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Setpoint Tracking-Error Control Design for Stabilization of Quadruple-Tank Process With Time Delay Using a Relaxing Port-Hamiltonian Formulation

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ABSTRACT

The nonlinearity of chemical processes primarily arises from strong interdependencies among state variables and the presence of time delays. This characteristic leads to complex dynamic behaviors such as non-minimum phase responses, posing significant challenges for control design. This study presents a novel control strategy that integrates Pade approximation of time delay with a tracking-error-based control law within a port-Hamiltonian (PH) framework. Specifically, the Pade approximation is employed to effectively handle the time delay by approximating exponential delay terms, thereby facilitating its incorporation into the control design and enhancing the robustness of the closed-loop system. Concurrently, the tracking-error-based feedback laws ensure the asymptotic convergence of system trajectories to the desired reference trajectories, even in the presence of non-minimum phase dynamics. The proposed methodology is illustrated through simulations on the quadruple-tank process - a benchmark nonlinear system with inherent time delays. Results demonstrate that the proposed control scheme successfully mitigates the adverse effects of time delay and achieves the closed-loop stabilization at the target equilibrium point.

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1. Introduction

The port-Hamiltonian (PH) framework is a well-established methodology for modeling physical and chemical processes through port variables, offering a structured representation of system dynamics [1], [2]. Noting that if a (nonlinear) system admits a PH formulation, its intrinsic structural properties such as energy exchange and dissipation can be systematically characterized via interconnection and damping matrices [2]. Moreover, a power-preserving inequality can be directly derived by taking the time derivative of the Hamiltonian storage function, which can be considered as one of key features of this framework. Additionally, this inequality reflects a fundamental physical principle, that is, the energy supplied by external sources to the system is always greater than or equal to internal energy accumulation. From this, by leveraging this property, various passivity-based control (PBC) strategies have been developed to address stabilization challenges in nonlinear systems. For instance, notable passivity-based approaches include energy-balancing PBC (EB-PBC) [3], interconnection and damping assignment PBC (IDA-PBC) [2], [4], and control by interconnection (CbI) [5]. Despite their effectiveness, the PBC design is often constrained by a separability condition, which requires the damping matrix to be symmetric and positive definite [6]. To overcome these limitations, recent studies [7], [8] have introduced a relaxed PH formulation combined with tracking-error-based control laws, thereby enhancing the applicability of passivity-based approach within the PH framework to a broader class of nonlinear systems that does not satisfy the strict separability condition. The efficacy of this approach has been demonstrated through applications to the quadruple-tank process and other benchmark nonlinear chemical systems without time delay. It is important to note that time delay remains a critical challenge in process control, as it can induce instability and significantly degrade closed-loop performance [9], [10]. The dynamics associated with time delays are nonlinear and complex, often leading to abnormal behaviors such as non-minimum phase behavior. As a result, compensating for time-delay effects is a critical and technically demanding problem in modern control theory.

In this study, the tracking-error-based control strategy developed within the framework of relaxing port-Hamiltonian (PH) representation [7], [8] is extended to stabilize a class of linear systems incorporating time-delay states. It is important to remark that reference trajectories with certain structures here are generated by ordinary differential equations and a part of them are, then, assigned to the constant desired equilibrium points. This control method can be considered as a simplified version of the tracking-error PBC method in [11], where all reference trajectories are obtained by a set of generalized canonical form transformations and time-varying. To demonstrate the effectiveness of the proposed approach, the quadruple-tank process with time delay [12], [13] is utilized as a case study. Noting that the nonlinear dynamics of this process, exhibiting a non-minimum phase behavior, is exacerbated by the presence of time delays. And, these all together not only degrade achievable control performance but also increase the complexity of the control design, making it a challenging yet practically relevant benchmark [14], [15].

The proposed methodology is implemented through a two-step procedure:

1. Time-delay approximation: The time-delayed terms are approximated using first-order Pade approximations, yielding rational transfer function representations to achieve compensated systems usable for control design.
2. Control design and stabilization: The tracking-error-based control law within the relaxing PH framework [7], [8] is applied to the compensated systems having no time-delay terms to achieve the asymptotic stabilization at the desired set point.

