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DESIGN A DYNAMIC SLIDING MODE CONTROLLER FOR A BALL-BEAM SYSTEM

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ABSTRACT

This paper present the dynamic sliding mode controller to control a ball-beam system. These purposes compare results with Static and Fuzzy sliding mode control [2], [4-6] (SSMC and FSMC). A lot of studies is proposed to control nonlinear system as well as this ball on a beam system such as PD-Fuzzy, sliding mode control (SMC). However, every algorithm has advantages and disadvantages. Control parameters of the PD-Fuzzy controller can be chosen due to the expert's experiences and parameters of PD is fixed in the operating of the system which makes system non-flexible. SMC is response very good however appearance phenomenon chattering by Sign function in the sliding surface. This paper proposes a solution to reduce phenomenon chattering by algorithm dynamic sliding mode controller (DSMC). Simulations, as well as implementation results on real experiments, showed that the proposed control works well.

Keywords: *Sliding mode control; balance control; PD-Fuzzy; ball and beam; under-actuated systems.*

1. INTRODUCTION

The nonlinear control systems have been important research. Many approaches have been proposed to control the muti input multi output (MIMO) and single input single output (SISO) system. A ball-beam system is a non-stabilized and complicated single input multi output (SIMO) system which is highly nonlinear. So it is controlled by nonlinear algorithm [7-8] or intelligent control algorithms, such as neural network and fuzzy-logic controllers [9-10]. These papers deal with the dynamic sliding mode controller of the ball-beam system to reduce the chattering phenomenon.

The paper is organized as follow. Modeling of a ball-beam system is present in section 2. In Section 3, design controller dynamic sliding mode control. Simulation results are given and discussed in Section 4. Control results are shown in Section 5. Finally, some concluding remarks are given in Section 6.

2. A DYNAMIC MODEL OF A BALL-BEAM SYSTEM

Refer to Fig1 we can see, a ball is placed on a beam where it is allowed to roll with one degree of freedom along the length of the beam. The ball is rolled by gravity force and the change of angle the beam through the lever arm moment of the DC motor.

The mathematical structure of a ball beam system is described by equations as follow.

$$\ddot{r} = \frac{1}{k_4}\left(r\dot{\alpha}^2 - gsin\alpha\right) \quad (1)$$

$$\ddot{\alpha} = \frac{1}{\left(mr^2 + k_1\right)}\left[u - \left(2mr\dot{r} + k_2\right)\dot{\alpha} - \left(mgr + \frac{L}{2}Mg\right)cos\alpha\right] \quad (2)$$

Where:

$$k_1 = \frac{R_m J_m L}{K_m K_g d} + J_B$$

$$k_2 = \frac{L}{d}\left(\frac{K_m K_b}{R_m} + K_b + \frac{R_m B_m}{K_m K_g}\right)$$

$$k_3 = 1 + \frac{K_m}{R_m}$$

$$k_4 = \frac{7}{5}$$

m	Mass of the ball
L	Length of the beam
η	Effective of motor

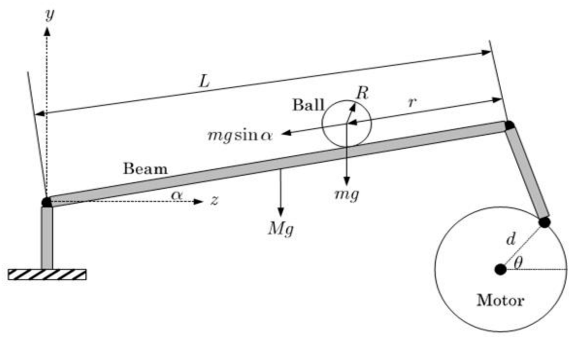

Figure 1. *Schematic diagram a ball-beam system*

$v_{in}(t)$: voltage supply to the motor

$u(t) = k_3 V_{in}(t)$: control input to the Ball on a beam system

Table1. *Parameters of the ball on a beam system*

Symbol	Description
R_m	Armature the resistance of the motor
K_g	Gear ratio
K_m	Motor torque constant
J_m	Effective moment of inertial
$\theta(t)$	Servo gear angle
$\alpha(t)$	Beam angle
$r(t)$	Ball position
g	Gravitational constant
M	Mass of the beam

