ANALYSES ON RESPONSE TIME OF MAGNETOHYDRODYNAMIC GENERATORS FOR THE LIQUID WORKING FLUID

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ABSTRACT

Magneto-hydro-dynamic generators is a great application to measure the velocity and flow rate for the liquid metal cooled fast breeder reactors because the decrease of working fluid flow caused by the non-uniformity must be immediately identified to initiate reactor processes. There are many methods to examine the time response and the time constant of the eddy current loop generated in the liquid metal with inductance and resistance by means of the magnetic field. In this study, a specific estimation is carried out using the interaction of magnetic field considering the delay of process caused by the eddy current.

Keywords: *magneto-hydro-dynamics; response time; liquid metal; working fluid flow; time constant.*

1. INTRODUCTION

In the electromagnetic flow-meter (EMF), the flow rate of the liquid metal can be calculated by using the magneto-hydro-dynamic (MHD) effect [1]. Working fluid flows in the channel, as shown in Fig. 1, to which a magnetic field H_m is supplied.

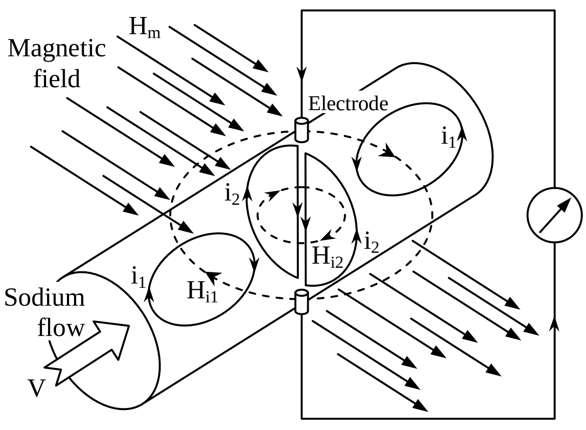
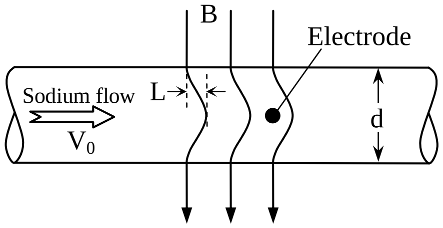
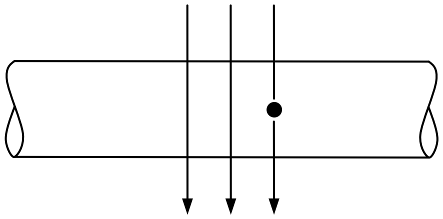


Figure 1. *Induced eddy current and magnetic field in EMF*

The movement of the liquid (for example sodium) causes an electric field following the law of induction. The voltage which is generated on the surface is taken out by the electrodes and measured by the meter. The channel made by materials that conduct electricity and the magnetic field H_m of internal field produce two types of electrical eddy current: (1) those marked i_1 in Fig. 1, created in the boundary of the field in a plane perpendicular to it and parallel to the working fluid flow, sensed by the change in EMF between the two sides of the gradient appearing in the length of the magnetic field in these boundary areas, and (2) those marked i_2 created in a plane parallel to the magnetic field and perpendicular to the working fluid flow, sensed by the motion of this fluid through the magnetic field, and which come back the channel or the working fluid border region.



(a) Distortion of magnetic force lines in sodium flow caused by eddy current



(b) Magnetic force lines upon stoppage of sodium flow

Figure 2.*A magnetic flux L·B·d intersects the electrode when the fluid flow stops*

Any difference interacting in the working fluid speed will change the eddy currents and generate in accordance with Faraday's law of induction a gradient EMF in the round shaped by the electrodes and connections [2]. This gradient EMF will occur as a delay in the difference of output signal achieved from the appearing EMF [3].

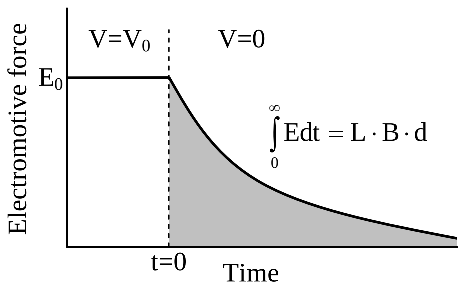


Figure 3. *Decline of electromotive force after step change in sodium flow rate*

To examine this delay, the simple assumption is utilized for considering. The H_m, sensed eddy currents i_1 and i_2 in Fig. 1, generate rotating magnetic field H_{i1} and H_{i2}. These rotating fields are applied on the originally supplied magnetic field H_m. which is thereby affected as illustrated in Fig. 2(a). In other words, the magnetic field H_m is changed into deviated form by the working flow. If the working flow stops, the eddy currents i_1 and i_2 will disappear, to save the lines of magnetic force to their original straight form as shown in Fig. 2(b). This difference in the magnetic field will change a flux L·B·d (L: Distance of displacement from the original straight magnetic force lines, B: Magnetic flux density of the applied field H_m, d: Diameter of pipe), to intersect the electrodes, to sense on them the transient Faraday EMF mentioned earlier [4]. Suppose the working flow stops instantaneously from velocity V_0 to 0 at time 0, then, as shown in Fig. 3, the EMF intensity, which was originally $E=V_0 \cdot B \cdot d$ before the step change brought to the working flow, drops to zero, tracing the curve depicted in Fig. 3. The shaded area underneath this curve represents the EMF output integrated in respect of time after the step change, and equals the change brought thereby to magnetic flux:

