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MODEL AND CONTROL ALGORITHM CONSTRUCTION FOR ROTARY INVERTED PENDULUM IN LABORATORY

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ABSTRACT

A rotary inverted pendulum is a popular and basic model in control theory. However, the price of this model is remarkably expensive and unsuitable for laboratory equipping in a limited condition of Vietnam. Recent self-created models do not satisfy the requirement in model parameters. It causes that the control designing method that is based on system parameters cannot be designed well. The paper proposes steps to create an inexpensive and easy-to-make rotary inverted pendulum for control laboratories in the condition of Vietnam. The model is clear in dynamic structure, system parameters. And the ability of the system in training in laboratories is proven by applying controllers which are designed through dynamic equations. Then, the survey on controllers-swing up and balancing- and system responses will show the effectiveness of this real model. The balancing LQR controller is selected and the swing-up controller is Furuta methods.

Keywords: *Rotary inverted pendulum; LQR algorithm; Swing up; Furuta method; balancing control.*

TÓM TẮT

Con lắc ngược quay là một mô hình quen thuộc và cơ bản trong lý thuyết điều khiển tự động. Tuy nhiên, giá tiền một mô hình là rất cao, không phù hợp điều kiện trang bị phòng thí nghiệm còn hạn chế ở Việt Nam. Các mô hình được tự chế tạo hiện nay thì không đáp ứng được thông số mô hình, dẫn tới việc áp dụng các giải thuật phụ thuộc vào mô hình không được thiết kế thành công. Bài báo đề xuất chế tạo mô hình con lắc ngược quay giá rẻ và dễ xây dựng cho các phòng thí nghiệm điều khiển trong hoàn cảnh Việt Nam. Mô hình rõ ràng trong cấu trúc, thông số hệ thống. Và khả năng ứng dụng hệ thống để hướng dẫn trong phòng thí nghiệm được chứng minh bằng cách áp dụng xây dựng giải thuật trên mô hình này với việc xây dựng giải thuật-swingup và cân bằng- dựa trên hệ động lực học hệ thống. Sau đó, khảo sát trên các bộ điều khiển và đáp ứng hệ thống sẽ cho thấy hiệu quả của mô hình thực này. Giải thuật cân bằng LQR được lựa chọn và giải thuật swing-up là phương pháp Furuta.

Từ khóa: *Con lắc ngược quay; giải thuật LQR; Swing up; phương pháp Furuta; điều khiển cân bằng.*

1. INTRODUCTION

A rotary inverted pendulum (RIP) is a classical object for control engineering. Because of its nonlinear model and simplicity in mechanical structure, it is usually used in research and technical education. Some algorithms have been developed on this model, such as fuzzy [1], adaptive PID [2], neuron network [3], sliding mode [4]... However, the price of a standard RIP is too high (the price from Quanser is 2,594 USD, equivalent to 59 million VND). Thence, producing a cheap RIP (approximately 3 million VND), which has all exact values of system parameters and work well based on theoretical background, is important for laboratories that have difficulties in infrastructure, such as in Vietnam.

This paper presents a direction for creating a cheap RIP. Thence, authors apply LQR, a balancing control algorithm, and Furuta, a classical swing up the controller, on this model. Both controllers are designed based on a mathematical model of a real system. Therefore, the ability of this real-time RIP is proved to be suitable for both simulation and experiment. In LQR control, components of control matrixes are selected in various ways to examine the suitability of experiment and theoretical points. Moreover, because of appropriate motor identification in this paper, energy-based swing up controller [5], is designed well from an exact model of a real system. Application of mathematics in balancing and swing up control proves that real-time RIP satisfies the requirement of a standard model for laboratories and it can be replicated for laboratories developing for control engineering in Vietnam. Besides, following study [6] of Quanser, this paper also implements other inadequate documents: study [7] represents algorithms but simulation and experiment are not shown, the study [8] introduces balancing control but swing up the controller is not inferred.

This paper concludes of 5 sections: Section 1 presents researching direction, section 2 describes mathematical model; section 3 lists control algorithms that are based on a standard model for a survey; section 4 shows simulation and experimental results and section 5 gives a conclusion to end the paper.

2. MATHEMATICAL MODEL

2.1 Structure description

Structure of RIP is shown in Figure 1. Angle of pendulum and position of arm are α and β, correspondingly. System parameters are listed in Table 2.

