JTE
HCMUTE
ISSN: 2615-9740

JOURNAL OF TECHNICAL EDUCATION SCIENCE
Ho Chi Minh City University of Technology and Education
Website:https://jte.hcmute.edu.vn/
Email:jte@hcmute.edu.vn

Multi-Objective Optimization of Vacuum Frying Process Conditions for Jackfruit Using the Restricted Area Method (RAM) With Combination Criteria R.

Lieu Le Quang[1], Liviu Giurgiulescu[2], Suc Nguyen Van[3], Dzung Nguyen Tan[3*]
[1]*College of Fisheries and Food Technology, Vietnam*
[2]*Technical University of Cluj Napoca, North Universitary Center of Baia Mare, Romania*
[3]*Ho Chi Minh City University of Technology and Education, Vietnam*

[*]Corresponding author. *Email: tandzung072@hcmute.edu.vn. Tel: 0918801670*

ARTICLE INFO

Received:	08/09/2025
Revised:	17/09/2025
Accepted:	22/09/2025
Published:	28/11/2025

KEYWORDS

Vacuum frying;
Vacuum frying technology;
Vacuum frying technique;
Vacuum-fried jackfruit;
Multi-objective optimization.

ABSTRACT

This study reports the development and solution of a multi-objective optimization problem using the restricted area method with Combination Criteria R, aimed at determining the optimal process conditions for vacuum frying for jackfruit. Under the optimal conditions, the obtained product simultaneously achieved the lowest possible energy consumption, while meeting the required moisture content, exhibiting an attractive bright yellow color, a crisp and porous texture, and maintaining acrylamide levels within the permissible limit. Experiments were conducted to establish five objective functions: y_1 (kWh/kg) – energy consumption; y_2 (%) – product moisture content; y_3 – color difference index between the product and the standard sample; y_4 – product porosity, y_5 (µg/kg) – acrylamide content after frying, all of which depend on three process variables: Z_1 (°C) – vacuum frying medium temperature; Z_2 (kPa) – vacuum frying chamber pressure; Z_3 (min) – vacuum frying time. Based on this, a multi-objective optimization problem (29) was formulated, solved, and experimentally validated. The results identified the optimal process conditions as: Z_1 = 96.97^{0}C; Z_2 = 9.65 kPa; Z_3 = 18.77 min, corresponding to y_1 = 2.10 kWh/kg; y_2 = 2.30%; y_3 = 0.88, y_4 = 0.24, y_5 = 76.68 µg/kg. These findings demonstrate that vacuum frying under the optimal conditions provides products with near-minimal energy consumption, desired moisture content, appealing bright yellow color, crisp and porous structure, and acrylamide levels lower than the permissible limit.

Doi: https://doi.org/10.54644/jte.2025.1996

1. Introduction

Vacuum frying is a process carried out under pressures lower than atmospheric pressure, which lowers the boiling point of water in the food and allows frying at temperatures lower than those of conventional frying. As a result, the rate of oil oxidation, the Maillard reaction, and other undesirable thermal degradations are suppressed; meanwhile, oil uptake is reduced, and the natural color, texture, and nutritional components (such as vitamins, polyphenols, carotenoids, etc.) are better preserved. Moreover, the formation of acrylamide and furan is significantly reduced compared to atmospheric frying, [1]-[2]. Both classical and recent studies on vacuum frying of raw materials such as potato, apple, carrot, and mango have demonstrated that the resulting products exhibit superior quality compared to those obtained under atmospheric frying [2]. Specifically, vacuum-fried products show lower oil uptake, brighter and more appealing color, crispier and more porous texture, and up to ~90–98% reduction in acrylamide content, depending on the type of raw material and processing conditions, [3]-[6].

Under vacuum conditions, the core temperature of the material rapidly reaches the boiling point of water corresponding to the frying pressure, and then remains almost constant until the moisture evaporation process is completed; thereafter, it gradually increases toward the oil temperature. This mechanism, together with the reduction in the number of pores within the product structure, is considered a key factor in limiting oil uptake during vacuum frying, especially after centrifugation is applied to remove excess surface oil, [1], [3], [7].

Results from numerous studies have confirmed that vacuum frying (at temperatures of 80–110 °C and pressures of 8–20 kPa) helps suppress oxidation and occurs at lower temperatures, thereby yielding products with higher L values (brighter color) and less browning compared to atmospheric frying [1], [3]. In addition, vacuum frying produces products with a crispier and more porous texture at the same moisture content, due to the formation of a favorable micro-porous structure and reduced tissue damage compared to conventional frying [1], [8]. This method also contributes to better retention of nutritional value, as heat- and oxygen-sensitive compounds (such as vitamins, carotenoids, and polyphenols) are significantly less degraded [2]. Notably, many studies have also reported that vacuum frying markedly reduces acrylamide content (by approximately 90–98% in potato chips, depending on the raw material and processing conditions), owing to the combined effects of lower frying temperature and reduced oxygen concentration in the frying medium [1], [5], [6], [9].

Jackfruit (*Artocarpus heterophyllus*) is a tropical fruit with high nutritional value. The major chemical composition per 100 g of fresh pulp includes: water (72–77 g), carbohydrates (glucose, fructose, sucrose, and starch) (18–25 g), protein (1.2–2.0 g), fat (0.1–0.6 g), dietary fiber (1.0–2.5 g), vitamins (A, C, and B group) (8–20 mg), and minerals such as K (200–450 mg), Ca (15–30 mg), Mg (25–45 mg), with an energy content of 90–110 kcal. Ripe jackfruit is rich in easily digestible sugars that provide rapid energy, and it also contains antioxidants such as carotenoids and polyphenols, which are beneficial to human health. The economic value of jackfruit has been increasing due to the growing domestic and export demands in diverse forms (fresh fruit, dried products, canned goods, and snacks). However, postharvest jackfruit is highly perishable because of its high moisture content and intense physiological activity; therefore, appropriate preservation methods such as refrigeration, packaging treatment, or immediate processing after harvest are required to maintain quality.

Traditionally, postharvest jackfruit has mainly been preserved by two methods: refrigeration and drying, for domestic consumption, trade, and export. However, these methods have not substantially enhanced the value of jackfruit. Recently, many enterprises have adopted vacuum frying technology to produce high-quality products, thereby contributing to an increase in the economic value of jackfruit.

