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TRAJECTORY TRACKING SLIDING MODE CONTROL FOR CART AND POLE SYSTEM

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ABSTRACT

Cart and Pole is a classical model in the control laboratories for testing control algorithm. Balancing control at equilibrium the point has been operated many times on this model with various methods. However, a control algorithm that makes system to track a suggested trajectory, when stability requirement is guaranteed by mathematics, is still opened. In this paper, the authors suggest using a sliding mode control – a nonlinear algorithm- to stabilize cart and pole system at an equilibrium point. Then, this algorithm controls the cart to track the trajectory of sine signal and pulse signal when still stabilizing pendulum on inverted position. Sliding Mode Control method is familiar in control and automation. In this paper, the abilities of Sliding Mode Control are shown its abilities in both simulation and experiment results. On Matlab/Simulink simulation, Sliding Mode Control proves its advantages over LQR control. Then, experiments show the results of applying a sliding control for real model .

Keywords: *cart and pole; sliding control; LQR control; balancing control; trajectory tracking control.*

1. INTRODUCTION

Cart and Pole (C&P) is a classical model in control engineering. By practising on this model, methods to stabilize SIMO (simple input multiple output) systems are developed [1-4]. Among those methods, LQR is an effective method due to its simple structure. Solving Ricatti equation by Matlab commands was designed to simplify the process of finding a feedback control matrix of this method. However, LQR just is a linear control algorithm and often used in the equilibrium problem [5], [6]. Therefore, this method just guarantees the stability of the system if its condition is near the equilibrium point. Some authors [3], [7] presented the tracking-LQR way for C&P by changing the equilibrium point to force the cart moving to follow the “new” equilibrium point. But, this way is not guaranteed by mathematics and if “new” equilibrium is far from an initial position, the system is un-stabilized. In order to solve this problem, in this paper, we propose a sliding mode control (SMC) method - nonlinear control algorithm- not only to stabilize the C&P but also control it tracking the sine and pulse trajectories. SMC has been used widely in many laboratories not only in Vietnam but also around the world [8], [9]. This means SMC is very popular and has high efficiency in the field of control- working well with many different nonlinear systems. Due to satisfying Lyapunov criteria, this method is proved to control well C&P in both simulation and real experiment.

This paper concludes 6 sections. Section 1 presents the topic of paper. Section 2 describes mathematical model of C&P. Section 3 lists both LQR and SMC methods for stabilizing and trajectory tracking of this model. Section 4 shows simulation results. Experimental results are shown in section 5. Then, section 6 ends paper by a conclusion.

2. MATHEMATICAL MODEL

From [10], mathematical structure of C&P is shown in Fig. 1 below.

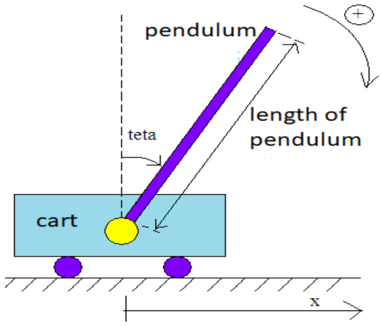

***Fig. 1** Mathematical structure of C&P*

Due to Euler-Lagrange method, dynamic equations of C&P are

$$\frac{d\left(\frac{\partial L}{\partial \dot{q}}\right)}{dt} - \frac{\partial L}{\partial q} = Q \qquad (1)$$

where: $L = T - V$ is Lagrange operator; $T = T_{pole} + T_{cart}$ is kinetic energy of system, $V = V_{pole}$ is potential energy of system, Q is the sum of external forces on system; $q = [x \quad \theta]^T$; $Q = [F \quad 0]^T$; x is position of cart (m); θ is angle of pendulum (rad); F is external force on cart (N)

By physical calculation [10], (1) is obtained as

$$(m+M)\ddot{x} + mC_1\ddot{\theta}\cos\theta - mC_1\dot{\theta}^2\sin\theta = F \qquad (2)$$

$$mC_1\ddot{x}\cos\theta + (J_1 + mC_1^2)\ddot{\theta} - mC_1 g\sin\theta = 0 \qquad (3)$$

where: C_1 is length of pendulum (m); m is mass of pendulum (kg); M is mass of cart (kg); J_1 is inertial moment of pendulum (kgm^2)

In the real model, voltage is signal to apply on the motor and then, force is caused to affect the cart. Therefore, in order to make simulation closed to real experiment, the voltage on the motor is selected as control input signal.