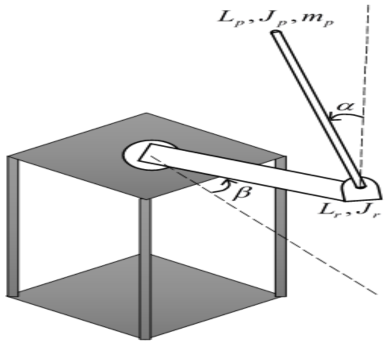

Figure 1. *Structure of RIP*

2.2 Mathematical model of RIP [10]

Total kinetic energy of system is

$$K=\frac{1}{2}J_r\dot{\beta}^2+\frac{1}{2}J_p\dot{\alpha}^2 +\frac{1}{2}m_p\left\{L_r{}^2\dot{\beta}^2+\frac{1}{4}L_p{}^2\left(\dot{\beta}^2-\dot{\beta}^2\cos^2\alpha\right)\right\} -\frac{1}{2}L_rL_pm_p\dot{\beta}\dot{\alpha}\cos\alpha \quad (1)$$

Total potential energy of system is

$$U=m_pgL_p\left(1-\cos\alpha\right) \quad (2)$$

Lagrange operator is

$$L=K-U \quad (3)$$

By Euler-Lagrange method, dynamic equations of system are

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{\beta}}\right)-\frac{\partial L}{\partial\beta}=\tau-B_r\dot{\beta}\\&\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{\alpha}}\right)-\frac{\partial L}{\partial\alpha}=-B_p\dot{\alpha}\end{aligned} \quad (4)$$

Thence, final dynamic equations of RIP are described as

$$\begin{aligned}&\left(m_pL_r{}^2+\frac{1}{4}m_pL_p{}^2-\frac{1}{4}m_pL_p{}^2\cos(\alpha)^2+J_r\right)\ddot{\theta}\\&-\left(\frac{1}{2}m_pL_pL_r\cos(\alpha)\right)\ddot{\alpha}\\&+\left(\frac{1}{2}m_pL_p{}^2\sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha}\\&+\left(\frac{1}{2}m_pL_pL_r\sin(\alpha)\right)\dot{\alpha}^2=\tau-B_r\dot{\theta}\end{aligned} \quad (5)$$

$$-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta}+\left(J_p+\frac{1}{4}m_pL_p{}^2\right)\ddot{\alpha}$$
$$-\frac{1}{4}m_pL_p{}^2\cos(\alpha)\sin(\alpha)\dot{\theta}^2 \quad (6)$$
$$-\frac{1}{2}m_pL_pg\sin(\alpha)=-B_p\dot{\alpha}$$

In order to make RIP in Figure 1 to suit real-time model, control signal is transformed from momen of DC motor to voltage that is applied to DC motor by formula (7) below

$$\tau=\frac{K_tV_a}{R_a}-\frac{K_tK_v\dot{\beta}}{Ra} \quad (7)$$

where K_t, R_a, V_a will be listed in Table 2.

By method of motor identification in [9], parameters of DC motor in Figure 4 are shown in Table 1 below

***Table 1**. Motor parameters*

L_m (H)	0.250868
K_v (V(rad/s))	0.064943
R_m (Ω)	6.835271
$\lvert T_f \rvert$ (Nm)	0.010764
J_m (kgm^2)	0.000134
C_m (Nm/(rad/sec))	0.000048

2.3 Energy based method for swing up control

To swing up pendulum from a downward position to upward position, energy-based method can be used to implement energy to make pendulum to an upward position. The total energy of system is

$$E=\frac{1}{2}ml^2\dot{\alpha}^2+mgl\cos\alpha \quad (8)$$

Energy of system at upward position is mgl. Thence, expected energy of system is

$$E_0=mgl \quad (9)$$

Relation of input u and energy E is

$$\dot{E}=mglu\dot{\alpha}\cos\alpha \quad (10)$$

Energy is increased if $u\dot{\alpha}\cos\alpha>0$. Therefore, to make pendulum to upward position, signal u should be selected in order to decrease $\left(E-E_0\right)^2$.

$$u=-u_{\max}\operatorname{sgn}\left(\left(E-E_0\right)\dot{\alpha}\cos\alpha\right) \quad (11)$$

$$u=\begin{cases}u_{\max}\ if\ \left(E-E_0\right)\dot{\alpha}\cos\alpha<0\\-u_{\max}\ if\ \left(E-E_0\right)\dot{\alpha}\cos\alpha>0\end{cases} \quad (12)$$