To date, many studies have been conducted on vacuum frying techniques and technologies. However, the development and solution of a multi-objective optimization problem to determine the optimal vacuum frying conditions—so that the product simultaneously achieves low energy consumption, the required moisture content, bright golden color, crisp and porous texture, and acrylamide levels within permissible limits—have not yet received adequate attention. This remains a practical issue in production that has not been thoroughly resolved.

Building upon this practical context, the present study focuses on formulating and solving a multi-objective optimization problem to determine the optimal vacuum frying conditions, thereby contributing to overcoming the current limitations in vacuum frying techniques and technologies.

2. Materials and Methods

2.1. Materials

- Yellow-fleshed jackfruit harvested from the Southeastern region of Vietnam was used. The ripe fruit has a bright yellow color, characteristic aroma, and sweetness. After harvest, the fruit was processed by removing the rind, fibrous tissues, and seeds; only the arils were collected, washed, and drained in preparation for frying. The proximate composition of jackfruit arils (per 100 g fresh weight) was as follows: moisture ~75.6 g, carbohydrates (glucose, fructose, sucrose, and starch) ~21.5 g, protein 1.68 g, lipid 0.43 g, dietary fiber 1.95 g, vitamins (A, C, and B group) ~9.2–16.7 mg, and minerals including potassium (212–430 mg), calcium (13–26 mg), magnesium (26–41 mg). The total energy content ranged from 92 to 107 kcal.
- The oil used as the heat transfer medium in vacuum frying is peanut oil that meets edible oil standards.

2.2. Research Equipment

- In this study, the main equipment used was a modern DVF-03 vacuum frying system, in which the frying process was controlled and monitored through a computer-based programmable system (It can be seen in Figure 1).

- LC-MS/MS (Liquid Chromatography – Tandem Mass Spectrometry, Model: Agilent 6460 Triple Quadrupole, Agilent Technologies, USA) was employed to determine the acrylamide content in the product.

- A spectrophotometer (Model: UV-1800 Spectrophotometer, Shimadzu, Japan) was used to determine the color spectrum.

- In addition, several other instruments were employed, including an infrared moisture analyzer for determining moisture content, a volumetric measuring device, an analytical balance, and a wattmeter for measuring energy consumption.

Figure 1. *DVF-03 vacuum frying system with automated control, operated and monitored through a computer-based programmable system [10]*

2.3. Determination of the technological parameters of the vacuum frying process

- To determine the temperature of the vacuum frying medium Z_1 (°C) and vacuum frying chamber pressure Z_2 (kPa), the DVF-03 vacuum frying system was equipped with temperature and pressure sensors, which enabled automatic measurement and control of these parameters throughout the frying process via computer-based monitoring, [10].

- To determine the vacuum frying time Z_3 (min), the system employed a computer-based timer preprogrammed in the control software, ensuring high accuracy, [10].

2.4. Determination of objective functions in the vacuum frying process

- **Determination of energy consumption**

A wattmeter was used to determine the energy consumption of the vacuum frying process of jackfruit. The energy consumption was calculated per 1 kg of vacuum-fried jackfruit product and was determined according to the following equation [10], [11]:

$$y_1 = P = \frac{U \times I \times \cos\varphi}{G}, \text{ kWh/kg} \tag{1}$$

In which: y_1 (kWh/kg) is the specific energy consumption per 1 kg of product; G (kg) is the mass of the jackfruit product; U (V), I (A), and cosφ are the voltage, current, and power factor of the power supply for the vacuum frying system, respectively, as measured by the integrated wattmeter.

- **Determination of moisture content of vacuum-fried jackfruit**

Moisture content of vacuum-fried jackfruit was determined using an analytical balance equipped with an infrared moisture analyzer. The moisture content of the product was calculated according to the following equation, [10], [12]:

$$y_2 = W(\%) = \frac{m_2 - m_0}{m_1 - m_0} \times 100,\ \% \tag{2}$$

In which: m_0 (g) – weight of the empty sample dish; m_1 (g) and m_2 (g) – weight of the sample before drying and weight of the sample after infrared radiation until a constant mass is reached, respectively, indicating that no further moisture is released and only dry matter remains.

- **Determination of the color difference index between the product and the reference sample**

$$y_3 = \Delta E^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \tag{3}$$

In which: ΔL^* denotes the difference in lightness; Δa^* denotes the difference along the green–red axis; Δb^* denotes the difference along the yellow–blue axis; ΔE^* is referred to as the total color difference between the product and the reference sample, which in this study is defined as the color attribute of the product, [11], [12].

- **Determination of product porosity**

$$y_4 = \varepsilon = 1 - \frac{\rho}{\rho_0} \tag{4}$$

Where: ρ (kg/m^3) – apparent density (calculated based on the external volume of the sample, without excluding the pore volume); ρ_0 (kg/m^3) – true density (calculated based on the external volume of the sample after excluding the pore volume); $0 \le y_4 < 1$, [11], [12].

- **Determination of true density (ρ_0) and apparent density (ρ) of the sample (kg/m^3):** First, the sample mass m_s (g) was measured. A volumetric device filled with edible oil was used to determine the initial volume V_1 (ml). The vacuum-fried jackfruit sample was then immersed in the oil and kept for a period of time to allow the oil to penetrate and fill the internal pores. The new volume V_2 (ml) was subsequently recorded. Thus, the true volume of the sample was calculated as follows:

$$V = V_2 - V_1 \text{ (ml)} \tag{5}$$

True density:

$$\rho_0 = \frac{m_s}{V} \text{ (g/ml or kg/m}^3\text{)} \tag{6}$$

Next, the oil-immersed sample was removed and drained (without drying, since drying would cause the oil inside the pores to evaporate). At this stage, the pores were completely filled with oil. The sample was then placed into another graduated container filled with cooking oil, in which the initial volume was V_1 (ml). After the sample was immersed, the oil volume increased to V_3 (ml). The apparent volume of the sample was therefore determined as:

$$V_s = V_3 - V_1 \text{ (ml)} \tag{7}$$

Apparent density:

$$\rho = \frac{m_s}{V_s} \text{ (g/ml or kg/m}^3\text{)} \tag{8}$$

- **Determination of acrylamide content in vacuum-fried products**

The acrylamide content in the product was determined using a modern analytical instrument, LC–MS/MS (Liquid Chromatography–Tandem Mass Spectrometry). The analytical procedure for acrylamide determination was as follows [13]:

- ✓ Prepare acrylamide standard solutions at different concentrations.
- ✓ Analyze by LC–MS/MS and record the peak area (from integration).