Also from [10], in the case that moment caused by DC motor is transferred into force F that affects the cart, relation between the voltage on DC motor and force on a cart is presented as below

$$F = \frac{d_l}{R}\left[\frac{K_t}{R_m}e - d_l\left(\frac{K_bK_t}{R_mR} + \frac{C_m}{R}\right)\dot{x} - \frac{J_md_l}{R}\ddot{x}\right] \qquad (4)$$

From (2)-(4), in the case that input signal of system is voltage, the dynamic equations of C&P are

$$M_f(q)\ddot{q} + V_{mf}(q,\dot{q})\dot{q} + G_f(q) = [k_1e \quad 0]^T \qquad (5)$$

where: $M_f(q) = \begin{bmatrix} m+M+k_3 & mC_1\cos\theta \\ mC_1\cos\theta & J_1+mC_1^2 \end{bmatrix}$;

$V_{mf} = \begin{bmatrix} k_2 & -mC_1\dot{\theta}\sin\theta \\ 0 & 0 \end{bmatrix}$; $G_f = \begin{bmatrix} 0 \\ -mC_1g\sin\theta \end{bmatrix}$

where: $k_1 = \frac{d_lK_t}{R_mR}$; $k_2 = \frac{d_l^2K_tK_b}{R^2R_m} + \frac{d_l^2C_m}{R^2}$; $k_3 = \frac{d_l^2J_m}{R^2}$; R is radius of pully on motor (m); d_l=1 is the rate of motion transmission; R_m is internal register of motor (ohm); L_m is resistance factor of motor (H); K_b is reactive constants of motor (V/(rad/sec)); K_t is moment constant of motor (Nm/A); J_m is inertial moment of rotor (kgm^2); C_m is viscosity constant of motor (Nmsec/rad); T_f is friction moment of motor (Nm).

Equations (2), (3) are written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + g_1(x).u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + g_2(x).u \end{cases} \qquad (6)$$

where: $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [\alpha \quad \dot{\alpha} \quad \beta \quad \dot{\beta}]^T$

To control tracking, the reference state variables are set as:

$$X_d = [x_{1d} \quad x_{2d} \quad x_{3d} \quad x_{4d}]^T$$

3. CONTROL ALGORITHM

3.1 LQR controller

LQR algorithm is a classical control method [11]. Mathematical proof through solving Ricatti equation guarantees stability about working point. But, an exact working region of the system cannot be defined exactly. In some models, this zone is very small and the stability of LQR controller is not guaranteed when the condition of the system is a little far from equilibrium. Some researches [3], [7] prove the effectiveness of this method in both simulation and experiment. Structure of LQR stabilizing controller is shown in Fig. 2. Control feedback matrix K is found by choosing control matrixes Q, R and calculating by Matlab commands. With that K, a structure of LQR trajectory tracking control is shown in Fig. 3.