In fact, as well as theory control moment of the motor is *u(t)* consider Fig2 and [3], [6] so we get:

$$u(t) = \eta . K_g . J_m \quad (3)$$

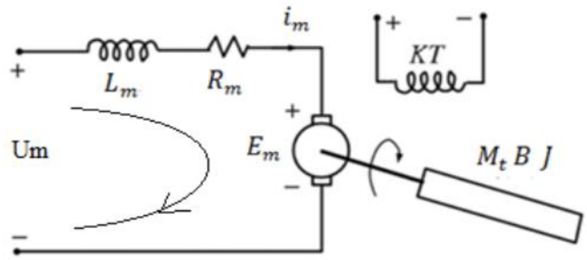

Figure 2. *Equation characteristic of DC motor*

The equilibrium of the system is working condition. So we get equation as follows:

$$\left(mgr_e + \frac{L}{2}Mg\right) cos\alpha_e = u_e \quad (4)$$

$$sin\alpha_e = 0 \quad (5)$$

3. DESIGN CONTROLLER DSMC FOR SYSTEM

Consider equation nonlinear of the system [4]

$$y^{(n)} = f(y, \dot{y}, \ddot{y}, y^{(n-1)}) + g(y, \dot{y}, \ddot{y}, y^{(n-1)})u$$

Set $x_1 = y, x_2 = \dot{y}, x_3 = \ddot{y}, \ldots x_n = y^{(n-1)}$

So system state equation is rewritten as follow:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \ldots. \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x) + g(x)u \end{cases} \quad (6)$$

Output signal $y = x_1$.

Define the sliding the surface as follows:

$$S = e^{(n-1)} + a_{n-2}e^{(n-2)} + \ldots + a_1\dot{e} + a_0 e$$

The control law is determined as follow

$$u = -\frac{1}{g(x)}\begin{bmatrix} f(x) + a_{n-2}(x_n - r^{(n-1)}) \\ + \ldots + a_1(x_3 - \ddot{r}) + a_0(x_2 - \dot{r}) - r^{(n)} + \\ \alpha sign(S) \end{bmatrix}$$

Apply to a ball-beam system we get:

From (1), (2) and (6) rewrite equation state of the system as follow:

$$\dot{x}_1 = x_2 \quad (7)$$

$$\dot{x}_2 = \frac{1}{k_4}\left(r{x_4}^2 - g\sin x_3\right) = f_2 \quad (8)$$

$$\dot{x}_3 = x_4 \quad (9)$$

$$\dot{x}_4 = \frac{1}{(m{x_1}^2+k_1)}\left[u - (2mx_1x_2 + k_2)x_4 - \left(mgx_1 + \frac{L}{2}Mg\right)\cos x_3\right] = f_4 \quad (10)$$

The sliding surface is determined [1]:

$$s_2 = \ddot{e}_\alpha + \lambda_1\dot{e}_\alpha + \lambda_2 e_\alpha + \lambda_3\dot{e}_r + \lambda_4 e_r$$
$$= \ddot{\alpha} + \lambda_1\dot{\alpha} + \lambda_2\alpha + \lambda_3\dot{r} + \lambda_4(r - r_d) \quad (11)$$

$s_2.\dot{s_2} < 0$ satisfy the Lyapunov stability criterion.

Where:

e: error of set signal and real signal

$$e_r = r - r_d, \dot{e_r} = \dot{r} - \dot{r_d} = \dot{r}$$

$$e_\alpha = \alpha - \alpha_e, \dot{e}_\alpha = \dot{\alpha}, \ddot{e}_\alpha = \ddot{\alpha}$$

$\lambda_1, \lambda_2, \lambda_3$, and λ_4 are scalars such that $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 < 0, \lambda_4 < 0, \lambda_1\lambda_2 + \frac{g}{k_4}\lambda_3 > 0$ and ${\lambda_1}^2\lambda_4 > \lambda_1\lambda_2\lambda_3 + \frac{g}{k_4}{\lambda_3}^2$ are chosen to satisfy the Routh–Hurwitz stability criterion.

The control law is determined:

$$\dot{u} = (mr^2 + k_1) * \left\{-f + \frac{2mr\dot{r}}{(mr^2+k_1)^2}u - \lambda_1\frac{u-(2mr\dot{r}+k_2)\dot{\alpha}-(mgr+\frac{L}{2}Mg)\cos\alpha}{(mr^2+k_1)} - \lambda_2\dot{\alpha} - \lambda_3\frac{r\dot{\alpha}^2-g\sin\alpha}{k_4} - \lambda_4\dot{r} - \Gamma_4 sgn(s_2)\right\} \quad (12)$$