- ✓ Construct a calibration curve by plotting peak area versus concentration.
- ✓ Analyze the sample, obtain the peak area, and interpolate on the calibration curve to determine the acrylamide concentration.
- ✓ Multiply by the extract volume and divide by the sample mass to calculate the acrylamide content (µg/kg or ng/g).

Finally, the acrylamide content in the product is determined according to the following equation:

$$y_5 = \text{Acrylamide}(\mu g/kg) = \frac{C_{ext} \times V_{ext} \times DF \times 1000}{m_s} \times \frac{1}{R_{recov}}, \ \mu g/kg \tag{9}$$

Where: C_{ext} (µg/mL) – the concentration of the extract; V_{ext} (ml) – the volume of the extract; DF – the dilution factor (DF = 1 indicates no dilution); m_s (g) – the sample mass; R_{recov} – the recovery correction factor (Rrecov = 1 indicates no recovery correction).

2.5. Method for Developing Mathematical Models of the Objective Functions

By employing a systematic approach to analyze the techniques and technology of vacuum frying, the results show that the objective functions of the process include: y_1 (kWh/kg) – energy consumption per kilogram of product; y_2 (%) – product moisture content; y_3 – total color difference between the product and the reference sample; y_4 – product porosity; y_5 (µg/kg) – acrylamide formed in the product after frying. These objective functions depend on three main technological factors: Z_1 (°C) – the temperature of the vacuum frying medium; Z_2 (kPa) – vacuum frying chamber pressure; Z_3 (min) – the vacuum frying time.

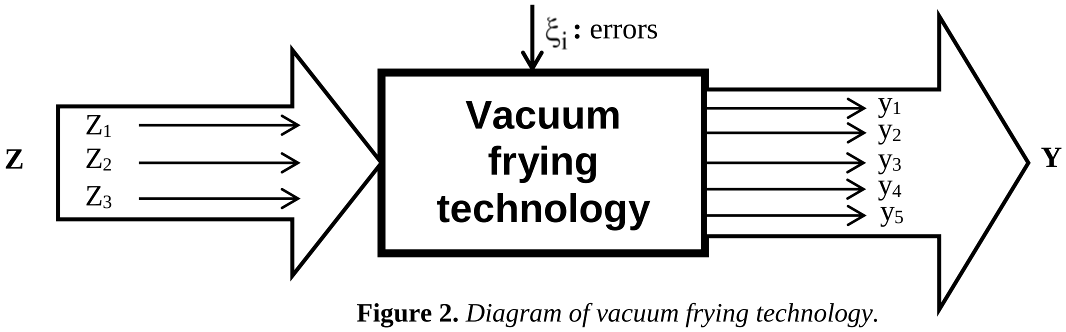

Figure 2. *Diagram of vacuum frying technology.*

To develop the mathematical models for the objective functions of the vacuum frying process (y_1, y_2, y_3, y_4, y_5) as dependent on the three main technological factors (Z_1, Z_2, Z_3), this study employed a second-order orthogonal experimental design. Denoting x_1, x_2, x_3 as the coded variables corresponding to the actual variables Z_1, Z_2, Z_3, the general quadratic regression equations can be expressed in the following form (It can be seen in Figure 2) [10]:

$$y_j = f_j(x_1, x_2, x_3) = b_0 + \sum_{u=1}^{k} b_u x_u + \sum_{u \neq i; u=1}^{k} b_{ui} x_u x_i + \sum_{u=1}^{k} b_{uu}\left(x_u^2 - \lambda\right) \tag{10}$$

$$\text{With } j = 1 \div 5$$

In which: x_1, x_2 and x_3 are the coded variables of the actual variables Z_1, Z_2 and Z_3, respectively, and are determined according to the following equations:

$$x_j = \frac{Z_j - Z_j^0}{\Delta Z_j}; \qquad Z_j = x_j.\Delta Z_j + Z_j^0 \tag{11}$$

$$Z_j^0 = (Z_j^{max} + Z_j^{min})/2; \ \ \Delta Z_j = (Z_j^{max} - Z_j^{min})/2; \ Z_j^{min} \le Z_j \le Z_j^{max}; \ j = 1 \text{ to } 3 \tag{12}$$

The number of experiments designed according to the second-order orthogonal experimental scheme is determined by the following equation:

$$N = n_k + n_* + n_0 = 2^k + 2k + n_0 = 18 \qquad (13)$$

where kis the number of technological factors influencing the vacuum frying process (Z_1, Z_2, Z_3), with k=3; and n_0 is the number of experiments at the center point, with $n_0 = 4$.

The value of the star point is determined by equation (14):

$$\alpha = \sqrt{\sqrt{N.2^{(k-2)}} - 2^{(k-1)}} = 1.414 \qquad (14)$$

The condition of the orthogonal matrix is determined by equation (15):

$$\lambda = \frac{1}{N}\left(2^k + 2\alpha^2\right) = \frac{3}{2} \qquad (15)$$

2.6. Multi-objective Optimization using the Restricted Area Method (RAM) with Combination Criteria R

When studying the techniques and technology of vacuum frying, it is crucial to determine the optimal process conditions. Under optimal conditions, the frying process not only produces a product with the lowest energy cost and appropriate moisture content to extend shelf life, but also ensures sensory quality and food safety. In other words, it is necessary to identify the frying environment temperature Z_1 (°C), frying environment pressure Z_2 (kPa), and frying time Z_3 (minutes) such that: the energy cost y_1 (kWh/kg) is minimized; the product moisture content y_2 (%) is reduced to the lowest acceptable level; the color difference index y_3 compared to the reference sample is minimized; the product porosity y_4 is maximized; and the acrylamide content y_5 (µg/kg) is minimized, while always remaining below the permissible limit to avoid potential carcinogenic risk [10].