The reminder of this paper is organized as follows. Section 2 provides a brief overview of the tracking-error-based control approach and the relaxing port-Hamiltonian (PH) formulation for nonlinear systems. Section 3 presents the theoretical extension of this control methodology to a class of linear systems with the inclusion of time-delayed terms. In Section 4, the stabilization of the quadruple-tank process is addressed as a case study to demonstrate the practical applicability of the proposed method, and Section 5 discusses the simulation results that validate the effectiveness of the control strategy. Finally, the conclusion is given in Section 6.

2. PH formulation and tracking-error-based feedback

2.1. An overview of the port-Hamiltonian representation

In this paper, we consider (nonlinear) systems that are affine in terms of the control input u and their dynamics can be written in the following form [16]:

$$\dot{x} = f(x) + G(x)u, \qquad x(t=0) = x_0, \tag{1}$$

where $x = x(t) \in \Omega \subset \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ represents the control signal. In addition, $f(x) \in \mathbb{R}^n$ and $G(x) \in \mathbb{R}^{n\times m}$ are (nonlinear) functions with respect to the vector field x.

It is postulated that the vector field $f(x)$, related to evolution of the system dynamics (1), satisfied the so-called separability condition [2], [6], [17]-[19], that is, it can be decomposed and expressed as the product of structural matrices, and the gradient of a Hamiltonian storage function as follows :

$$f(x) = \left[J(x) - R(x)\right]\frac{\partial H}{\partial x}, \tag{2}$$

where $J(x)$ and $R(x)$ are the two structure matrices fulfilling the conditions: $J(x) = -J^{\mathrm{T}}(x)$ and $R(x) = R^{\mathrm{T}}(x)$ with respect to the Hamiltonian function $H(x): \mathbb{R}^n \to \mathbb{R}$. From this, the system dynamics (1) with the output $y := G^{\mathrm{T}}(x)\dfrac{\partial H}{\partial x}$ can be rewritten into the following form:

$$\begin{cases} \dot{x} = \left[J(x) - R(x) \right] \dfrac{\partial H}{\partial x} + G(x)u, \\ y = G^{\mathrm{T}}(x) \dfrac{\partial H}{\partial x}, \end{cases} \tag{3}$$

thereby yielding the time derivative of $H(x)$ as below:

$$\dot{H}(x) = -\left(\frac{\partial H}{\partial x} \right)^{\mathrm{T}} R(x) \frac{\partial H}{\partial x} + u^{\mathrm{T}} y. \tag{4}$$

This work explores a broad category of extended PH models, characterized through the properties of the matrix $R(x)$, as presented in Definition 1 [20].

Definition 1. Any system, described by (3), is said to belong to an extended class of PH models, denoted by $\mathcal{H}_e$, if the matrix $R(x)$ meets one of the following conditions:

a. $R(x)$ is positive semi-definite, i.e., $\forall \upsilon \in \mathbb{R}^n \setminus \{\mathbf{0}_n\}, \upsilon^{\mathrm{T}} R(x) \upsilon \geq 0$,

b. $R(x)$ is positive semi-definite, i.e., $\forall \upsilon \in \mathbb{R}^n \setminus \{\mathbf{0}_n\}, \upsilon^{\mathrm{T}} R(x) \upsilon \leq 0$ or indefinite (i.e. neither positive nor negative semi-definite).

with $\mathbf{0}_n$ denoting the n-dimensional vector having all zero elements.

Fundamentally, the system dynamics (1) with the condition (a) is a standard PH systems, which fulfills the dissipation inequality[1]: $\dot{H}(x) \leq u^{\mathrm{T}} y$, and is considered here as a non-relaxing PH model, denoted by $\mathcal{H}_{e,\mathrm{nr}}$. On the other hand, the system dynamics (1) belongs to a class of relaxing PH models under the condition (b), denoted by $\mathcal{H}_{e,\mathrm{r}}$.