Where:

$$f = \frac{2mr\dot{r}}{\left(mr^2+k_1\right)^2}\left[\left(2mr\dot{r}+k_2\right)\dot{\alpha} + \left(mgr + \frac{L}{2}Mg\right)\cos\alpha\right]$$
$$-\frac{1}{\left(mr^2+k_1\right)}[\left(2mr\dot{r}+k_2\right)\ddot{\alpha} + 2m\left(\dot{r}^2 + \frac{r^2\dot{\alpha}^2 - gr\sin\alpha}{k_4}\right)\dot{\alpha} \quad (13)$$
$$-\left(mgr + \frac{L}{2}Mg\right)\dot{\alpha}\sin\alpha + mg\dot{r}\cos\alpha$$

4. RESULTS OF SIMULATION

Based on the parameters of real experimental, we get parameters as follow:

$m=60.10^{-3}kg$, $M=250.10^{-3}kg$, $g=9.81$ m/s^2, $L=0.6m$, $d=0.08m$, $R=12.10^2m$, $K_g=42/12$, $R_m=6.83\ \Omega$, $B=3.374.10^{-4}Nm(rad/s)$, $J_m=5.0246.10^{-4}Kg.m^2$

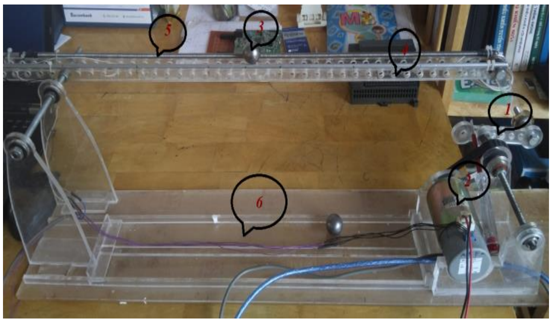

***Figure 3.** Experimental model*

1-Level of the arm, 2-DC Servo motor, 3-ball, 4-the beam, 5-Sensor to determine the ball position, 6-Base.

Parameters λ_1=42; λ_2=442; λ_3=-362.5; λ_4=-322.5; Γ_4=12 are selected such as the root of a polynomial.

$$\Delta = (s) = s^4 + \lambda_1 s^3 + \lambda_2 s^2 - \lambda_3\frac{g}{k_4}s - \lambda_4\frac{g}{k_4} = 0$$

The control structure of a ball-beam system is based on control law (12).

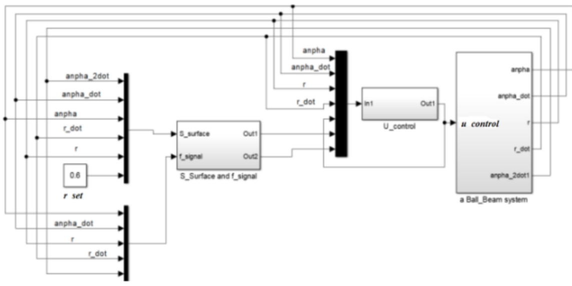

***Figure 4.** Control structure of a ball-beam system*

In this section, we will compare results simulation of DSMC with [2]-FSMC, [4-5]-SSMC.

With conditions to simulate modelling

alpha_init=-0.0927; is initial condition of beam angle.

r_init=0.2; is initial condition of the ball position.