Thus, in the technique and technology of vacuum frying, a multi-objective optimization problem has been established, in which the objective functions y_1, y_2, y_3, and y_5 are to be minimized, while y_4 is to be maximized. To convert all objective functions into a minimization form, which facilitates solving the problem, the objective functions are redefined as follows: $I_1 = y_1$; $I_2 = y_2$; $I_3 = y_3$; $I_4 = 1 - y_4$; $I_5 = y_5$, With this definition, when y_4 reaches its maximum value, I_4 attains its minimum. Therefore, the multi-objective optimization problem can be formulated as follows: Find the solution (x_1^{opt}, x_2^{opt}, x_3^{opt}) $\in \Omega_x$ = [-1.414, 1.414] such that [10]:

$$\begin{cases} I_{1\min} = f_1(x_1^{opt}, x_2^{opt}, x_3^{opt}) = \mathrm{Min}\{f_1(x_1, x_2, x_3)\} < C_1 \\ I_{2\min} = f_2(x_1^{opt}, x_2^{opt}, x_3^{opt}) = \mathrm{Min}\{f_2(x_1, x_2, x_3)\} < C_2 \\ I_{3\min} = f_3(x_1^{opt}, x_2^{opt}, x_3^{opt}) = \mathrm{Min}\{f_3(x_1, x_2, x_3)\} < C_3 \\ I_{4\max} = 1 - f_4(x_1^{opt}, x_2^{opt}, x_3^{opt}) = \mathrm{Min}\{1 - f_4(x_1, x_2, x_3)\} < C_4 \\ I_{5\min} = f_5(x_1^{opt}, x_2^{opt}, x_3^{opt}) = \mathrm{Min}\{f_5(x_1, x_2, x_3)\} < C_5 \end{cases} \qquad (16)$$

Where x_1, x_2, x_3 are the coded variables corresponding to the technological factors Z_1, Z_2, Z_3 and C_j (j = 1 ÷ 5) are the optimal values of the objective functions constrained by techno-economic conditions.

Now, by solving the system of equations (16) for each objective function sequentially, the optimal solutions (x_1^{optj}, x_2^{optj}, x_3^{optj}) $\in \Omega_x$ = [-1.414, 1.414] can be determined such that $I_j = I_{j\min}$ for j = 1 ÷ 5.

- **Case 1:** Consider the five solution sets (x_1^{optj}, x_2^{optj}, x_3^{optj}) for j = 1 ÷ 5. If these five solution sets coincide, i.e., (x_1^{optj}, x_2^{optj}, x_3^{optj}) ≡ (x_1^{opt}, x_2^{opt}, x_3^{opt}), thì (x_1^{opt}, x_2^{opt}, x_3^{opt}) is the solution of the multi-objective optimization problem (16). This optimal scheme is called the utopian optimal scheme, and the utopian point (or utopian objective vector) is defined as: $I^{UT} = (I_{1\min}, I_{2\min}, I_{3\min}, I_{4\min}, I_{5\min})$.

- **Case 2:** In practice, it is rare for all five solutions to coincide, meaning that the utopian optimal solution does not exist. However, the utopian point can still be determined: $I^{UT} = (I_{1min}, I_{2min}, I_{3min}, I_{4min}, I_{5min})$. In this context, the problem is to find a common optimal solution for all five objective functions such that the Pareto-optimal objective vector $I^{PR} = (I_{1min}^{PR}, I_{2min}^{PR}, I_{3min}^{PR}, I_{4min}^{PR}, I_{5min}^{PR})$ is as close as possible to the utopian point. Therefore, in this case, the restricted area method with combination criterion R is used to determine the solution of the multi-objective optimization problem (16).

Set up the combination criterion R as follows: In practice, many multi-objective optimization problems are subject to constraints on the values of the individual objective functions $f_j(Z)$, in other words, these constraints define the feasible range of the component objective functions $f_j(x)$ as dictated by the technological problem, where $x = (x_1, x_2, x_3)$ aries within the defined domain Ω_x.

$$\text{Notation:} \quad f_j(x) = f_j(x_1, x_2, x_3) < C_j, \ \forall j = 1 \div 5 \tag{17}$$

Usually, for a minimization problem, C_j is the upper bound of the component objective function $f_j(x)$, whereas for a maximization problem, C_j is the lower bound of $f_j(x)$, When the value of the objective function $f_j(x)$ lies outside its feasible range, it is considered a restricted area.

The conditions in (17) define the restricted area:

$$C = \{f_j(x) > C_j\} \text{ for the objective function } f_j(x) \tag{18}$$

The restricted area method is proposed to solve the multi-objective optimization problem (16), with the combination optimality criterion R(x) defined by the following expression:

$$R(x) = \sqrt[5]{r_1(x).r_2(x).r_3(x).r_4(x).r_5(x)} = \sqrt[5]{\prod_{j=1}^{5} r_j(x)} \tag{19}$$

$$\text{In which:} \quad r_j(x) = \frac{C_j - f_j(x)}{C_j - f_{jmin}} \qquad \text{khi} \quad f_j(x) \le C_j \tag{20}$$

$$r_j(x) = 0 \qquad\qquad \text{khi} \quad f_j(x) > C_j \tag{21}$$

From (20), it can be seen that when $f_j(x) \to f_{jmin}$ then $r_j(x) \to r_{jmax} = 1$. Thus, using the combination optimality criterion R(x) the multi-objective optimization problem (16) can be reformulated as follows: find a solution $x^R = \left(x_1^R, x_2^R, x_3^R\right) \in \Omega_x$ such that the objective function R(x) reaches its maximum value within the feasible domain Ω_f.

$$R_{max} = R\left(x^R\right) = \max\left\{R(x)\right\} = \max\left\{\sqrt[5]{\prod_{j=1}^{5} r_j(x)}\right\} \tag{22}$$

Where $x = (x_1, x_2, x_3) \in \Omega_x$

It is easy to see that $0 \le R(x^R) \le 1$, where $R(x^R) = 1$ when the optimal solution is exactly the utopian solution x^{UT} and $R(x^R) = 0$ when even a single value $f_j(x)$ iolates inequality (17), meaning that the point f(x) falls within the restricted area CCC, as defined in (18).

3. Results and discussion

3.1. Set up a table of technological factors affecting the vacuum frying process

The experiment was conducted at a vacuum frying environment pressure of $Z_2 = 8$ kPa and a vacuum frying time of $Z_3 = 25$ minutes. The results show that when the frying temperature $Z_1 < 85^0$C, the porosity of the fried jackfruit reaches $y_4 = 0.16$, resulting in a product density of $I_4 = 1 - y_4 = 1 - 0.16 = 0.84$, which is greater than $C_4 = 0.80$ (the maximum allowed porosity of the product is 0.2, meaning the product density must be below 0.8). Therefore, the product does not meet the porosity requirement. Conversely, when the frying temperature $Z_1 > 115^0C$, the color difference index of the product reaches $y_3 = 2.85$, exceeding $C_3 = 1.0$ (the limit for the color difference index of the product). Hence, the product fails to meet the color quality requirement.