In practice, it may be challenging to formulate the system dynamics (1) into a non-relaxing PH model (i.e. $\in \mathcal{H}_{e,\mathrm{nr}}$). Hence, the positive semi-definiteness of $R(x)$ should not be strictly enforced, thereby possibly resulting in a relaxing PH model (i.e. $\in \mathcal{H}_{e,\mathrm{r}}$). In any case, the extended structured form in Definition 3.1, particularly associated with a quadratic Hamiltonian function, remains valuable for control design through a trajectory tracking approach, as demonstrated in the following section.

2.2. Tracking-error-based control via the (relaxing) PH structure

Given a reference trajectory x_d either passing through the set-point or consisting of the desired (optimal) profile, the following proposition shall propose a specific structure for x_d, which ensures the convergence of x towards x_d.

Proposition 1 [8], [20]: Assume that

- the nonlinear system (1) can be formulated into the extended PH models (i.e. either $\in \mathcal{H}_{e,\mathrm{nr}}$ or $\in \mathcal{H}_{e,\mathrm{r}}$) with respect to the quadratic storage function, i.e.,

$$H(x) := \frac{1}{2} x^{\mathrm{T}} R_{di} x, \tag{5}$$

where R_{di} is an arbitrary positive definite symmetric (constant) matrix, i.e. $R_{di} \geq 0$.

[1] It reveals that the increase of energy inside the system is always smaller or equal to the total amount of energy that is supplied by the external source from a physical perspective. Additionally, the system is lossless, that is the equality sign occurs, provided that the damping matrix $R(x)$ with respect to the natural dissipation becomes null.

- the dynamics of x_d is governed by the following differential equation:

$$\dot{x}_d = [J(x) - R(x)]\frac{\partial H(x_d)}{\partial x_d} + R_I(x)\frac{\partial \Psi(e)}{\partial e} + G(x)u, \tag{6}$$

with the quadratic function $\Psi(e) := \frac{1}{2}e^{\mathrm{T}}R_{di}e$, where $e := x - x_d$ is defined as the error state vector and $R_I(x) = R_I^{\mathrm{T}}(x) \geq 0$ is a generalized damping injection matrix. Then,

i. when $x(t) \in \mathscr{H}_{e,\mathrm{nr}}$, the system trajectory x exponentially converges to the reference one x_d, if $R_I(x)$ is suitably selected such that the following condition:

$$R_I(x) > 0, \tag{7}$$

is met.

ii. Otherwise, when $x(t) \in \mathscr{H}_{e,\mathrm{r}}$, there is an asymptotic stabilization of x toward the reference one x_d, provided that $R_I(x)$ is chosen to fulfil the following inequality constraint:

$$R(x) + R_I(x) = \left(R(x) + R_I(x)\right)^{\mathrm{T}} > 0. \tag{8}$$

Remark 1: For the sake of deriving the feedback law u with $\dim(u) = m$, a sub-vector of x_d, containing only m components, is appropriately selected. This selection ensures that the resulting $m \times m$ submatrix, extracted from rows of the matrix $G(x)$ corresponding to m chosen elements of x_d, is of full rank. More importantly, it guarantees that the closed-loop system can be fully regulated using the computed control input u, thereby satisfying a specific form of the zero-state detectability (ZSD) condition as discussed in [20].

3. Approximation of time delay and stabilization for a class of linear systems

3.1. Time-delay compensation via Pade Approximation

We consider here a class of affine linear systems with time delay terms, described as follows:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau_1) + Bu(t), \qquad x(0) = 0, \tag{9}$$

where $A_0 \in \mathbb{R}^{n\times n}, A_1 \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$ are the known constant matrices, and τ_1 is the known and constant time delay.