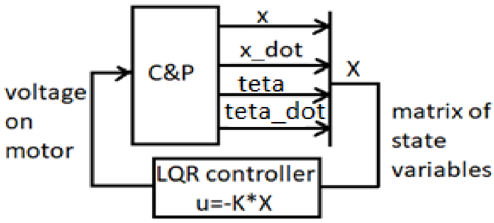

Fig. 2 *LQR stabilizing control for C&P*

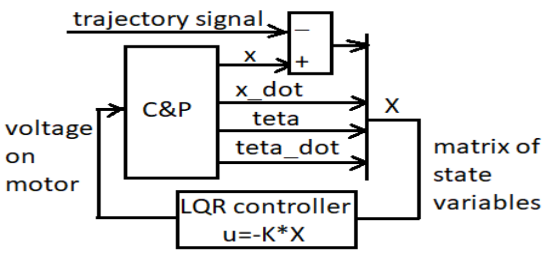

Fig. 3 *LQR trajectory tracking control for C&P*

In tracking control (Fig. 3), the feedback signal of the position of cart is deviated by minus an amount of value of trajectory signal. By this way, the equilibrium is changed along with trajectory. This change forces the cart to move along with trajectory. This method is not guaranteed by mathematics.

3.2 SMC controller

In [12], an incremental SMC is presented. Section 3.2 presents this method for applying in stabilizing and tracking control for C&P in (7) to (22) below.

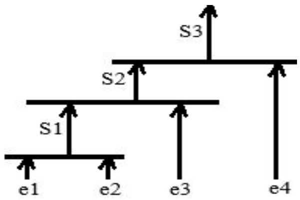

Fig. 4 *Structure of SMC control*

Sliding surfaces are defined as below

$$s_1 = c_1 e_1 + c_2 e_2 \quad (7)$$

$$s_2 = s_1 + c_3 e_3 \quad (8)$$

$$s_3 = s_2 + c_4 e_4 \quad (9)$$

where: $e_i = x_i - x_{id}$ (i = 1, 2, 3, 4) is the error between variable x_i and reference signal x_{id}.

Based on (6), the derivative values of sliding surfaces in (7)-(9) are listed as below

$$\dot{s}_1 = c_1\dot{e}_1 + c_2\dot{e}_2 = c_1(\dot{x}_1 - \dot{x}_{1d}) + c_2(\dot{x}_2 - \dot{x}_{2d}) = c_1(\dot{x}_1 - \dot{x}_{1d}) + c2\big(f_1(x) - g_1(x)u_1 - \dot{x}_{2d}\big) \quad (10)$$

$$\dot{s}_2 = \dot{s}_1 + c_3\dot{e}_3 = \dot{s}_1 + c_3(x_4 - \dot{x}_{3d}) = c_1(\dot{x}_1 - \dot{x}_{1d}) + c_2(f_1(x) + b_1(x)u_2 - \dot{x}_{2d}) + c_3(x_4 - \dot{x}_{3d}) \quad (11)$$

$$\dot{s}_3 = \dot{s}_2 + c_4\dot{e}_4 = c_1(\dot{x}_1 - \dot{x}_{1d}) + c_2(f_1(x) + b_1(x)u_3 - \dot{x}_{2d}) + c_3(x_4 - \dot{x}_{3d}) + c_4(f_2(x) + b_2(x)u_3 - \dot{x}_{4d}) \quad (12)$$

Control signals for each sliding surface are:

$$u_i = u_{eq(i)} + u_{sw(i)} \quad \text{(i=1, 2, 3)} \quad (13)$$

where: $u_{eq(i)}$ is control signal to keep components following sliding surface; $u_{sw(i)}$ is control signal to move components to sliding surface.

Let $\dot{s}_i = 0$, then, from (10)-(12), we obtain

$$u_{eq1} = \frac{-\left(c_1\left(x_2 - \dot{x}_{1d}\right) + c_2 f_1\left(x\right)\right)}{c_2 b_1\left(x\right)} \tag{14}$$

$$u_{eq2} = \frac{-\left(c_1\left(x_2 - \dot{x}_{1d}\right) + c_2 f_1\left(x\right) + c_3 x_4\right)}{c_2 b_1\left(x\right)} \tag{15}$$

$$u_{eq3} = \frac{-\left(c_1\left(x_2 - \dot{x}_{1d}\right) + c_2 f_1\left(x\right) + c_3 x_4 + c_4 f_2\left(x\right)\right)}{c_2 b_1\left(x\right) + c_4 b_2\left(x\right)} \tag{16}$$