The experiment was conducted at a vacuum frying environment temperature of $Z_1 = 100^0C$ and a vacuum frying time of $Z_3 = 25$ minutes. The results show that when the vacuum frying environment pressure $Z_2 < 5$ kPa, the energy cost of the fried jackfruit reaches $y_1 = 3.8$ kWh/kg, which is higher than $C_1 = 3.0$ kWh/kg (the product cost limit requires it to be below 3.0 kWh/kg). Therefore, the product has a high energy cost, resulting in a high price, and does not meet the techno-economic requirements of the vacuum frying technology. Conversely, when the frying environment pressure $Z_2 > 11$ kPa, the Acrylamide content in the fried product reaches $y_5 = 126$ µg/kg, exceeding $C_5 = 100$ µg/kg (the product safety limit; exceeding this threshold is undesirable since Acrylamide is a carcinogenic substance). Hence, the product fails to meet the safety quality requirement.

The experiment was conducted at a vacuum frying environment temperature of $Z_1 = 100^0C$ and a vacuum frying environment pressure of $Z_2 = 8$ kPa. The results show that when the vacuum frying time $Z_3 < 17$ minutes, the moisture content of the fried jackfruit reaches $y_2 = 4.2\%$, which is higher than $C_2 = 2.5\%$ (the storage requirement limit requires it to be below 2.5%). Therefore, the product does not meet the preservation requirement. Conversely, when the vacuum frying time $Z_3 > 33$ minutes, the energy cost of the fried jackfruit reaches $y_1 = 4.2$ kWh/kg, which is higher than $C_1 = 3.0$ kWh/kg (the product cost limit requires it to be below 3.0 kWh/kg). As a result, the product has a high energy cost, leading to a high price, and thus does not satisfy the techno-economic requirements of the proposed vacuum frying technology.

From the experimental results, a table of technological factors affecting the vacuum frying process has been established (see Table 1).

Table 1. *The technological factors affecting the vacuum frying process*

Parameters	Levels					Deviation ΔZ_i
	$-\alpha$ (-1.414)	Low (-1)	Central (0)	High (+1)	$+\alpha$ (1.414)	
Z_1 (^0C)	85.86	90	100	110	114.14	10
Z_2 (kPa)	5.17	6	8	10	10.83	2
Z_3 (min)	17.93	20	25	30	32.07	5

3.2. Developing mathematical models y_1, y_2, y_3, y_4, y_5 to describe the vacuum frying proces

To construct the statistical mathematical models for the objective functions y_1, y_2, y_3, y_4, y_5 as dependent on the three technological factors x_1, x_2, x_3, this study employed a second-order orthogonal experimental design. The technological factors affecting the vacuum frying process are presented in Table 1, while the second-order orthogonal experimental matrix is given in Table 2.

Table 2. *The second-order orthogonal design matrix was constructed with $k = 3$ and $n_0 = 4$*

	N	Z_1 (^0C)	Z_2 (kPa)	Z_3 (min)	x_1	x_2	x_3	y_1	y_2	y_3	y_4	y_5
2^k	1	110	10	30	1	1	1	2.91	1.63	1.38	0.295	99.23
	2	90	10	30	-1	1	1	2.25	2.22	1.11	0.242	78.49
	3	110	6	30	1	-1	1	2.87	1.49	1.30	0.295	79.47
	4	90	6	30	-1	-1	1	2.16	2.08	0.86	0.230	79.57
	5	110	10	20	1	1	-1	2.04	2.27	1.22	0.278	86.81
	6	90	10	20	-1	1	-1	1.87	2.10	0.86	0.234	76.47
	7	110	6	20	1	-1	-1	2.17	2.13	1.12	0.229	67.05

	8	90	6	20	-1	-1	-1	1.82	1.97	0.66	0.167	77.55
2k	9	114.14	8	25	1.414	0	0	2.94	1.73	1.48	0.347	81.41
	10	85.86	8	25	-1.414	0	0	2.35	2.03	0.61	0.172	74.17
	11	100	10.83	25	0	1.414	0	2.56	2.47	0.96	0.314	85.59
	12	100	5.17	25	0	-1.414	0	2.67	2.28	0.86	0.224	72.38
	13	100	8	32.07	0	0	1.414	3.02	1.71	1.40	0.273	82.27
	14	100	8	17.93	0	0	-1.414	2.37	2.08	0.84	0.208	72.06
n_0	15	100	8	25	0	0	0	2.83	2.16	0.79	0.212	71.82
	16	100	8	25	0	0	0	2.74	2.21	0.78	0.216	72.63
	17	100	8	25	0	0	0	2.67	2.17	0.80	0.228	70.29
	18	100	8	25	0	0	0	2.69	2.19	0.86	0.207	73.05

The vacuum frying experiments were carried out according to the second-order orthogonal design scheme presented in Table 2. The results for the objective functions (y_1, y_2, y_3, y_4, y_5) were determined, and these values are presented in Table.

From the data in Table 2, the results were transferred to Table 3 for processing and for constructing the mathematical models (regression equations) of the objective functions (y_1, y_2, y_3, y_4, y_5).

Table 3. *Experimental data processing*

	N	x_0	x_1	x_2	x_3	x_1x_2	x_1x_3	x_2x_3	x_1^2 - 2/3	x_2^2 - 2/3	x_3^2 - 2/3	y_1	y_2	y_3	y_4	y_5
2^k	1	1	1	1	1	1	1	1	0.333	0.333	0.333	2.91	1.63	1.38	0.295	99.23
	2	1	-1	1	1	-1	-1	1	0.333	0.333	0.333	2.25	2.22	1.11	0.242	78.49
	3	1	1	-1	1	-1	1	-1	0.333	0.333	0.333	2.87	1.49	1.30	0.295	79.47
	4	1	-1	-1	1	1	-1	-1	0.333	0.333	0.333	2.16	2.08	0.86	0.230	79.57
	5	1	1	1	-1	1	-1	-1	0.333	0.333	0.333	2.04	2.27	1.22	0.278	86.81
	6	1	-1	1	-1	-1	1	-1	0.333	0.333	0.333	1.87	2.10	0.86	0.234	76.47
	7	1	1	-1	-1	-1	-1	1	0.333	0.333	0.333	2.17	2.13	1.12	0.229	67.05
	8	1	-1	-1	-1	1	1	1	0.333	0.333	0.333	1.82	1.97	0.66	0.167	77.55
2k	9	1	1.414	0	0	0	0	0	1.333	-0.667	-0.667	2.94	1.73	1.48	0.347	81.41
	10	1	-1.414	0	0	0	0	0	1.333	-0.667	-0.667	2.35	2.03	0.61	0.172	74.17
	11	1	0	1.414	0	0	0	0	-0.667	1.333	-0.667	2.56	2.47	0.96	0.314	85.59
	12	1	0	-1.414	0	0	0	0	-0.667	1.333	-0.667	2.67	2.28	0.86	0.224	72.38
	13	1	0	0	1.414	0	0	0	-0.667	-0.667	1.333	3.02	1.71	1.40	0.273	82.27
	14	1	0	0	-1.414	0	0	0	-0.667	-0.667	1.333	2.37	2.08	0.84	0.208	72.06
n_0	15	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	2.83	2.16	0.79	0.212	71.82
	16	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	2.74	2.21	0.78	0.216	72.63
	17	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	2.67	2.17	0.80	0.228	70.29
	18	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	2.69	2.19	0.86	0.207	73.05