It is important to remark that, in the Laplace domain with s being a variable, the term $x(t-\tau_1)$ can be expressed by:

$$\mathscr{L}\{x(t-\tau_1)\} = e^{-\tau_1 s}\mathscr{L}\{x(t)\}, \tag{10}$$

which yields the following equality:

$$\mathscr{L}\{x(t-\tau_1)\} = \frac{1 - \frac{\tau_1 s}{2}}{1 + \frac{\tau_1 s}{2}}\mathscr{L}\{x(t)\}, \tag{11}$$

by applying the first order Pade approximation to $e^{-\tau_1 s}$ [10], [21]. Then, by adding $\mathscr{L}\{x(t)\}$ to both sides of (11) and rearranging the terms, one can obtain the following equation:

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$$s\mathscr{L}\{\mathrm{x}(t)\} = -\frac{2}{\tau_1}\mathscr{L}\{\mathrm{x}(t)\} + \frac{4}{\tau_1}\mathscr{L}\{x(t)\}, \tag{12}$$

with the new state vector $\mathrm{x}(t)$ defined by: $\mathrm{x}(t) := x(t) + x(t-\tau_1)$. Consequently, by taking the inverse Laplace transform of (12), the dynamics of $\mathrm{x}(t)$ can be represented by the following differential equation:

$$\dot{\mathrm{x}}(t) = -\tau_1^{/}\mathrm{x}(t) + 2\tau_1^{/}x(t), \qquad \mathrm{x}(0) = 0, \tag{13}$$

with $\tau_1^{/} = \dfrac{2}{\tau_1}$. As a result, the time-delay linear system (9) can be approximated in a non-delayed form as follows:

$$\begin{cases} \dot{x}(t) = \left(A_0 - A_1\right)x(t) + A_1\mathrm{x}(t) + Bu(t), \\ \dot{\mathrm{x}}(t) = -\tau_1^{/}\mathrm{x}(t) + 2\tau_1^{/}x(t). \end{cases} \tag{14}$$

Remark 2: The state vector $\mathrm{x}(t)$ carries no physical meaning and is only used to tackle state delays. Furthermore, the system (14), obtained by approximating the original system (9) with time delay via the Pade approximation, is called a compensated system. Indeed, it clearly has a non-delay form that is usable for control design.

3.2. Delay-dependent stabilization

By using the Pade approximation, the stabilization problem of the system (9) is achieved by stabilizing the compensated system (14) under feedback laws. The following proposition proposes a relaxing PH formulation along with the damping injection R_I to ensure the convergence of the system state $\bar{x} = \begin{bmatrix} x \\ \mathrm{x} \end{bmatrix}$ towards the reference trajectory $\bar{x}_d = \begin{bmatrix} x_d \\ \mathrm{x}_d \end{bmatrix}$.

Proposition 2: The compensated system (14) can be formulated into a relaxing PH representation with the quadratic storage function $H(\bar{x}) = \frac{1}{2}\bar{x}^{\mathrm{T}}R_{di}\bar{x}$, where the constant interconnection and damping matrices, namely J and R, are expressed as follow:

$$J = \begin{bmatrix} 0 & \dfrac{A_1 - 2\tau_1^{/}\mathbf{I}_{n\times n}}{2} \\ \dfrac{-A_1 + 2\tau_1^{/}\mathbf{I}_{n\times n}}{2} & 0 \end{bmatrix} \text{and}\, R = \begin{bmatrix} A_1 - A_0 & -\dfrac{A_1 + 2\tau_1^{/}\mathbf{I}_{n\times n}}{2} \\ -\dfrac{A_1 + 2\tau_1^{/}\mathbf{I}_{n\times n}}{2} & \tau_1^{/}\mathbf{I}_{n\times n} \end{bmatrix}, \tag{15}$$

with $\mathbf{I}_{n\times n}$ denoting an identity square matrix having the dimension of n. Moreover, if the damping injection $R_I = \begin{bmatrix} \Gamma & 0 \\ 0 & \Lambda \end{bmatrix}$, where the submatrices Γ and Λ are appropriately chosen to satisfy the following inequality constraints:

$$A_1 - A_0 + \Gamma > 0, \tag{16}$$

$$\left(A_1 - A_0 + \Gamma\right)\left(\tau_1^{/}\mathrm{I}_{n\times n} + \Lambda\right) - \frac{\left(A_1 + 2\tau_1^{/}\mathrm{I}_{n\times n}\right)^2}{4} > 0, \tag{17}$$

is assigned, the dynamics of $\bar{x}$ asymptotically converges to the reference trajectory $\bar{x}_d$, governed by:

$$\dot{\bar{x}}_d = (J-R)\frac{\partial H(\bar{x}_d)}{\partial \bar{x}_d} + R_I \frac{\partial \Psi(e)}{\partial \bar{e}} + Bu(t), \tag{18}$$

with $\bar{e} = \bar{x} - \bar{x}_d$ and $G(x) = \begin{bmatrix} B \\ \mathbf{0}_{n\times n} \end{bmatrix}$.