Lyapunov function is defined as

$$V = \frac{1}{2} s_3^2 \tag{17}$$

Then, derivative of Lyapunov function is

$$\dot{V} = s_3 \dot{s}_3 \tag{18}$$

where:

$$\begin{aligned}\dot{s}_3 &= c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 + c_4\dot{e}_4 \\ &= c_1\left(x_2 - \dot{x}_{1d}\right) + c_2\left(f_1\left(x\right) + b_1\left(x\right)\left(u_{sw3} + u_{eq3}\right)\right) + \\ &\quad + c_3 x_4 + c_4\left(f_2\left(x\right) + b_2\left(x\right)\left(u_{sw3} + u_{eq3}\right)\right)\end{aligned}$$

$$\begin{aligned}\dot{s}_3 &= c_1\left(x_2 - \dot{x}_{1d}\right) + c_2 f_1\left(x\right) + c_3 x_4 \\ &+ c_4 f_2\left(x\right) + \left(c_2 b_1\left(x\right) + c_4 b_2\left(x\right)\right)u_{eq3} \\ &+ \left(c_2 b_1\left(x\right) + c_4 b_2\left(x\right)\right)u_{sw3}\end{aligned} \tag{19}$$

Substituting (16) into (19), we obtain

$$\dot{s}_3 = \left(c_2 b_1\left(x\right) + c_4 b_2\left(x\right)\right)u_{sw3} \tag{20}$$

Because V>0, in order to satisfy Lyapunov criteria, $\dot{V}$ should be chosen to be negative. Therefore, we choose u_{sw3} as (21) to make $\dot{V} < 0$:

$$u_{sw3} = \frac{-ks_3 - \eta sign\left(s_3\right)}{c_2 b_1\left(x\right) + c_4 b_2\left(x\right)} \tag{21}$$

where: $k, \eta = const > 0$

Thence, the control signal $u_3 = u_{eq3} + u_{sw3}$ guarantees $s_3 \xrightarrow{t\to\infty} 0$. Besides, $s_1, s_2 \xrightarrow{t\to\infty} 0$ and $e_1, e_2 \xrightarrow{t\to\infty} 0$.

Thence, SMC signal for total C&P is

$$u = u_{eq} + u_{sw} \tag{22}$$

where:

$$u_{eq} = \frac{-\left(c_1\left(x_2 - \dot{x}_{1d}\right) + c_2 f_1\left(x\right) + c_3 x_4 + c_4 f_2\left(x\right)\right)}{c_2 b_1\left(x\right) + c_4 b_2\left(x\right)}$$

and $u_{sw} = \dfrac{-ks_3 - \eta sign\left(s_3\right)}{c_2 b_1\left(x\right) + c_4 b_2\left(x\right)}$

4. SIMULATION

4.1 Condition of Simulation

The C&P system coefficients for simulation are:

$$M = 0.39\ ;\quad m = 0.23\ ;\quad C_1 = 0.48\ ; \quad R = 0.24;\ g = 9.81;\ J_1 = \frac{m^2 + 0.003^2}{12}. \tag{23}$$

Coefficients of DC motor are

$$\begin{aligned} &K_b = 0.086164500636167; \\ &R_m = 11.944421124154792; \\ &J_m = 0.000059833861116\ ; \\ &C_m = 0.000067435629646\ ; \\ &R_m = 0.015. \end{aligned} \tag{24}$$

These coefficients are closed to the real model in Section 5 for experiment. Therefore, the simulation results are expected to be closed to experimental results. With LQR and SMC that are designed in Section 3, control parameters of these controller can be chosen through genetic algorithm. Thence, control parameters of LQR controller are