***Table 2**. System parameters*

Para-meter	Description	Unit	Value
m_p	Mass of pendulum	kg	0.025
L_p	Length of pendulum	m	0.23
L_r	Length of arm	m	0.16
J_r	Inertia moment of arm	kg.m	0.0019
J_p	Inertia moment of pendulum	kg.m	0.000466
R_a	Armature resistor	Ω	6.835721
K_t	Motor torque constant	Nm/A	0.0198
K_v	Motor voltage constant	Vs/rad	0.0198
B_r	Friction of arm	No unit	0.0017
B_p	Friction of pendulum	No unit	≈ 0
g	Gravity acceleration	m/s^2	9.81

3. BALANCING AND SWING UP CONTROL

3.1 LQR control

Linear quadratic regulator (LQR) is a linear controller. But, because the nonlinear model of RIP, dynamic equations have to be linearized around working point (upward position).

$$\dot{x}=Ax+BV_a \quad (13)$$

$$\begin{bmatrix}\dot{\alpha}\\\ddot{\alpha}\\\dot{\beta}\\\ddot{\beta}\end{bmatrix}=\begin{bmatrix}0&1&0&0\\\frac{\partial\ddot{\alpha}}{\partial\alpha}&\frac{\partial\ddot{\alpha}}{\partial\dot{\alpha}}&\frac{\partial\ddot{\alpha}}{\partial\beta}&\frac{\partial\ddot{\alpha}}{\partial\dot{\beta}}\\0&0&0&1\\\frac{\partial\ddot{\beta}}{\partial\alpha}&\frac{\partial\ddot{\beta}}{\partial\dot{\alpha}}&\frac{\partial\ddot{\beta}}{\partial\beta}&\frac{\partial\ddot{\beta}}{\partial\dot{\beta}}\end{bmatrix}\begin{bmatrix}\alpha\\\dot{\alpha}\\\beta\\\dot{\beta}\end{bmatrix}+\begin{bmatrix}0\\\frac{d\ddot{\alpha}}{dV_a}\\0\\\frac{d\ddot{\beta}}{dV_a}\end{bmatrix}V_a \quad (14)$$

From (5), (6), (7) and, $\ddot{\alpha}$ and $\ddot{\beta}$ are found and then substituted into (14) to generate matrixes A và B in (15), (16) (We consider that α , $\dot{\alpha}$, β , $\dot{\beta}$, V_a are 0 at balancing position).

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 103.4346 & 0 & 0 & -0.4417 \\ 0 & 0 & 0 & 1 \\ 13.0311 & 0 & 0 & -0.3175 \end{bmatrix} \quad (15)$$

$$B=\begin{bmatrix} 0 & 6.31 & 0 & 4.53 \end{bmatrix}^T \quad (16)$$

Matrixes Q and R are selected as unit matrixes

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \text{R=1} \quad (17)$$

In Matlab, command K= dlqr (A, B, Q, R) used to calculate feedback control matrix K from matrixes A, B of the linearized model and weighing matrixes Q, R. With matrixes in (15), (16), (17), feedback control matrix is

$$K=\begin{bmatrix} 46.2600 & 4.7839 & -0.8791 & -1.3477 \end{bmatrix} \quad (18)$$

3.2 Swing up control

Swing up is used to move pendulum to initial downward position $-\pi$ to balancing position before pendulum is balanced by LQR method. From (12), we recognize that arm rotates in a positive direction when $u=u_{\max}$ and in a negative direction when $u=-u_{\max}$ (coordinate is shown in Figure 1). When position of arm is over [-30,30] degrees, an effect to motion of the arm is in opposite direction. When the position of the pendulum is in the range of [-15,15] degrees, balancing LQR control is activated.

4. SIMULATION AND EXPERIMENT

In Figure 2, a RIP that is created with the price of 3,000,000 VND concludes of mica structure and an L-form iron bar as a pendulum. System parameters in Table 2 are the same in both simulation and experiment.

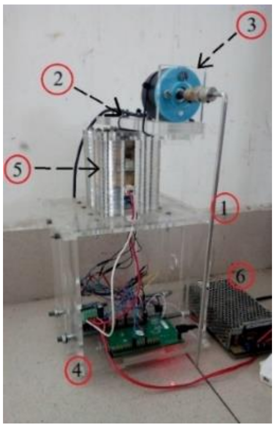

Figure 2. *Real-time RIP*

Figure 3. *RIP in balancing position*

Components in Figure 2 are

(1): Pendulum.

(2): Arm.

(3): Encoder 200 pulse/round to calculate α .