Using Microsoft Excel 2024 to process the data in Table 3, the coefficients b_0, b_u, b_{ui} and b_{uu} of equation (10) were determined. The coefficients in the regression equation (mathematical model) were tested for significance using Student's criterion, while the adequacy of the equation compared with the experimental data was evaluated using Fisher's criterion. On this basis, the mathematical models (regression equations) for the objective functions (y_1, y_2, y_3, y_4, y_5) were developed and presented in equations (23), (24), (25), (26), and (27).

$$\begin{aligned} y_1 = f_1(x_1, x_2, x_3) &= 2.815 + 0.227x_1 + 0.267x_3 \\ &\quad + 0.106x_1x_3 - 0.163x_1^2 - 0.178x_2^2 - 0.138x_3^2 \end{aligned} \tag{23}$$

$$\begin{aligned} y_2 = f_2(x_1, x_2, x_3) &= 2.186 - 0.106x_1 + 0.067x_2 - 0.132x_3 \\ &\quad - 0.189x_1x_3 - 0.153x_1^2 + 0.097x_2^2 + 0.147x_3^2 \end{aligned} \tag{24}$$

$$y_3 = f_3(x_1, x_2, x_3) = 0.841 + 0.32x_1 + 0.064x_2 + 0.132x_3 + 0.095x_1^2 + 0.135x_3^2 \tag{25}$$

$$\begin{aligned} y_4 = f_4(x_1, x_2, x_3) &= 0.233 + 0.039x_1 + 0.021x_2 + 0.021x_3 \\ &\quad - 0.013x_2x_3 + 0.012x_1^2 + 0.016x_2^2 \end{aligned} \tag{26}$$

$$\begin{aligned} y_5 = f_5(x_1, x_2, x_3) &= 72.13 + 2.56x_1 + 4.67x_2 + 3.61x_3 + 5.21x_1x_2 \\ &\quad + 2.6x_1x_3 + 2.763x_1^2 + 3.339x_2^2 + 2.425x_3^2 \end{aligned} \tag{27}$$

3.3. Solving the one-objective optimization problems

The single-objective optimization problems are formulated as follows: Find the solution (x_1, x_2, x_3) $\in \Omega_x = \{-1.414 \le x_1, x_2, x_3 \le 1.414\}$ such that the objective functions y_1, y_2, y_3, y_4, y_5 in equations (23), (24), (25), (26), and (27) attain the following values: $y_{1min} = \min f_1(x_1, x_2, x_3)$; $y_{2min} = \min f_2(x_1, x_2, x_3)$; $y_{3min} = \min f_3(x_1, x_2, x_3)$; $y_{4max} = \max f_4(x_1, x_2, x_3)$; $y_{5min} = \min f_5(x_1, x_2, x_3)$.

By using the Excel–Solver software, the solutions of equations (23), (24), (25), (26), and (27) were obtained and summarized in Table 4.

Table 4. *Solutions of the single-objective optimization problems*

y_j	Optimal values of y_j	values of $x_1^{j\ opt}$	values of $x_2^{j\ opt}$	values of $x_3^{j\ opt}$
y_{1min}	1.37	-1.414	1.414	-1.414
y_{2min}	0.861	1.414	-0.345	1.414
y_{3min}	0.58	-1.211	-1.414	-0.489
y_{4max}	0.37	1.414	1.414	1.414
y_{5min}	65.42	1.414	-1.414	-1.414

The results in Table 4 show that the solutions of the single-objective optimization problems do not coincide. This indicates that the utopian optimal scheme does not exist; however, the utopian point has been determined as: $f^{UT} = (f_{1min}, f_{2min}, f_{3min}, f_{4max}, f_{5min}) = (1.37, 0.861, 0.58, 0.37, 65.42)$.

3.4. Solving the multi-objective optimization problem by the restricted area method

To facilitate solving the multi-objective optimization problem, the component objective functions need to be transformed into the same form (either minimization or maximization). Therefore, the objective functions (y_1, y_2, y_3, y_4, y_5) are reformulated as follows:

$$\begin{gathered} I_1 = y_1 = f_1(x_1, x_2, x_3);\ I_2 = y_2 = f_2(x_1, x_2, x_3);\ I_3 = y_3 = f_3(x_1, x_2, x_3); \\ I_4 = 1 - y_4 = 1 - f_4(x_1, x_2, x_3);\ I_5 = y_5 = f_5(x_1, x_2, x_3); \end{gathered} \tag{28}$$

With the above reformulation, the multi-objective optimization problem can be stated as follows: Find the solution $x = (x_1^{opt}, x_2^{opt}, x_3^{opt}) \in \Omega_x = [-1.414, 1.414]$ such that:

$$\begin{cases} I_{j\min} = f_j(x_1^{opt}, x_2^{opt}, x_3^{opt}) = \mathrm{Min}\{f_j(x_1, x_2, x_3)\} < C_j \\ -1.414 \le x_1, x_2, x_3 \le 1.414; \quad j = 1 \div 5 \end{cases} \tag{29}$$

With the above definitions: $I_{1min} = y_{1min}$; $I_{2min} = y_{2min}$; $I_{3min} = y_{3min}$; $I_{4min} = 1 - y_{4max}$; $I_{5min} = y_{5min}$. Accordingly, the utopian point is rewritten as: $I^{UT} = (I_{1min}, I_{2min}, I_{3min}, I_{4min}, I_{5min}) = (1.37, 0.861, 0.58, 0.63, 65.42)$.