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ R = 1;\ K = \begin{bmatrix} -7.8256 \\ -2.1168 \\ 36.1042 \\ 5.9293 \end{bmatrix}^T \tag{25}$$

SMC control parameters are

$c_1 = 232.921;\ \ c_2 = 254.559;\ \ c_3 = -910.366\ ;$

$$c_4 = -279.992\,;\quad k = 150\,;\quad \eta = 0.2. \tag{26}$$

The initial values of variables of C&P are selected as

$$x_{init} = \dot{x}_{init} = \dot{\theta}_{init} = 0\,;\quad \theta_{init} = 0.6(rad) \tag{27}$$

4.2 Stabilizing control

Simulation results are shown in Fig. 5, Fig. 6.

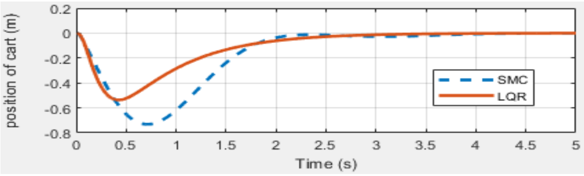

***Fig. 5** Position of Cart (m)*

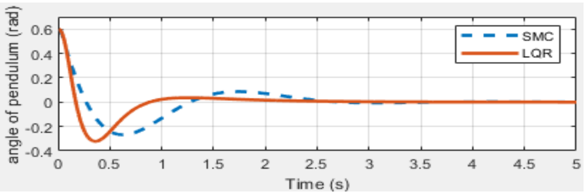

***Fig. 6** Angle of Pendulum (rad)*

From Fig. 5 and Fig. 6, both controllers operate well. The LQR method has better settling time and has small overshoot compares to SMC. This proves that when only controlling the linear operating system around the equilibrium point, LQR gives better results.

If the initial value of angle of pendulum in (27) is chosen as 0.8 (rad) instead of 0.6 (rad), then, simulation results are shown in Fig. **7** and Fig. **8** below.

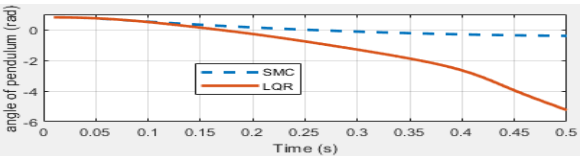

***Fig. 7** Angle of Pendulum in first 0.5s*

In Fig. *7*, under LQR controller, after 0.5s, the angle of pendulum moves to the value of 5(rad). In this situation, the system is uncontrollable and the pendulum is unbalanced. But, under SMC controller, the pendulum is kept balanced. If the period of examining in Fig. *7* is extended from 0.5s to 5s, the response of angle of pendulum under SMC controller is shown in Fig. *8*. It is obvious that the SMC can balance well the pendulum even the initial value of pendulum is far from equilibrium point (in that same situation, LQR method cannot control well - Fig. *6*).

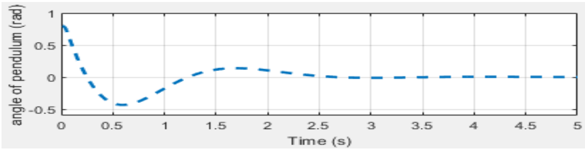

***Fig. 8** Angle of pendulum (rad) under SMC controller in first 5s*

Along with the angle of pendulum in Fig. 8, the position of cart is also shown in Fig. 9. In this figure, the SMC proves that it is suitable for SIMO system when both angles of pendulum and position of cart are stabilized at one place well by only one control input- voltage on DC motor.

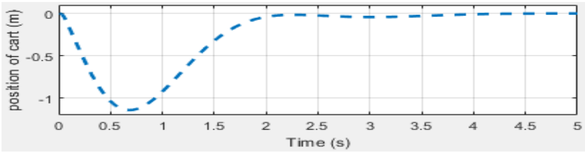

***Fig. 9** Position of Cart (m) under SMC controller in first 5s*

4.3 Tracking control

From controller designing in Section 3 and parameters of control methods and C&P system in Section 4.1, the simulation of tracking control is shown in Fig. 10 to Fig. 13.

