[20] as follows:

$$\text{Lognormal distribution } f(s) = \frac{1}{s \times \sigma \sqrt{2\pi}} \exp\left\{-\frac{(\ln s - \mu)^2}{2\sigma^2}\right\}, \text{ with } s > 0 \quad (3)$$

$$\text{Where, } \hat{\mu} = \frac{1}{n} \sum_{i=1}^n \ln(s_i) \quad \text{và} \quad \hat{\sigma} = \frac{1}{n} \sum_{i=1}^n (\ln(s_i) - \hat{\mu})^2 \quad (4)$$

$$\text{Beta distribution } f(s) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \times s^{(\alpha-1)} \times (1-s)^{(\beta-1)}, & \text{with } 0 \leq s \leq 1, \alpha \geq 0, \beta \geq 0 \\ 0 & \text{, other cases.} \end{cases} \quad (5)$$

In which, α and β are the parameters of the Beta distribution calculated using the mean and standard deviation of the random variable as follows:

$$\alpha = \frac{\mu \times \beta}{1 - \mu}, \quad \beta = (1 - \mu) \times \left(\frac{\mu(1 + \mu)}{\sigma^2} - 1 \right) \quad (6)$$

$$\text{Where, } \mu = \frac{1}{N_d} \sum_{j=1}^{N_d} d_j, \quad \sigma = \sqrt{\frac{1}{N_d} \sum_{j=1}^{N_d} (d_j - \mu)^2} \quad (7)$$

$$\text{Weibull distribution } f(s) = \frac{k}{c} \left(\frac{s}{c}\right)^{k-1} \times \exp\left[-\left(\frac{s}{c}\right)^k\right] \quad (8)$$

Where, s is the solar irradiance, k is the shape parameter, and c is the scale parameter, where $k > 0$ and $c > 0$.

2.3. Goodness-of-Fit Metrics

The selection of an appropriate distribution for the model plays a decisive role; therefore, the following methods are proposed: Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and the Coefficient of Determination (R^2), as implemented in [21]. These methods are expressed as follows:

$$\text{Mean Squared Error: } MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (9)$$

$$\text{Root Mean Squared Error: } RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2} \quad (10)$$

$$\text{Mean Absolute Error: } MAE = \frac{1}{n} \sum_{i=1}^n |Y_i - \hat{Y}_i| \quad (11)$$

Where, n is the number of predictions, Y_i is the actual value, \hat{Y}_i is the predicted value.

$$\text{Coefficient of Determination (R}^2\text{): } R^2 = 1 - \frac{SSR}{TSS} \quad (12)$$

Where, SSR denotes the Sum of Squares of Residuals, and TSS denotes the Total Sum of Squares. The R^2 value ranges from 0 to 1.

These quantitative criteria serve as the basis for identifying the optimal distribution model, which is then used to generate irradiance input samples for power simulation in Simulink.

3. Results

The model is based on a 250 – kW photovoltaic (PV) system, which is used for simulation and analysis to determine the grid-integrated output power from solar irradiance data. The system is configured with an MPPT controller to maximize output power. The DC power is then converted into three – phase AC through a three – level IGBT inverter, with harmonic filtering performed by an inductor and capacitor at the inverter output. The generated energy is supplied to both loads and the grid, ensuring stable operation through current and voltage control, as illustrated in Figure 2.