(4): Control processor STM32F407 that is programmed by Matlab and H-bridge BTS7960 to control DC motor.

(5): DC motor that is combined with encoder 200 pulse/round to calculate β (Figure 4).

(6): Voltage power supply 24VDC.

Figure 4. *DC motor that is combined with encoder 200 pulse/round of RIP (commercial price is 200,000 VND)*

4.1 Simulation results

With control matrix in (18) and initial values of RIP are $\alpha=0.1(rad)$; $\dot{\alpha}=0(rad)$; $\beta=0.1(rad)$; $\dot{\beta}=0(rad)$, simulation results are shown in Figure 5.

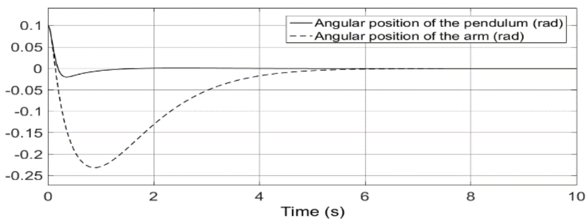

Figure 5. *Angles of pendulum and arm under LQR control*

Simulating of swing up process of pendulum with parameters in Table 2, m_p = 0.025(kg), L_p = 0.23(m), g = 9.8(m/s), initial values of system variables are $\left[\alpha(0),\dot{\alpha}(0)\right]^T$ = $\left[\pi,0\right]^T$ and using LQR controller for balancing control, through Figure 6, it spends 2s to move pendulum to the upward position and keep it balanced there.

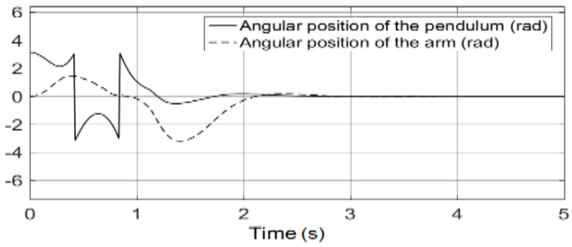

Figure 6. *Angles of pendulum and arm under swing up and balancing LQR control*

4.2 Experimental results

4.2.1 Swing up

Experimental results show that it spends 5s to swing up pendulum from a downward position to a upward position (Figure 7).

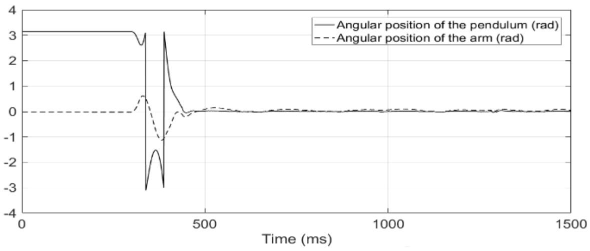

Figure 7. *Angles of pendulum and arm under swing up and balcncing LQR control in experiment*

4.2.2 Analysis of system responses under changes in matrixes Q and R

Through trial and error test in experiment, weighing matrixes Q và R are selected as in (19) for good performance.

$$Q=\begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & q_{33} & 0 \\ 0 & 0 & 0 & q_{44} \end{bmatrix}=\begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{R=1} \quad (19)$$

$$K=\begin{bmatrix}103.7946 & 10.5908 & -18.6361 & -7.4061\end{bmatrix} \quad (20)$$

In matrix Q, components q_{11}, q_{22}, q_{33}, q_{44} sequentially relate to the angle of the pendulum, angle velocity of pendulum, angle of arm, angle velocity of arm. With K as in (20), experimental results are in Figure 8 to Figure 11.