4.3.1. Trajectory is pulse signal

In Fig. 10, a period of trajectory is the 20s, both LQR and SMC controllers work well in making system tracking the pulse signal. The settling time is the same for both control methods. But, the overshoot is smaller in the case of SMC. Thence, the SMC gives a better response of position of a cart than LQR.

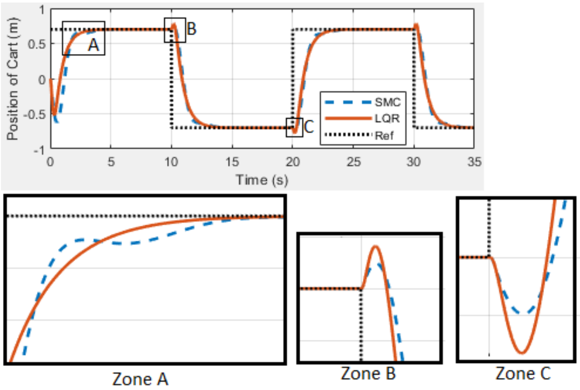

Fig. 10 *Position of Cart (m) following the trajectory of pulse (Ref) (period is 20s)*

Following Fig. 10, the response of pendulum is shown in Fig. 11 below.

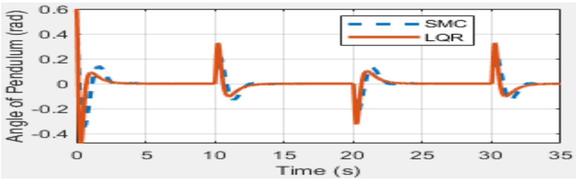

Fig. 11 *Angle of Pendulum(rad) following the trajectory of pulse (Ref) (period is 20s)*

In Fig. 11, the vibration of the pendulum under SMC is smaller than under LQR. Besides, the settling time is the same in both cases.

4.3.2. Trajectory is sine signal

If trajectory is sine signal which has period of 20s. Then, the simulation responses of system under SMC and LQR are shown in Fig. 12 to Fig. 13 below.

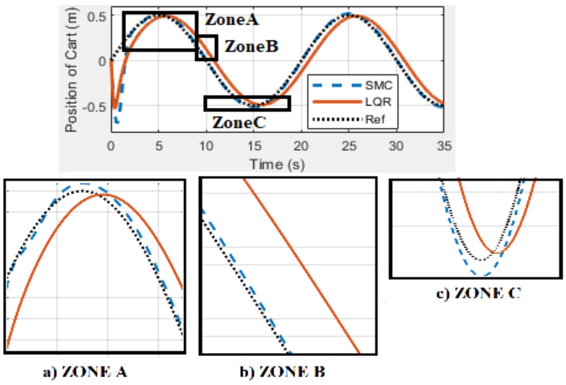

Fig. 12 *Position of Cart following the trajectory of sine (Ref) (period is 20s)*

In Fig. 12, the position of cart does not track well the trajectory under LQR. If using LQR method, there is a delay time (about 1s if the period of trajectory is 20s). Besides, there is no day time of response of cart under SMC. Thence, tracking control under SMC is better.

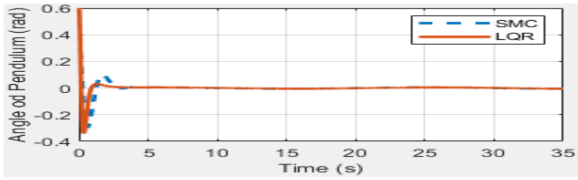

Fig. 13 *Angle of Pendulum following the trajectory of sine (Ref) (period is 20s)*

In Fig. 13, because both methods control well system following the trajectory, the angle of a pendulum is stabilized well with the same settling time and overshoot.