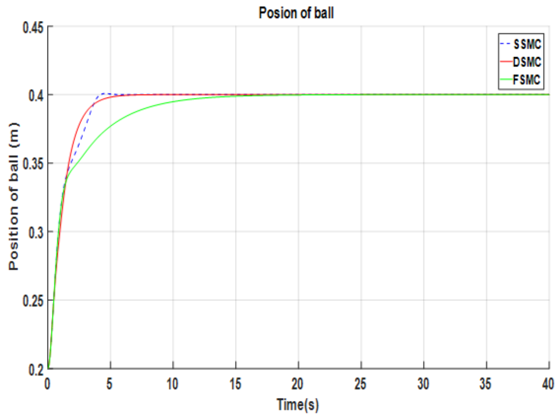

Figure 5. *Result of ball position at r_set = 0.4*

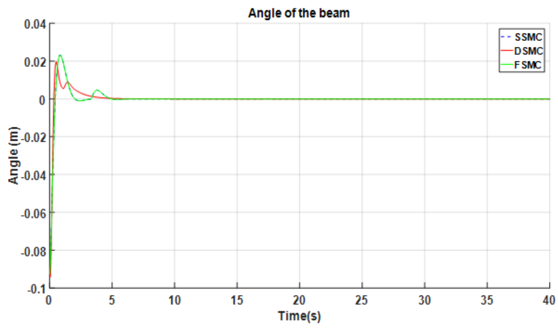

Figure 6. *Result of beam angle at r_set= 0.4*

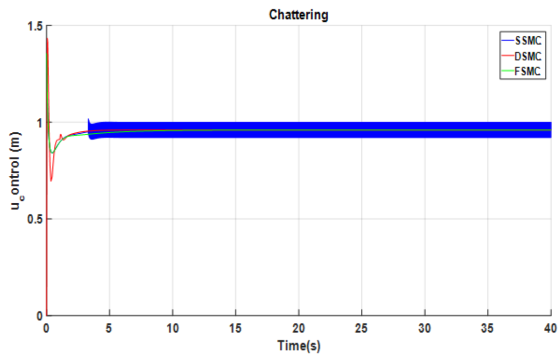

Figure 7. *Result of u_control at r_set = 0.4*

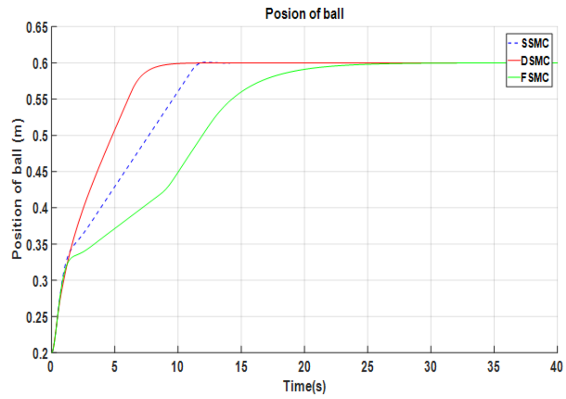

Figure 8. *Result of ball position at r_set = 0.6*

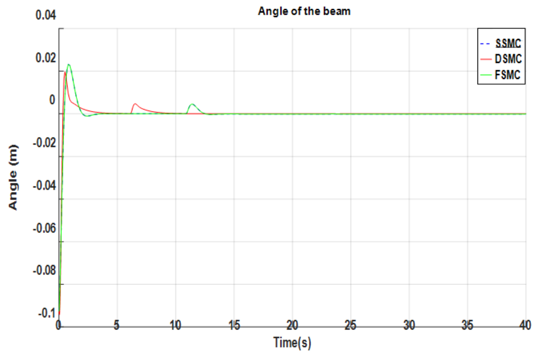

Figure 9. *Result of beam angle at r_set = 0.6*

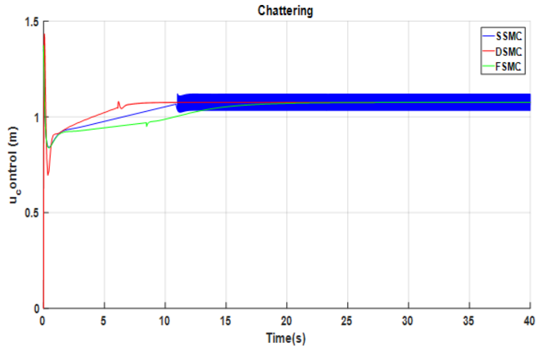

Figure 10. *Result of u_control at r_set = 0.6*

Table2. *Evaluate results of DSMC, SSMC, FSMC*

Controller	**Response**	**Overshot**	**Chattering**
DSMC	Fast	Non	Little
SSMC	Medium	Non	Many
FSMC	Slow	Non	Little

From Fig5-Fig10, we can see that the output y = r (t) is converged to desired signal r_set=0.4 about 5s and r_set=0.6 about 9s. It can response good of the ball position with anywhere of r_set. The chattering is reduced with controller DSMC and FSMC. Of course, if r_set of the ball position as far r_init is more time to respond.

5. RESULTS OF EXPERIMENTAL CONTROL

From the results of the simulation, we will control real experimental as Fig3. Here, the authors used DSP STM32F407VGT, tool Waijung 15.04 to support compiler to C/C++ through Matlab/Simulink with sample time 0.01s. With *r_set* is a desire position of the ball. Data collection are collected throughout the Terminal software.