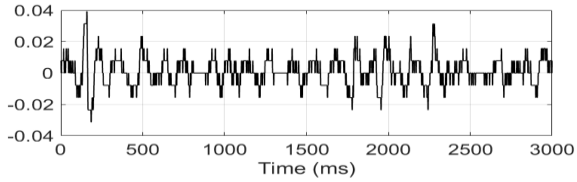

Figure 8. *Angle of pendulum (rad)*

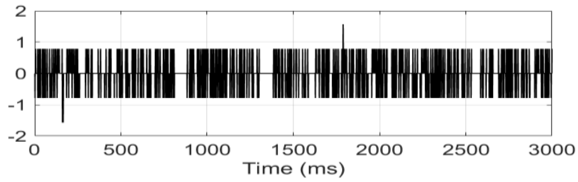

Figure 9. *Angle velocity of pendulum (rad/s)*

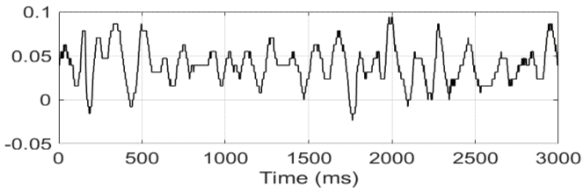

Figure 10. *Angle of arm (rad)*

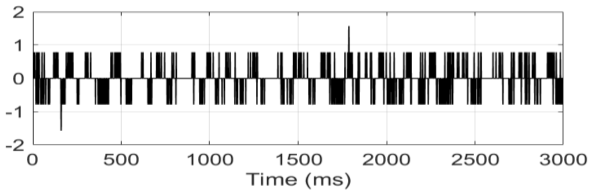

Figure 11. *Angle velocity of arm (rad)*

From matrixes Q, R in (19), components q_{11}, q_{22}, q_{33}, q_{44} of matrix Q and matrix R are changes sequentially to examine their effect on response of system.

Case 1: Increase q_{11} = 10000

$$Q=\begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{R=1} \quad (21)$$

$$K_1=\begin{bmatrix}143.9241 & 12.6167 & -17.8804 & -8.7311\end{bmatrix} \quad (22)$$

Figure 12 shows that pendulum deviates far from balancing position because control input signal is increased to focus on pendulum angle. Thence, arm also deviates far from standard position (Figure 17).

Case 2: Increase q_{22} = 100

$$Q=\begin{bmatrix}1000 & 0 & 0 & 0\\ 0 & 100 & 0 & 0\\ 0 & 0 & 500 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \text{R=1} \quad (23)$$

$$K_2=\begin{bmatrix}118.2634 & 14.0747 & -15.3651 & -7.2480\end{bmatrix} \quad (24)$$

Angle velocity of pendulum increases (Figure 23) when increasing q_{22}. This makes pendulum to deviate from balancing position (overshoot) and arm is un-stabilzed and deviates far from balancing position.

Case 3: Increase q_{33} = 2000

$$Q=\begin{bmatrix}1000 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 2000 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \text{R=1} \quad (25)$$

$$K_3=\begin{bmatrix}140.7661 & 14.7013 & -36.3857 & -12.2462\end{bmatrix} \quad (26)$$

When increasing q_{33}, controller focuses on arm controlling to balancing position (Figure 19). It causes that pendulum angle is not focused and deviates far from balancing position.

Case 4: Increase q_{44} = 100

$$Q=\begin{bmatrix}1000 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 500 & 0\\ 0 & 0 & 0 & 100\end{bmatrix}, \text{R=1} \quad (27)$$

$$K_4=\begin{bmatrix}156.1865 & 16.4435 & -16.0521 & -11.2380\end{bmatrix} \quad (28)$$

In Figure 30, angle of arm deviates far from balancing position. Thence, voltage input signal is increased and it causes that system is unstabilized.

Case 5: Increase R=10

$$Q=\begin{bmatrix}1000 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 500 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \text{R=10} \quad (29)$$

$$K_5=\begin{bmatrix}62.3470 & 6.3839 & -6.1475 & -3.1180\end{bmatrix} \quad (30)$$

When increasing R, voltage control signal is not big enough to response to keep pendulum not far from balancing position. Therefore, arm and pendulum vibrates in long period around working point (Figure 16, Figure 21).