Since it is impossible to find a solution that simultaneously satisfies all objective function values (y_{1min}, y_{2min}, y_{3min}, y_{4max}, y_{5min}) or ((I_{1min}, I_{2min}, I_{3min}, I_{4min}, I_{5min}). the idea of the multi-objective optimization problem (29) is to find the Pareto optimal solution $x^R = (x_1^R, x_2^R, x_3^R)$, at which the Pareto efficiency: $I^{PR} = (I_1^{PR}, I_2^{PR}, I_3^{PR}, I_4^{PR}, I_5^{PR})$ is both closest to the utopian point and farthest from the restricted area, where I_j ($1 \div 5$) must satisfy the technological conditions and the initial requirements.

$$I_1 < C_1 = 3.0;\ \ I_2 < C_2 = 2.5;\ \ I_3 < C_3 = 1.0;\ \ I_4 < C_4 = 0.8;\ \ I_5 < C_5 = 100 \tag{30}$$

The combination optimality criterion R or $R(I_1, I_2, I_3, I_4, I_5) = R(y_1, y_2, y_3, y_4, y_5) = R(x_1, x_2, x_3) = R(x)$ is rewritten as follows:

$$\begin{cases} R(x) = R(x_1, x_2, x_3) = \sqrt[5]{\prod_{j=1}^{5} r_j(x_1, x_2, x_3)} = \sqrt[5]{\prod_{j=1}^{5} r_j(x)} \\ \Omega_x = \{-1.414 \le x_1,\ x_2,\ x_3 \le 1.414\};\ x = (x_1, x_2, x_3) \end{cases} \tag{31}$$

$$\text{With:} \quad r_j(x) = \left(\frac{C_j - I_j(x)}{C_j - I_{j\min}} \right) \quad \text{when} \quad I_j(x) < C_j \tag{32}$$

$$r_j(x) = 0 \qquad \text{when} \quad I_j(x) \ge C_j \tag{33}$$

At this stage, the multi-objective optimization problem is restated as follows: Find the Pareto optimal solution $x^R = (x_1^R, x_2^R, x_3^R) \in \Omega_x$ such that $R(x_1^R, x_2^R, x_3^R) = \mathrm{Max}\{R(x_1, x_2, x_3)\}$.

The maximum value of (31) was determined by using the Excel – Solver sofware:

$$R(x)_{max} = \mathrm{Max}\{R(x_1, x_2, x_3)\} = R(x_1^R, x_2^R, x_3^R) = 0.312$$

With: $x_1^R = -0.303$; $x_2^R = 0.826$; $x_3^R = -1.246$;

Then, transforming into real variables:

$Z_1^{opt} = 96.97^0$C; $Z_2^{opt} = 9.65$ kPa; $Z_3^{opt} = 18.77$ minutes

Substituting x_1^R, x_2^R, x_3^R into these equations (28), the results were obtained as:

$I_1^{PR} = 2.10$; $I_2^{PR} = 2.30$; $I_3^{PR} = 0.88$; $I_4^{PR} = 0.76$; $I_5^{PR} = 76.68$

Substituting x_1^R, x_2^R, x_3^R into these equations (23), (24), (25), (26) and (27), the results were obtained as:

$y_1^{PR} = 2.10$; $y_2^{PR} = 2.30$; $y_3^{PR} = 0.88$; $y_4^{PR} = 0.24$; $y_5^{PR} = 76.68$

Where: $f^{PR} = y^{PR} = (y_1^{PR}, y_2^{PR}, y_3^{PR}, y_4^{PR}, y_5^{PR}) = (2.10, 2.30, 0.88, 0.24, 76.68)$ called the optimal Paréto effect.

The results show that at $R(x)_{max} = 0.876$, the Pareto optimal solution is: $x_1^R = -0.303$; $x_2^R = 0.826$; $x_3^R = -1.246$, corresponding to the technological parameters: optimal vacuum frying temperature $Z_1^{opt} = 96.97^0$C; optimal vacuum frying pressure $Z_2^{opt} = 9.65$ kPa and optimal frying time $Z_3^{opt} = 18.77$ minutes. Under these optimal conditions, the energy consumption for the vacuum frying process is $y_1^{PR} = 2.10$ kWh/kg; the moisture content of vacuum-fried jackfruit is $y_2^{PR} = 2.30$ %; the color difference index is

y_3^{PR} = 0.88; the porosity of the product is y_4^{PR} = 0.24; and the acrylamide content in the product is y_5^{PR} = 76.68 µg/kg. Compared with the experimental results in Table 2, these outcomes are consistent and meet the objectives set forth in the optimization problem.

3.5. Experiment to test the results of multi-objective optimization problem

Vacuum frying of jackfruit was carried out using technological parameters corresponding to the Pareto optimal solution: vacuum frying temperature $Z_1^{opt} = 96.97^0$C; vacuum frying pressure Z_2^{opt} = 9.65 kPa and vacuum frying time Z_3^{opt} = 18.77 minutes. The results obtained were: energy consumption of 1kg product y_1 = 2.10 kWh/kg; moisture content of the vacuum-fried jackfruit y_2 = 2.30%; color difference index y_3 = 0.88; product porosity y_4 = 0.24; and acrylamide content in the product y_5 = 76.68 µg/kg.

Figure 3. *Vacuum-fried jackfruit under optimal conditions ($x_1^R = -0.303$; $x_2^R = 0.826$; $x_3^R = -1.246$)*

These results are close to the Pareto optimal values: y_1^{PR} = 2.10 kWh/kg; y_2^{PR} = 2.30%; y_3^{PR} = 0.88; y_4^{PR} = 0.24; y_5^{PR} = 76.68. Thus, the Pareto optimal solution ($x_1^R = -0.303$; $x_2^R = 0.826$; $x_3^R = -1.246$) determined from the multi-objective optimization problem is fully consistent with the experimental results of vacuum frying of jackfruit. As shown in Figure 3, the vacuum-fried jackfruit under optimal conditions has a natural bright yellow color, does not shrink, exhibits crisp porosity, and retains its characteristic flavor. Therefore, the vacuum frying process under optimal conditions produces a high-quality product.