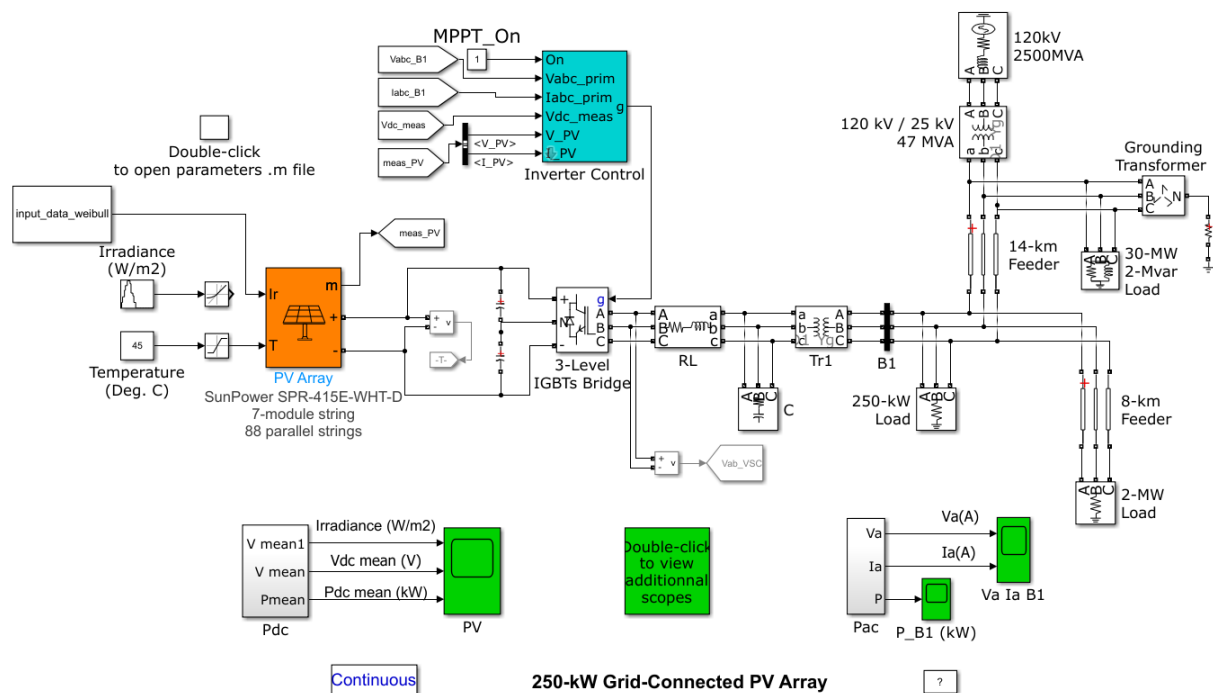


Figure 2. Simulation model for determining the output power of the 250 – kW PV system.

The simulation model has a DC – side nominal voltage of 480 V and is connected via a three – phase inverter and a 250 – kVA transformer (250 V / 25 kV) to deliver power to the 25 – kV medium – voltage grid. Mixed load branches of 250 kW, 2 MW and 30 MW, together with a reactive component of 2 MVar, are connected and conveyed over transmission lines of 8 km and 14 km. Detailed configuration and technical specifications of the 250 – kW PV system are presented in Table 1.

Table 1. Technical specifications of the 250 – kW grid – connected PV system.

Specification	Value
Module type	SPR-415E-WHT-D
Number of modules	7 × 88 (string × parallel)

Total maximum power	~252.2 kW
Voc/Isc	85.3 / 6.09 (V/A)
Vmp/Imp	72.9 / 5.69 (V/A)
System frequency	60 Hz
Rated voltage (AC/DC)	480 V (DC), 25 kV (AC – grid)
Transformer capacity	250 kVA
Fixed load	250 kW
Large load	2 MW, 30 MW
Reactive power component of load	2 MVar

First, the input data must be preprocessed and smoothed prior to simulation based on (1) and (2) before performing the probability density function calculations. The results of this preprocessing step are shown in Figure 3 below:

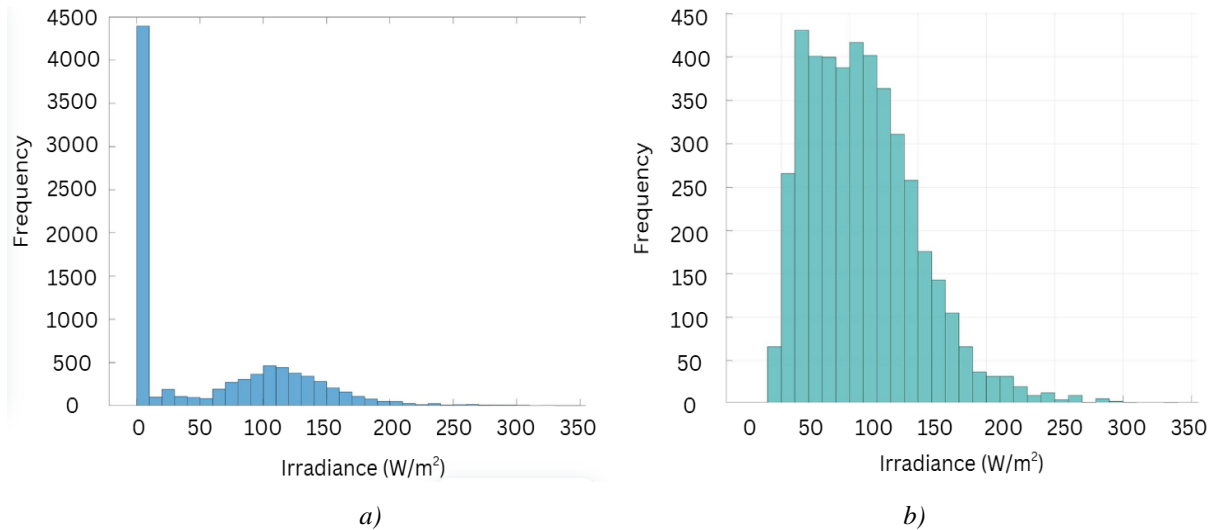


Figure 3. Chart of input data before and after data processing: (a) Raw data; (b) Processed data.

Through the solar irradiance preprocessing steps, the parameters of the three probability distributions – Beta, Lognormal, and Weibull – are presented in Table 2, based on Eqs. (4), (6) and (7), respectively. The parameters are estimated from the processed irradiance data using the method of moments and Maximum Likelihood Estimation (MLE).

Table 2. Parameter values for solar irradiance data.

Distribution	Mean (μ)	Standard deviation (σ)	Parameter 1	Parameter 2
Lognormal	$\mu = 4.637$	$\sigma = 0.368$	–	–
Beta	$\mu = 0.191$	$\sigma = 0.113$	$\alpha = 2.102$	$\beta = 8.879$
Weibull	–	–	$k = 2.718$	$c = 124.172$

The PDFs of each distribution were constructed from the estimated parameters and directly compared with the histogram of the actual data. The comparison chart is presented in Figure 4, which demonstrates a high degree of similarity between the theoretical distribution curves and the empirical distribution for all three distributions.

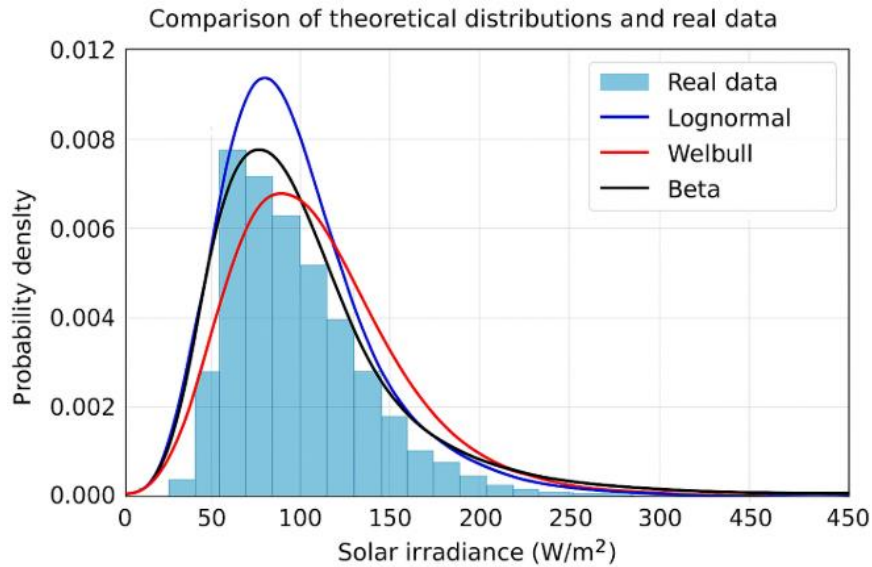


Figure 4. Actual solar irradiance data and probability density function (PDF) curves.