Proof:

It can be clearly seen that the compensated system (14) can be written in the following matrix form:

$$\dot{\bar{x}} = \begin{bmatrix} A_0 - A_1 & A_1 \\ 2\tau_1^{/}\mathbf{I}_{n\times n} & -\tau_1^{/}\mathbf{I}_{n\times n} \end{bmatrix}\bar{x} + \begin{bmatrix} B \\ \mathbf{0}_{n\times n} \end{bmatrix}\begin{bmatrix} u \\ 0 \end{bmatrix}, \tag{19}$$

which can be then formulated into an extended class of PH models (3) with two structural matrices J and R as follows:

$$J = \frac{Q - Q^{\mathrm{T}}}{2} \text{ and } R = -\frac{Q + Q^{\mathrm{T}}}{2}, \tag{20}$$

with the matrix $Q := \begin{bmatrix} A_0 - A_1 & A_1 \\ 2\tau_1^{/}\mathbf{I}_{n\times n} & -\tau_1^{/}\mathbf{I}_{n\times n} \end{bmatrix}$. Moreover, it is straightforward to verify that the positive definite condition of the matrix R is not met according to Definition 1; therefore, the resulting PH formulation with the structure matrices J and R, given by (15), is indeed a relaxing PH one, i.e. $\bar{x} \in \mathscr{H}_{e,\mathrm{r}}$. The first part of the proof follows straightforwardly.

Essentially, by applying the Schur complement lemma [22], it can be seen that the inequality constraints (16) and (17) guarantees that the principal minors' determinants of the symmetric matrix $R + R_I$ are all positive, thereby verifying the positive definiteness of $R + R_I$. The latter completes the proof by invoking Proposition 1, where the notations x and x_d are adapted for $\bar{x}$ and $\bar{x}_d$, respectively.

4. Case study: Quadruple tank process with time delay

4.1. Mathematical model

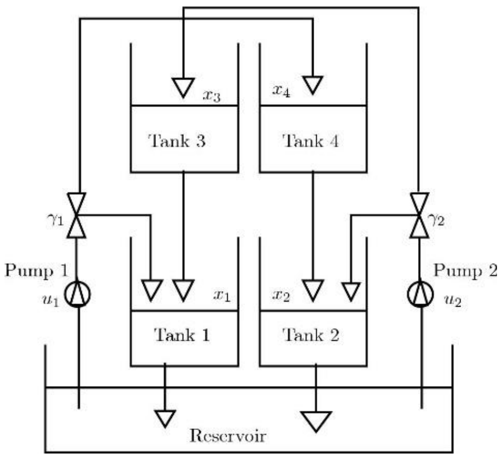

Figure 1. *Schematic diagram of a quadruple-tank process.*

This section provides a brief overview of the quadruple-tank process, a representative system featuring time delays, to illustrate the effectiveness of the proposed method. Figure 1 presents the

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schematic diagram of this process. Also, its nonlinear mathematical model is expressed as follows [12], [21]:

$$\begin{cases} \dfrac{dh_1}{dt} = -\dfrac{a_1}{A_1}\sqrt{2gh_1} + \dfrac{a_3}{A_1}\sqrt{2gh_3(t-\tau_1)} + \dfrac{\gamma_1}{A_1}v_1, \\ \dfrac{dh_2}{dt} = -\dfrac{a_2}{A_2}\sqrt{2gh_2} + \dfrac{a_4}{A_2}\sqrt{2gh_4(t-\tau_1)} + \dfrac{\gamma_2}{A_2}v_2, \\ \dfrac{dh_3}{dt} = -\dfrac{a_3}{A_3}\sqrt{2gh_3} + \dfrac{1-\gamma_2}{A_3}u_2, \\ \dfrac{dh_4}{dt} = -\dfrac{a_4}{A_4}\sqrt{2gh_4} + \dfrac{1-\gamma_1}{A_4}u_1, \end{cases} \tag{21}$$