Change in angle of pendulum in cases

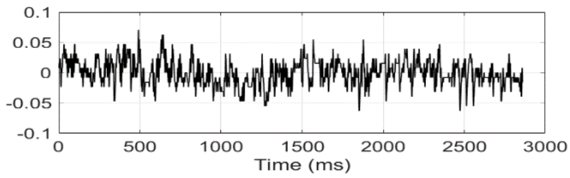

Figure 12. *Angle of pendulum corresponding to K_1 (rad)*

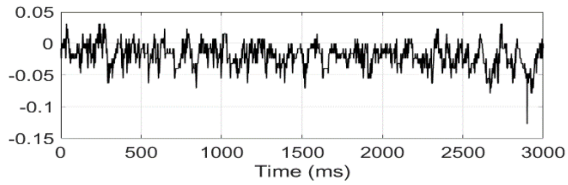

Figure 13. *Angle of pendulum corresponding to K_2 (rad)*

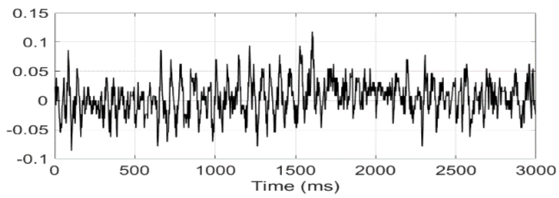

Figure 14. *Angle of pendulum corresponding to K_3 (rad)*

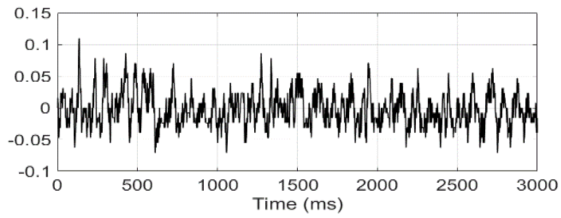

Figure 15. *Angle of pendulum corresponding to K_4 (rad)*

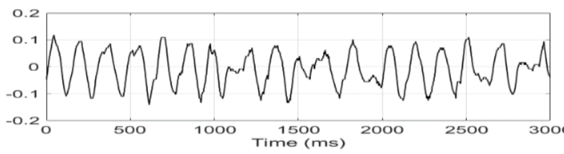

Figure 16. *Angle of pendulum corresponding to K_5 (rad)*

Change of angle of arm in cases

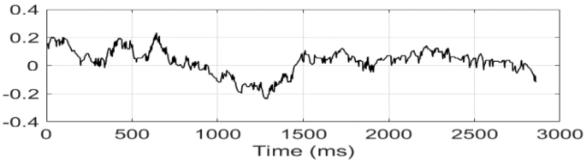

Figure 17. *Angle of arm corresponding to K_1 (rad)*

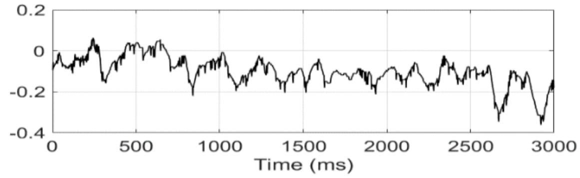

Figure 18. *Angle of arm corresponding to K_2 (rad)*

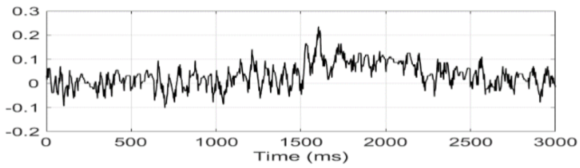

Figure 19. *Angle of arm corresponding to K_3 (rad)*

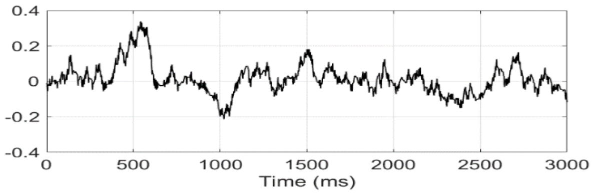

Figure 20. *Angle of arm corresponding to K_4 (rad)*

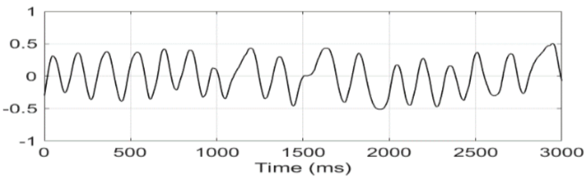

Figure 21. *Angle of arm corresponding to K_5 (rad)*

Change of angle velocity of pendulum in cases

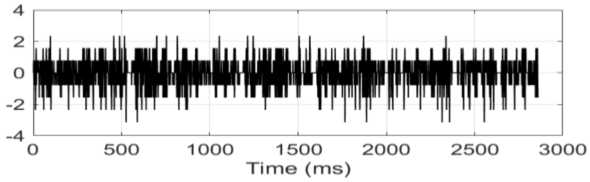

Figure 22. *Angle velocity of pendulum corresponding to K_1 (rad/s)*

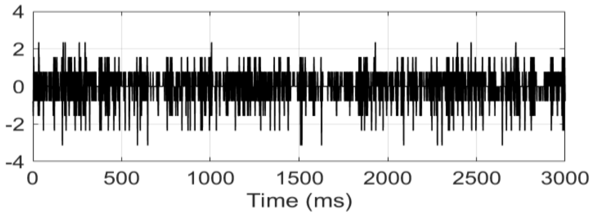

Figure 23. *Angle velocity of pendulum corresponding to K_2 (rad/s)*

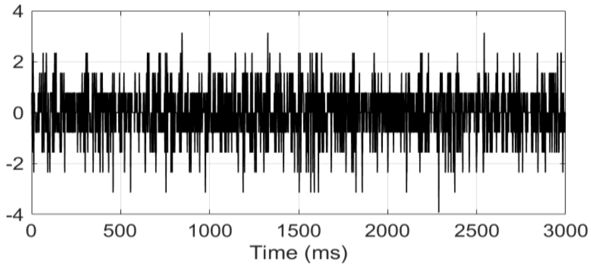

Figure 24. *Angle velocity of pendulum corresponding to K_3 (rad/s)*

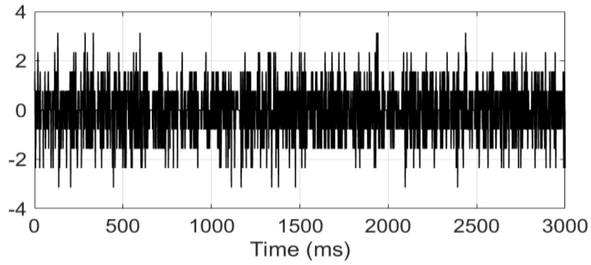

Figure 25. *Angle velocity of pendulum corresponding to K_4 (rad/s)*

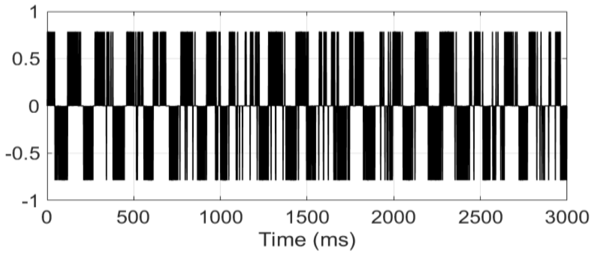

Figure 26. *Angle velocity of pendulum corresponding to K_5 (rad/s)*

Change of angle velocity of arm in cases

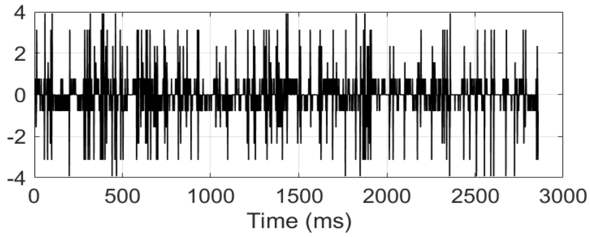

Figure 27. *Angle velocity of arm corresponding to K_1 (rad/s)*

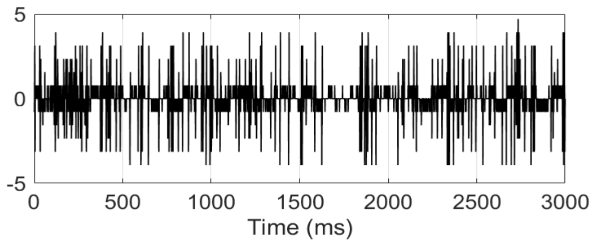

Figure 28. *Angle velocity of arm corresponding to* K_2 *(rad/s)*

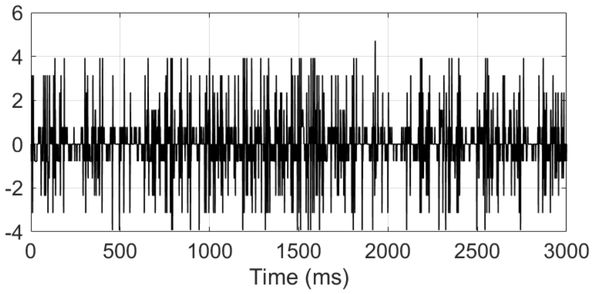