Statistical indices for evaluating the goodness-of-fit between distribution models and actual data are calculated using Equations (9) – (12) and summarized in Table 3.

Table 3. Results of the goodness-of-fit test.

	Weibull	Lognormal	Beta
MSE	5.952×10^{-7}	3.4198×10^{-7}	1.3716×10^{-7}
RMSE	0.00077149	0.00058479	0.00037035
MAE	0.0004559	0.00030046	0.00022172
R²	0.94868	0.97052	0.98817

The summary table of test results shows that the Beta distribution achieves the highest performance among the three models considered. Specifically, this model yields the lowest RMSE value of 0.00037035 and the smallest MAE of 0.00022172, along with an R² value of 98.8%, the highest among the surveyed distributions.

Although the Lognormal distribution also demonstrates a relatively good fit with an R² of 97%, its error values – RMSE of 0.00058479 and MAE of 0.0003046 – are still significantly higher, especially in the higher value ranges of the distribution.

In contrast, the Weibull model shows the largest error with an RMSE of 0.00077149 and an R² of 95%, reflecting poorer fitting capability in both the central and tail regions of the data.

These results confirm that the Beta distribution not only ensures a good fit across the entire data range but also demonstrates stability in controlling deviations. Therefore, the Beta distribution is selected as the optimal model for generating input solar radiation data series for the PV system power simulation process.

The effectiveness of the solar radiation sampling model is evaluated using hourly solar radiation data collected over one year from the NSRDB database at Thu Duc City, time zone UTC + 7. The inverse CDF sampling method is applied to generate samples from the probability distributions. This process helps ensure the continuity of the model. The parameters of the distributions used to calculate solar radiation samples from 7:00 AM to 5:00 PM are shown in Figure 5. The block diagram of the solar radiation sampling process is presented in Figure 2.

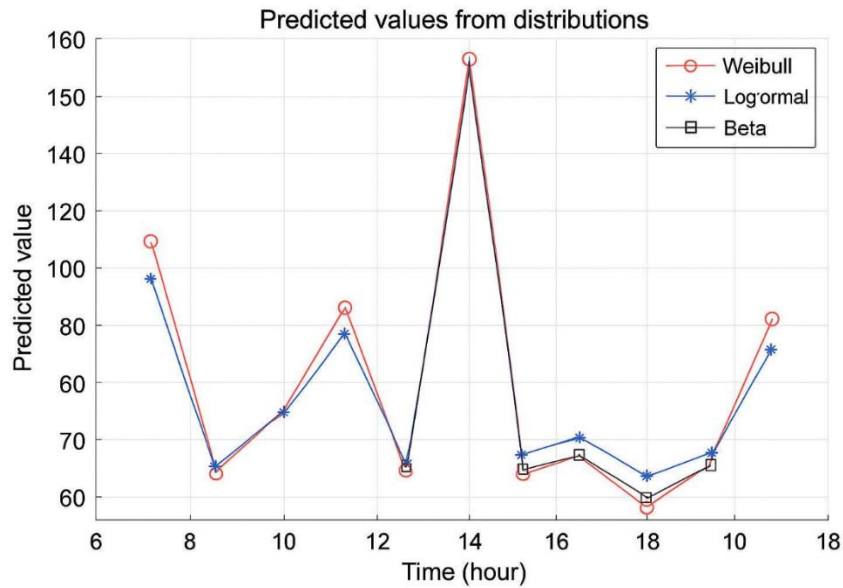


Figure 5. The results of solar radiation sampling at different times of the day using the Weibull, Lognormal, and Beta distribution sampling methods.

The output power of the PV system over the course of the day, using the solar radiation series generated from the optimal Beta distribution, is shown in Figure 6.