where $h_i, i \in \{1,2,3,4\}$ is the water level of tank i and $v_j, j \in \{1,2\}$ is the input flow of the pump j. In this model, A_i and a_i represent the cross-sections of the i^{th} tank and pipes on the i^{th} tank, respectively, while γ_1 and γ_2 denote the split fractions of the two valves splitting water flows into two different directions and g is the gravitational constant. In addition, τ_1 as the time delay of the system indicates the necessary amount of time for the liquid flowing from tank 3 to tank 1 and from tank 4 to tank 1[2]. Numerical values of other parameters are given in the appendix [8].

4.2. Linearized model

In this subsection, the term $h^e = \begin{bmatrix} h_1^e & h_2^e & h_3^e & h_4^e \end{bmatrix}^T$ denotes the initial steady-state water level of each tank in the quadruple-tank process, described in Figure 1. Then, the deviation forms of state variables can be defined by $\Delta h_i := h_i - h_i^e, i \in \{1,2,3,4\}$. As a result, linearizing the nonlinear model (16) at h^e subsequently yields the linearized model, which can be written in the following matrix form:

$$\dot{x} = A_0 x(t) + A_1 x(t-\tau_1) + Bu(t), \tag{22}$$

with the vector x given by $x = \begin{bmatrix} \Delta h_1 & \Delta h_2 & \Delta h_3 & \Delta h_4 \end{bmatrix}^T$ and the matrices A_0, A_1 and B represented by:

$$A_0 = \begin{bmatrix} -\dfrac{1}{\xi_1} & 0 & 0 & 0 \\ 0 & -\dfrac{1}{\xi_2} & 0 & 0 \\ 0 & 0 & -\dfrac{1}{\xi_3} & 0 \\ 0 & 0 & 0 & -\dfrac{1}{\xi_4} \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 & \dfrac{a_3}{A_1\xi_3} & 0 \\ 0 & 0 & 0 & \dfrac{a_4}{A_2\xi_4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} \dfrac{\gamma_1}{A_1} & 0 \\ 0 & \dfrac{\gamma_2}{A_2} \\ 0 & \dfrac{1-\gamma_2}{A_3} \\ \dfrac{1-\gamma_1}{A_4} & 0 \end{bmatrix} \tag{23}$$

With the constant parameters $\xi_i = \dfrac{A_i}{a_i}\sqrt{\dfrac{2h_i^e}{g}}, i \in \{1,2,3,4\}$, respectively.

[2] In general, the transport time delay from tank 3 to tank 1 is not necessarily equal to the one from tank 4 to tank 2 but for sake of simplicity, in this paper, these time delays are assumed to be similar.

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4.3. Controller design

To implement the control design in Proposition 2, the linearized system with time-delay terms (23) is initially transferred to a compensated system having the matrix form (19). Then, a two-step procedure is applied as below:

(i) the PH representation of the compensated system with the quadratic Hamiltonian function $H(\bar{x}) = \dfrac{1}{2}\bar{x}^{\mathrm{T}} R_{di} \bar{x}$ is obtained, where the structural matrices J and R are computed by using (15) with the matrices A_0 and A_1 given in (23),

(ii) two submatrices Γ and Λ of $R_I\left(\bar{x}\right) = \mathrm{diag}(\Gamma, \Lambda)$ in this situation can be chosen as follows:

$$\Gamma = \Lambda = \begin{bmatrix} \tau_1^{/} + \theta_1 & 0 & \dfrac{a_3}{2A_1T_3} & 0 \\ 0 & \tau_1^{/} + \theta_2 & 0 & \dfrac{a_4}{2A_2T_4} \\ 0 & 0 & \tau_1^{/} + \theta_3 & 0 \\ 0 & 0 & 0 & \tau_1^{/} + \theta_4 \end{bmatrix}, \tag{24}$$

where $\theta_1, \theta_2, \theta_3$ and θ_4 are the tuning control parameters. The reference trajectory $\bar{x}_d$ is subsequently generated by (18).