Figure 29. *Angle velocity of arm corresponding to* K_3 *(rad/s)*

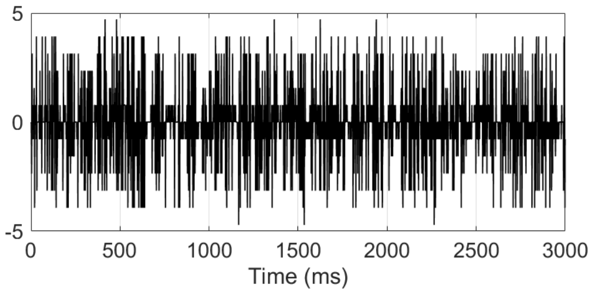

Figure 30. *Angle velocity of arm corresponding to* K_4 *(rad/s)*

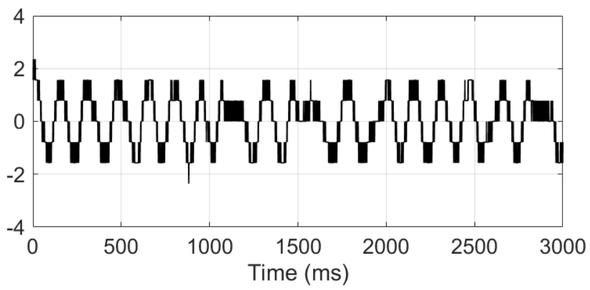

Figure 31. *Angle velocity of arm corresponding to* K_5 *(rad/s)*

From the sequential changes of q_{11}, q_{22}, q_{33}, q_{44} and R in experiment, it is recognized that when one component is increased, controller only focus on stabilizing the variable that relates to that component. Other varibles seem to be more unstabilized.

After 5 cases of survey, responses of system suit the law of changing parameters of LQR control. Thence, LQR can be designed well and work well on this standard model. However, because equipments are all bought in market for student and mica structure is weak, the model vibrates when operating. This phenomenon is shown in Figure 8, 10, 12, 13, 14, 17, 18, 19. Moreover, cheap encoder on DC motor have low resolution as 200 PPR, bigger than 1 degree. This causes that the feedback of sensor is not accurate enough and controlling is affected.

5. CONCLUSION

Through identifying process, controller designing, response analysis on both simulation and experiment, acceptable results prove that real-time RIP in this paper is a standard has exact dynamic equations and system pararmeters. This standard real-time model is suitable for testing controllers that are designed based on exact model although it is cheap and some hardware limitations still exist. This model is appropriate to be equipped with low expense for control laboratories in Vietnam with low expense. Identification gives adequate parameters of system and dynamic equations are exact. This success helps authors to design both balancing and swing up controller for system. Contemporarily, survey of LQR controller on this model gives results that suits the theoretical points of LQR method.

REFERENCES