5 EXPERIMENT

5.1 Introduction of hardware

An experimental C&P model is presented in Fig.14. The system concludes of a slider (number 1) that slides horizontal on a scroll bar. The role of slider is the cart. On this cart, an encoder is placed and a metal bar (number 3) that takes the role of pendulum is connected to the axis of the encoder on the slider. A DC motor (number 4) controls the motion of slider through a bully and belt. All these components are placed on a hard solid base (number 2). The electrical components of C&P are presented in Fig. 15. STM32F4 is used as the controller board due to its cheap price, the ability of being embedded by Matlab tool, its high speed of operation.

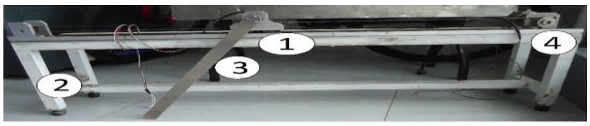

Fig.14 *Experimental model*

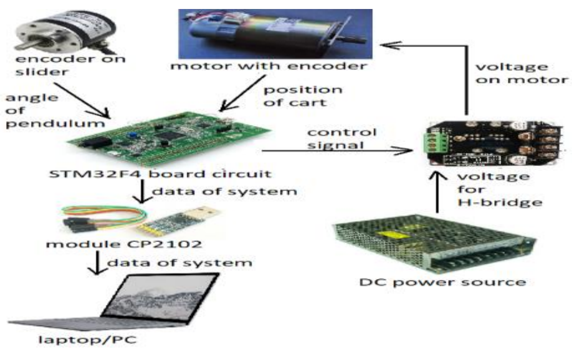

Fig. 15 *Connection of electrical components*

5.2 Stabilizing control

With the same controllers and system parameters in Section 4.2, the LQR controller is unable to real model system. But, the SMC controller still can balance system with the simulation results in Fig. 16 and Fig. 17.

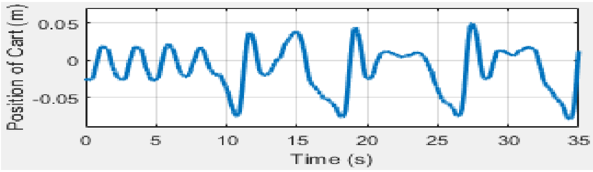

Fig. 16 *Position of Cart (m)*

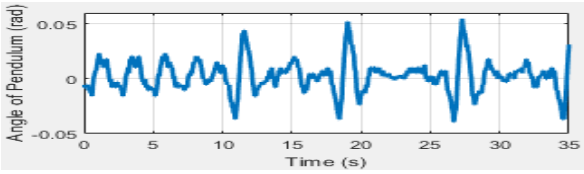

Fig. 17 *Angle of Pendulum (rad)*

Because the real model is not homogeneous with the simulation model (Actually, the pendulum is not completely homogeneity in all length. There is the wrongness in identifying the parameters of DC motor. There are the effects of friction of cart's motion…), the control parameters are only acceptable. In this case, the SMC parameters are proved to be more under over the uncertain of real model than the LQR parameters. These explanations can be used to describe why only SMC gives successful results in stabilizing real model.

5.3 Tracking control

Only the controller that can stabilize the equilibrium can be examined in the ability of tracking control because tracking control is developed after successful stabilization. From Section 5.2, only SMC can stabilize successfully the C&P system. Thence, SMC is object for this section.

5.3.1. Trajectory of pulse signal

When the period of pulse signal is 20s, the experimental responses of real C&P are shown in Fig. 18 and Fig. 19.