In what follows, for sake of illustration, we aim to control the deviated water levels of tank 1 and 2 from the states $x_j^{\mathrm{in}}, j \in \{1, 2\}$ at the time t_1 to the final ones x_j^{f} at the time t_2. On this basis, the reference trajectories $x_{d,1}$ and $x_{d,2}$ can be particularly assigned to the following predefined functions of time, as presented by:

$$\begin{cases} x_{d,j}(t) = x_j^{\mathrm{in}} + \left[10\left(\dfrac{t-t_1}{\delta}\right)^3 - 15\left(\dfrac{t-t_1}{\delta}\right)^4 + 6\left(\dfrac{t-t_1}{\delta}\right)^5\right]\left(x_j^{\mathrm{f}} - x_j^{\mathrm{in}}\right) & \text{for } t_1 \le t \le t_2 \\ x_{d,j}(t) = x_j^{\mathrm{f}} & \text{for } t > t_2 \end{cases}, \; j = 1, 2, \tag{25}$$

with $\delta = t_2 - t_1$, which enables to derive the feedback laws $u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}$ as follows:

$$u_1 = \frac{\mathrm{A}_1}{\gamma_1}\left[\frac{1}{\delta_1} x_{d,1} + \frac{a_3}{\mathrm{A}_1\delta_3} x_{d,3} - \frac{a_3}{\mathrm{A}_1\delta_3} \mathrm{x}_{d,3} - \left(\tau_1^{/} + \theta_1\right)\left(x_1 - x_{d,1}\right) - \frac{a_3}{2\mathrm{A}_1\delta_3}\left(x_3 - x_{d,3}\right) + \dot{x}_{d,1}\right], \tag{26}$$

$$u_2 = \frac{\mathrm{A}_2}{\gamma_2}\left[\frac{1}{\delta_2} x_{d,2} + \frac{a_4}{\mathrm{A}_2\delta_4} x_{4d} - \frac{a_4}{\mathrm{A}_2\delta_4} \mathrm{x}_{d,4} - (\tau_1^{/} + \theta_2)(x_2 - x_{d,4}) - \frac{a_4}{2\mathrm{A}_2\delta_4}(x_4 - x_{d,4}) + \dot{x}_{d,2}\right]. \tag{27}$$

5. Simulations

For implementing the feedback laws (26) and (27), all state variables are assumed to be measurable via sensors, and the process is initially located at $h^e = \begin{bmatrix} 25 & 25 & 25 & 25 \end{bmatrix}^{\mathrm{T}}$ (cm). The control parameters are selected as: $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 35$, $x_1^{\mathrm{in}} = x_2^{\mathrm{in}} = 0$ at $t_1 = 0$ and $\bar{x}_1^{\mathrm{f}} = \bar{x}_2^{\mathrm{f}} = 1(\mathrm{cm})$ at $t_2 = 250\,(\mathrm{sec})$.

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The transient responses of deviation variables, given in Figure 2, show that the system trajectories asymptotically converge to the set point $\overline{x}_1^{\mathrm{f}} = \overline{x}_2^{\mathrm{f}} = 1$ from the initial states $x_1^{\mathrm{in}} = x_2^{\mathrm{in}} = 0$ after $250\,(\mathrm{sec})$. Furthermore, the representation of control input, namely u_1 and u_2, is shown in Figure 3, revealing the physical admissibility of control signals in terms of their dynamics and amplitudes. Additionally, there are the overshoots of both u_1 and u_2, playing a key role as control actions to enforce the system trajectories to follow the reference ones. From this, the negative effects of the time delay can be effectively handled. From the perspective of process system engineering, both Pumps 1 and 2 as the actuators are regulated to increase their flow rates, which is instrumental in addressing the shortage of water in tanks 1 and 2, caused by the transport delays from tank 3 to tank 1 and from tank 4 to tank 2.