[1] Jen-Hsing Li, *Composite Fuzzy Control of a Rotary Inverted Pendulum*, 2013 IEEE International Symposium on Industrial Electronics

[2] T. C. Kuo, *Adaptive PID with sliding mode control for the rotary inverted pendulum system*, 2009 IEEE/ASME International Conference on Advanced Intelligent Mechatronics.

[3] Nguyen Duc Quyen, Ngo Van Thuyen, Nguyen Quang Hoc, Nguyen Duc Hien, *Rotary inverted pendulum and control of rotary inverted pendulum by artificial neural network*, Proc. Natl. Conf. Theor. Phys. 37 (2012), pp. 243-249.
[4] Philippe Faradja, Guoyuan Qi, Martial Tatchum, *Sliding mode control of a Rotary Inverted Pendulum using higher order differential observer*, 2014 14th International Conference on Control, Automation and Systems (ICCAS 2014).
[5] K. Furuta and M. Iwase, *Swing-up time analysis of pendulum*, Bulletin of the Polish Academy of Sciences Technical Sciences Vol. 52, No. 3, 2004.
[6] Velchuri Sirisha and Dr. Anjali. S. Junghare, *A Comparative study of controllers for stabilizing a Rotary Inverted Pendulum*, International Journal of Chaos, Control, Modelling and Simulation (IJCCMS) Vol.3, No.1/2, June 2014.
[7] Vũ Chấn Hưng, Đặng Thành Phu, Hoàng Văn Tuấn, *Điều khiển hệ thống con lắc ngược quay*, Tạp chí khoa học và công nghệ tập 43, số 3, 2005.
[8] Viroch Sukontanakarn and Manukid Parnichkun, *Real-Time Optimal Control for Rotary Inverted Pendulum*, American Journal of Applied Sciences 6 (6): 1106-1115, 2009 ISSN 1546-9239.
[9] Ho Trong Nguyen, Nguyen Minh Tam, Nguyen Van Dong Hai, *Application of genetic algorithm in optimization controller for cart and pole system*, Journal of Technical Education Science No.44A, Oct-2017.
[10] Quanser rotary inverted pendulum workbook.
[11] Minho Park, Yeoun-Jae Kim and Ju-Jang Lee, *Swing-up and LQR stabilization of rotary inverted pendulum*, The Sixteenth International Symposium on Artificial Life and Robotics 2011(AROB 16th '11).

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