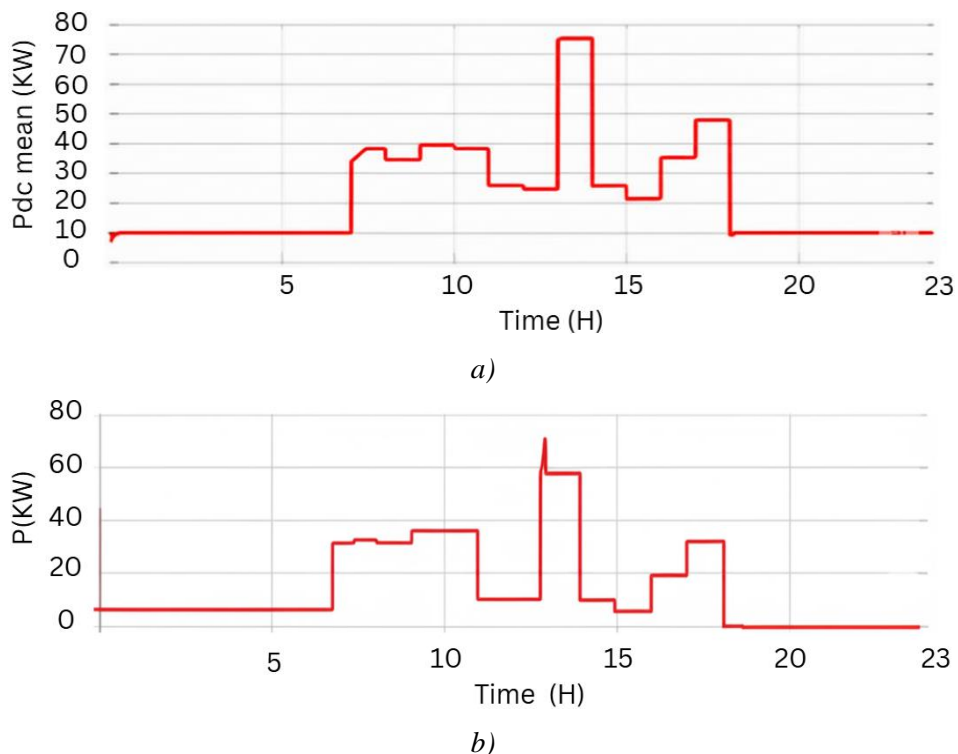


Figure 6. Power chart for a typical day: (a) Output power at the PV array; (b) Output power at Bus B1.

The output power at the PV array is shown in Figure 6 (a), with the average power (Pdc mean) ranging from 0 to 70 kW over the 20 hour observation period. The peak power occurs around midday, reflecting alignment with optimal solar radiation intensity. The power fluctuations confirm the uncertainty inherent in renewable energy sources, which is effectively modeled using the Beta distribution. Figure 6 (b) presents the power at Bus B1, where the power peaks at approximately 75–80 kW around midday, with variations well controlled thanks to the efficient MPPT controller that transfers power from the PV array to the grid. These results reinforce the role of the Beta distribution in optimizing

and stabilizing the output power of the solar power system, providing a scientific basis for operational planning and performance optimization of solar power systems under complex weather conditions.

Although the output power values obtained from the simulation are ideal, they may deviate from actual PV system measurements due to inverter conversion losses, MPPT tracking limitations, transformer losses, and the impact of temperature on the panels. However, the comparative assessment from the probability distributions remains valid. This is due to the suitability of the Weibull, Lognormal, and Beta distributions in representing the random nature of solar radiation, which is independent of the hardware characteristics of the PV system. Therefore, statistical evaluation ensures that the conclusions drawn about the distribution performance remain relevant to the output power, even in the presence of differences between simulated and actual PV output.

4. Discussions

The analysis results indicate that the Beta distribution demonstrates superior capability in modeling the characteristics of solar radiation in the study area compared to the Weibull and Lognormal distributions. The Beta distribution achieves the lowest RMSE and MAE values, while the coefficient of determination (R^2) reaches 98.8%, reflecting a very high degree of fit with the empirical distribution. The Weibull distribution ranks second, exhibiting a relatively high R^2 but a larger RMSE, primarily due to moderate deviations near the peak of the distribution. In contrast, the Lognormal distribution shows the poorest performance, with noticeable errors in the tail region, resulting in higher RMSE and MAE values.