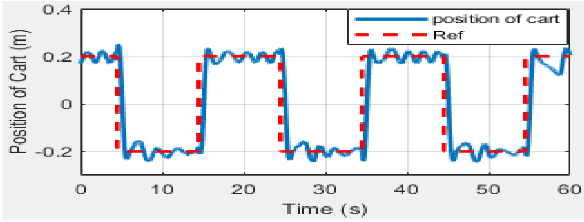

Fig. 18 *Position of Cart under SMC when trajectory is pulse signal (Ref) (period is 20s)*

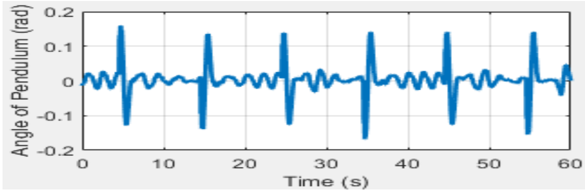

Fig. 19 *Angle of Pendulum under SMC when trajectory is pulse signal (period is 20s)*

From these figures, SMC controller is proved to control system tracking the pulse system (with the same period as in simulation). If the period is decreased to 10s, the experimental results are shown in Fig. 20 and Fig.21. In these figures, SMC shows the ability to make system tracking higher frequency trajectory of pulse signal.

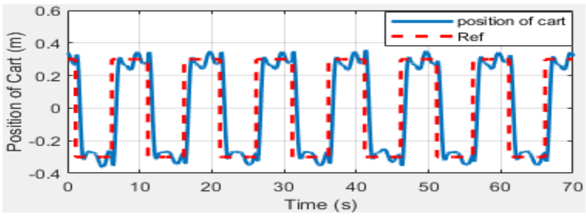

Fig. 20 *Position of Cart under SMC when trajectory is pulse signal (Ref) (period is 10s)*

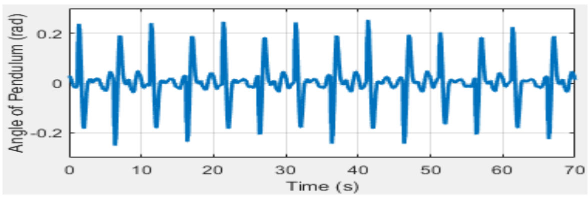

Fig. 21 *Angle of Pendulum under SMC when trajectory is pulse signal (period is 10s)*

5.3.2. Trajectory of sine signal

The sine trajectory is easier than pulse trajectory for system to follow due to its twisty shape. In this experiment, period of pulse signal is 20s, the experimental responses of real C&P are shown in Fig. 22 and Fig. 23.

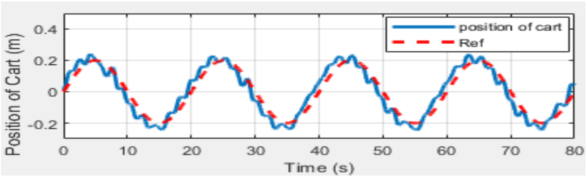

Fig. 22 *Position of Cart under SMC when trajectory is sine signal (Ref) (period is 20s)*

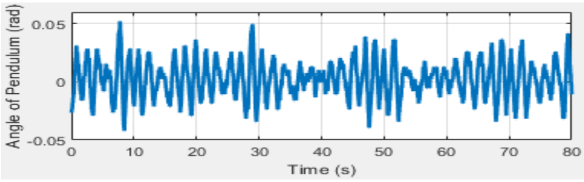

Fig. 23 *Angle of Pendulum under SMC when trajectory is sine signal (period is 20s)*

In these figures, SMC still shows its ability to track the system following the sine signal well.

6 CONCLUSION

In this paper, a method of SMC is presented to control C&P system stabilizing an equilibrium point and tracking pulse and sine signal in both simulation and experiment. LQR control is also presented to have a comparison between kind of control methods. In simulation, LQR controller cannot stabilize system if values of variables are far from equilibrium point when SMC can. The delay time when tracking under LQR control proves that tracking control on simulation shows better results under SMC than under LQR. Otherwise, on real model, under uncertain of real system, only SMC can work. Therefore, SMC shows it ability to practice on real model.

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