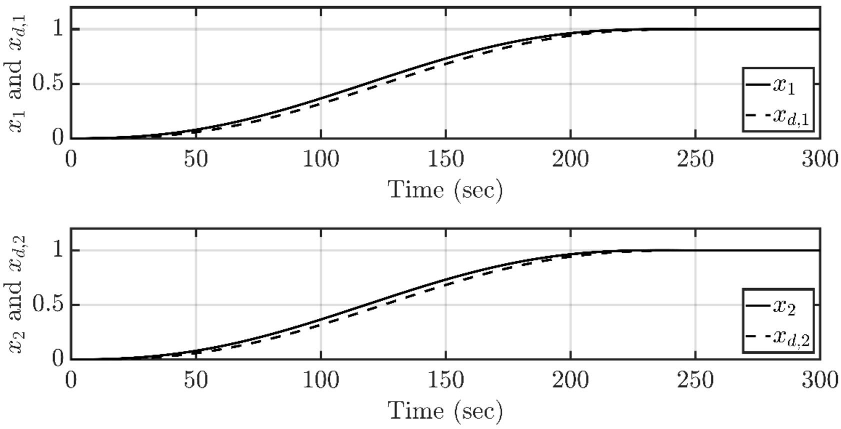

Figure 2. *Transient responses of the linearized system under effects of time delay.*

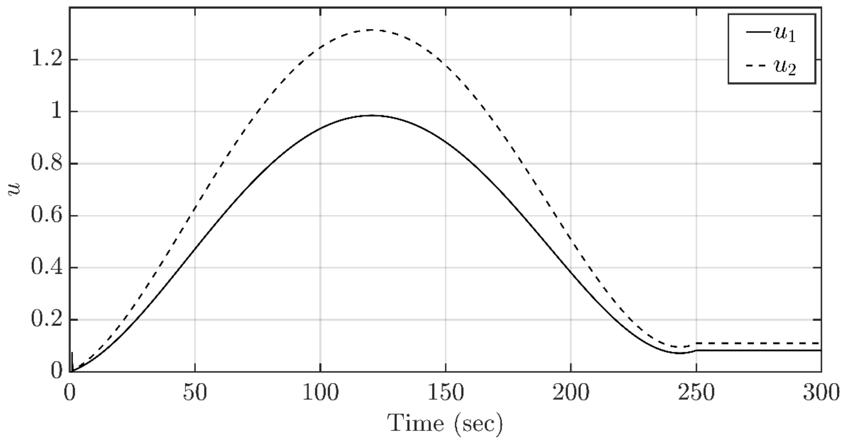

Figure 3. *Representation of the control inputs.*

6. Conclusions

In this paper, the tracking-error-based control method proposed in [7], [8] is extended to address the stabilization of a class of linear systems with time-delay effects. To manage the time-delay dynamics, the first-order Pade approximation is employed to transform the original delayed system into a compensated delay-free system, thereby facilitating the control design and mitigating the adverse effects of the delay. Stabilization of the closed-loop system is achieved by constructing a suitable reference trajectory structure and designing an appropriate damping injection matrix, thereby ensuring the convergence of the system trajectory to the desired reference. The effectiveness of the proposed approach is demonstrated through simulation studies on the quadruple-tank process with time delay. The results show that the proposed controller successfully regulates the flow rates of the two pumps—used as control inputs—to compensate for the water level deficits in tanks 1 and 2 caused by transport delays, ultimately stabilizing the system at the desired equilibrium point. As a direction for future work, the treatment of time delays in nonlinear systems will be investigated, moving beyond Pade approximation techniques to develop more general and robust control strategies.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Appendix.

Table A1. *Physical and operating paramters for the quadruple-tank process.*

Parameter	Symbol	Value
Cross section of pipes on the tank 1, 2, 3, 4 (cm^2)	(a_1, a_2, a_3, a_4)	(0.233, 0.233, 0.127, 0.127)
Cross section of tank 1, 2, 3, 4 (cm^2)	$(\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3, \mathrm{A}_4)$	(50.3, 50.3, 28.3, 28.3)
Split fraction of value 1 and 2 (%)	(γ_1, γ_2)	(40, 30)
Gravitational constant (cm/s^2)	g	981
Time delay (sec)	τ_1	10

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