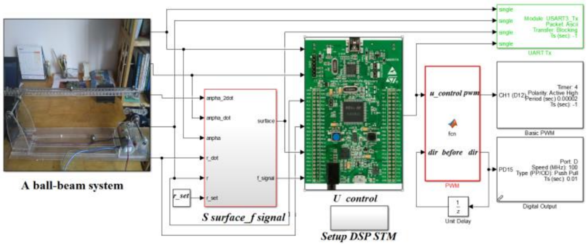

Figure 11. *The control structure of real experimental*

Results control:

r_set = 0.3m

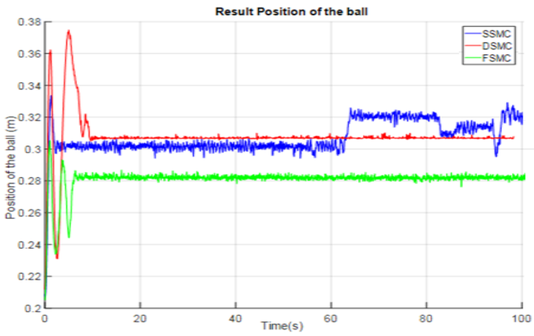

Figure 12. *Result of ball position (m) at r_set=0.3*

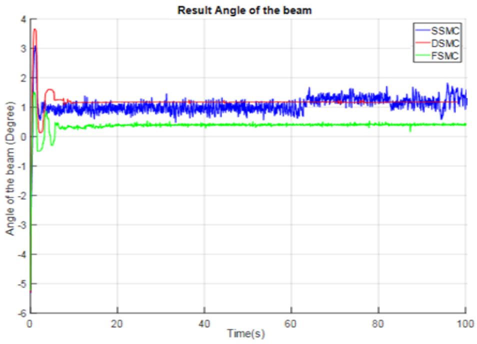

Figure 13. *Result of beam angle (rad) at r_set=0.3*

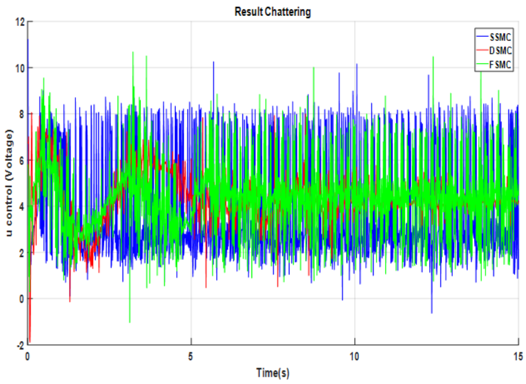

Figure 14. *Result of u_control signal (V) at r_set=0.3*

r_set = 0.4m

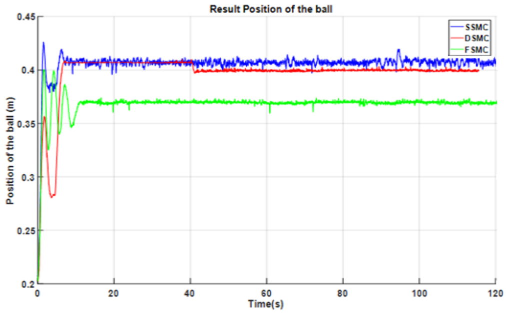

Figure 15. *Result of ball position (m) at r_set=0.4*

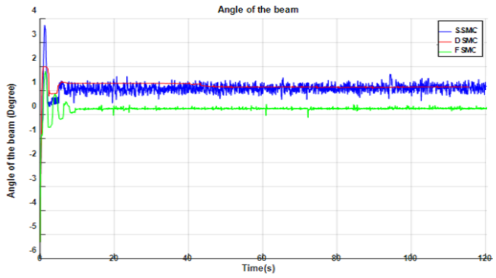

Figure 16. *Result of beam angle (rad) at r_set=0.4*

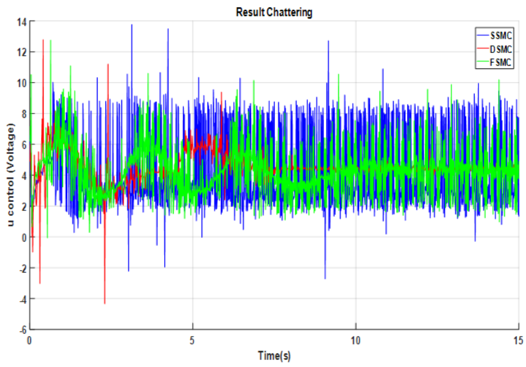

Figure 17. *Result of u_control signal (V) at r_set=0.4*

Similar simulation, results of implementation showed that DSMC controller has better results SSMC and FSMC about time to response *r_set* position of the ball, non-overshot and chattering of the control signal is reduced. DSMC can control the ball in any position on the beam. With FSMC is response ball position not well that indicated fig12, fig15.

6. CONCLUSION

In this paper, we proposed DSMC controller purpose to compare results control with FSMC and SSMC controllers of a ball-beam system from simulation to experiment. All results showed that DSMC is better working. The chattering is removed and the ball position can response anywhere on the beam by DSMC controller. But there are differences in the response time between under real model and simulation.

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