This study shows differences in product color compared to the research by Tanushree Maity, A. S. Bawa & P. S. Raju [11]. This discrepancy is mainly due to differences in jackfruit varieties and soil conditions between India and Vietnam, which lead to variations in chemical composition, thereby affecting the product's color. Conversely, when compared with the study by Akshata M.G and Shrikant B.S [12], the results are fully consistent regarding moisture content, porosity, and acrylamide formation in the product. Additionally, according to Patra et al. [8], the acrylamide content in vacuum-fried products is below 120 µg/kg; the results of this study also show similarity with these experimental data.

From the analysis and comparison of this study's results with previously published works, it can be concluded that the Pareto optimal solution ($x_1^R = -0.303$; $x_2^R = 0.826$; $x_3^R = -1.246$), corresponding to the technological parameters: vacuum frying temperature $Z_1^{opt} = 96.97^0$C; vacuum frying pressure Z_2^{opt} = 9.65 kPa and vacuum frying time Z_3^{opt} = 18.77 minutes represents the optimal vacuum frying process. Under this optimal condition, the energy consumption for vacuum frying is y_1^{PR} = 2.10 kWh/kg; he moisture content of the vacuum-fried jackfruit is y_2^{PR} = 2.30%; the color difference index is y_3^{PR} = 0.88; the product's porosity is y_4^{PR} = 0.24; and the acrylamide content in the product is y_5^{PR} = 76.68 µg/kg.

4. Conclusions

The multi-objective optimization of the vacuum-frying process for jackfruit using the restricted area method with the established combination criterion R yielded the Pareto optimal solution ($x_1^R = -0.303$; $x_2^R = 0.826$; $x_3^R = -1.246$). This solution corresponds to the technological parameters: vacuum frying temperature of 96.97^0C; vacuum frying pressure of 9.65 kPa and vacuum frying time of 18.77 minutes, and has been successfully applied in practical production.

At this optimal condition, the obtained product exhibits the lowest energy cost (2.10 kWh/kg), a moisture content meeting the requirement below 2.5% (2.30%), the lowest color difference index (0.88), the highest porosity (0.24), and a minimum acrylamide content (76.68 μg/kg). These results indicate that the vacuum-fried jackfruit achieves high product quality.

Acknowledgments

The author thanks Head of Lab B108 “Food Engineering and Technology”, Department of Food Technology, Faculty of Chemical and Food Technology, HCMC University of Technology and Education, Vietnam, for their support and help on carrying out the experiments.

Conflict of Interest

The authors declare no conflict of interest

REFERENCES

[1] Y. Wang *et al.*, “Effect of new frying technology on starchy food quality,” *Foods*, vol. 10, no. 8, p. 1852, Aug. 2021, doi: 10.3390/foods10081852.
[2] M. E. Sosa-Morales *et al.*, “Reviewing the effects of vacuum frying on frying medium and foods,” *Int. J. Food Sci. Technol.*, vol. 57, no. 6, pp. 3278–3295, 2022.
[3] J. Garayo and R. Moreira, “Vacuum frying of potato chips,” *J. Food Eng.*, vol. 55, pp. 181–191, 2002.
[4] S. L. Shyu and L. S. Hwang, “Effect of processing conditions on the quality of vacuum-fried apple chips,” *Food Res. Int.*, vol. 34, no. 2, pp. 133–142, 2001.
[5] B. Belková *et al.*, “Impact of vacuum frying on quality of potato crisps and acrylamide,” *Food Chem.*, vol. 241, pp. 51–59, 2018.
[6] I. Govindaraju *et al.*, “Dietary acrylamide: A detailed review,” *Foods*, vol. 13, no. 4, p. 556, Feb. 2024, doi: 10.3390/foods13040556.
[7] K. Kim and R. Moreira, “Vacuum frying vs. conventional frying,” *Eur. J. Lipid Sci. Technol.*, 2013, doi: 10.1002/ejlt.201300272.
[8] A. Patra *et al.*, “Evaluation of effect of vacuum frying on textural properties,” *LWT*, vol. 162, no. B, p. 112074, Dec. 2022, doi: 10.1016/j.foodres.2022.112074.
[9] Z. A. Ahmed and N. K. Mohammed, “Investigating influencing factors on acrylamide content in fried potatoes and mitigating measures: a review” *Food Prod. Process. Nutr.*, 2024, doi: 10.1186/s43014-023-00212-6.
[10] T. C. Thanh, L. D. T. Khanh, and D. N. Tan, “Designing and manufacturing a vacuum frying system with intelligent controlling,” *JTE*, no. 77, Jun. 2023, doi: 10.54644/jte.77.2023.1374.
[11] T. Maity, A. S. Bawa, and P. S. Raju, “Effect of preconditioning on physicochemical, microstructural, and sensory quality of vacuum-fried jackfruit chips,” *Dry. Technol.*, vol. 36, no. 1, pp. 1–12, 2018, doi: 10.1080/07373937.2017.1300590.
[12] A. M. Gokhale and S. B. Swami, “Effect of different process parameters and methods of jackfruit chips preparation on quality of chips: A review,” *Int. J. Food Ferment. Technol.*, vol. 14, no. 2, pp. 675–687, Dec. 2024, doi: 10.30954/2277-9396.02.2024.14.
[13] H. T. Feleke and T. Y. Belete, “LC-MS/MS triple quadrupole multiresidue analysis of pesticides from soils of vegetable farms: A report from Ethiopia,” *Appl. Environ. Soil Sci.*, Feb. 2025, doi: 10.1155/aess/4830881.

Lieu Le Quang
College of Fisheries and Food Technology, Vietnam
Phone: 0915844515
Email: liecdntsmb2@gmail.com. ORCID: https://orcid.org/0009-0000-8722-8873

Liviu Giurgiulescu
Chemistry-Biology Department
Technical University of Cluj Napoca, North Universitary Center of Baia Mare, Romania.
Email: giurgiulescu@gmail.com. ORCID: https://orcid.org/0000-0002-6482-3052

Suc Nguyen Van
Department of Food Technology; Faculty of Chemical and Food Technology;
HCM City University of Technology and Education, Vietnam;
No 01, Vo Van Ngan Street, Thu Duc Ward, HCM City.
Phone: 0905354758
Email: sucnv@hcmute.edu.vn. ORCID: https://orcid.org/0000-0002-5617-6553

Dzung Nguyen Tan
Department of Food Technology; Faculty of Chemical and Food Technology;
HCM City University of Technology and Education, Vietnam;
No 01, Vo Van Ngan Street, Thu Duc Ward, HCM City.
Phone: 0918801670
Email: tandzung072@hcmute.edu.vn. ORCID: https://orcid.org/0000-0002-2482-2098