The superior performance of the Beta distribution can be attributed to its flexibility in modeling random variables bounded within the interval (0, 1), which corresponds to the natural limits of normalized solar radiation. In contrast, although the Weibull distribution is widely used in wind and solar studies, its distribution shape is less adaptable to data characterized by high peakedness and near – symmetrical distribution. The Log-normal distribution exhibits a long right tail, causing the model to overestimate the frequency of high solar radiation values compared to observed reality. This is evident in the PDF graph, where the Lognormal curve exceeds the empirical distribution in the extreme region, and is further supported by the highest RMSE and MAE values among the three models, indicating greater deviation in the tail region of the distribution.

Integrating the parameters of the Beta distribution into the PV model in Simulink enables the reproduction of output power variations that closely reflect real conditions. The small fluctuations observed in the daily average power indicate a nonlinear relationship between solar radiation variability and PV power output, underscoring the importance of accurately modeling the radiation distribution for effective power forecasting and system operation optimization.

More importantly, the results highlight how uncertainties in solar radiation propagate into PV system behavior. Variability in irradiance translates into fluctuations in output power, which in turn can cause voltage instability, frequency deviations, and reduced power quality in grid-connected systems. Without explicitly considering these uncertainties, the stability of PV-integrated power grids may be overestimated. By employing probability distributions, the study provides a quantitative framework to evaluate such risks and supports more reliable system planning and operation.

The research process was carried out in three steps: data preprocessing, parameter estimation of the three distributions using the Maximum Likelihood Estimation (MLE) method, and evaluation through RMSE, MAE, and R^2 indices to select the optimal model, followed by PV power simulation based on solar radiation series generated from the selected distribution. This approach not only enhances the reliability of simulation results but also helps reduce the risk of inaccurate power output estimation in the operation of PV – integrated power grids.

The limitations of this study lie in the assumption of an ideal PV model, neglecting inverter conversion losses, MPPT tracking errors, transformer losses, and temperature effects. In reality, inverter inefficiencies reduce AC output, MPPT algorithms may deviate under fast irradiance changes, transformer impedance causes extra dissipation, and higher temperatures lower PV efficiency. These factors explain why simulated PV output can appear higher and more stable than actual measurements.

Nonetheless, as this work focuses on comparing irradiance probability distributions rather than control strategies, the relative evaluation among Weibull, Beta, and Lognormal distributions remains valid.

Future research should prioritize enhancing the PV model by integrating realistic inverter performance, MPPT algorithm dynamics, transformer characteristics, and thermal effects, thereby enabling simulated outputs to more accurately reflect actual operating conditions. Expanding the dataset across diverse regions, climatic conditions, and extended temporal scales, in combination with evaluating additional probability distributions (e.g., Gamma, Pearson, Burr), would allow for a more comprehensive assessment of model applicability. Furthermore, coupling probabilistic PV power models with scheduling, reserve allocation, and energy storage strategies is expected to improve the robustness and reliability of PV-integrated grid systems.

5. Conclusions

This study has demonstrated the effectiveness of using probability distribution functions such as Weibull, Beta and Lognormal to model the uncertainty of solar radiation. Instead of assuming a fixed average input value, this approach allows for a more realistic representation of the random nature and continuous fluctuations of solar radiation in practice.

The integration of probability distributions into the solar power system simulation on Simulink has provided a clear visualization of the impact of uncertainty on output power, with high accuracy confirmed by statistical indicators such as low RMSE and MAE values and an R^2 coefficient of up to 0.988.

By explicitly modeling uncertainty, the study also clarifies how variability in solar irradiance can influence PV system stability. Output fluctuations caused by random irradiance changes may propagate into the grid, affecting voltage and frequency stability. Recognizing these effects is crucial for designing adaptive control strategies, planning grid integration, and ensuring reliable operation of PV-based energy systems.

The research results not only improve the reliability of PV system power forecasting but also hold practical significance in risk assessment, operational optimization and grid development planning amid increasingly complex climate variability. Modeling uncertainty is a crucial step toward achieving a stable, flexible and well – adaptive solar power system that can respond effectively to changing environmental conditions.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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