$$\int_0^\infty E dt = L \cdot B \cdot d \qquad (1)$$

Consequently, the response time can be determined by

$$\tau = \int_0^\infty E dt \Big/ E_0 = L/V_0 \qquad (2)$$

which equals to the time constant when the output signal is shown by the first order lag. Thus, the problem of determining τ is reduced to finding L, that is the distance by which the magnetic force lines are entrained by the fluid flow in steady state [5].

2. NUMERICAL MODEL

According to research report [3] electromagnetic part of the flowmeter is derived from reduced Maxwell equations

$$\mathrm{rot}H = J \qquad (3)$$

$$\mathrm{div}B = 0 \qquad (4)$$

where H is vector of magnetic field intensity, B is vector of magnetic field induction, J is vector of current density.

$$\mathrm{rot}E = 0 \tag{5}$$

$$\mathrm{div}J = 0 \tag{6}$$

where E is vector of electric field intensity. Material properties are represented by equation

$$E = H\mu_0 \tag{7}$$

$$J = E\sigma \tag{8}$$

where μ_0 is permeability of vacuum, is specific conductance of measured liquid. Vector functions of electric and magnetic field are expressed by means of scalar electric ϕ_e and magnetic potentials ϕ_m

$$E = -\mathrm{grad}\phi_e \tag{9}$$

$$H = -\mathrm{grad}\phi_m \tag{10}$$

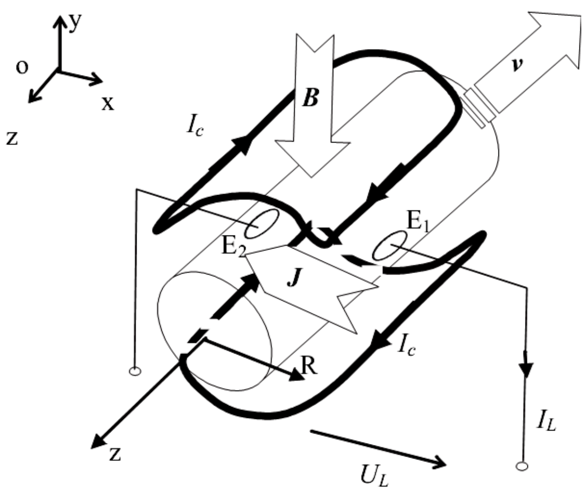


Figure 4. *Principle of the induction MHD flowmeter*

Final current density from (6) is influenced by velocity V of the flowing ions solution and outer magnetic field

$$J = \sigma(E + V \times B) \tag{11}$$

If electrodes E_1 and E_2 have different electrical potentials (Fig. 4) then current density J is created in the Ω area according to (11) and current I_L flows in the ion solution

$$I_L = \iint_{S_e} J \cdot dS = \iint_{S_e} \sigma(E + V \times B) \cdot dS \tag{12}$$

where S_e is the directed area of electrodes E_1 and E_2 into space Ω. We obtain voltage between flowmeter electrodes E_1, E_2 from

$$U_L = \int_{E_1}^{E_2} E \cdot dl \tag{13}$$

where electric field intensity is derived from the force F which affects a charge q. Current density J(V) depends on immediate ion velocity between E_1 and E_2. After modification voltage on electrodes is

$$U_L = \iiint_{\Omega} \left(\frac{J(V)}{I_L} \times B \right) \cdot (V_{i0} + V) dV \tag{14}$$

The model of fluid velocity distribution is derived for incompressible fluid and was able to formulate from balance of forces the Navier-Stokes equation for the fluid element

$$\frac{\partial V}{\partial t} + V \cdot \mathrm{grad}V = A - \frac{1}{\rho}\mathrm{grad}p + \upsilon \cdot \Delta V \tag{15}$$

where A is an external acceleration and υ kinematic viscosity. Next step was derivation of finite element methods (FEM) and finite volume methods (FVM) model [3]. The final term for output voltage on flowmeter electrodes which was evaluated is

$$U_L \cong \left(\frac{1}{|ik^+|} + \frac{1}{|ik^-|} \right) \frac{1}{2F_c^2 I_L} \cdot \sum_{e=1}^{N_\Omega} \frac{|J_2|}{\Delta V_e^2 \sqrt{\left(|V_{m,e}|^2 + \left(\frac{|V_{ok,e}^+| + |V_{ok,e}^-|}{2} \right)^2 \right)}} (J_e \times B_e) \cdot \left(J_e \left(\frac{1}{|ik^+|} + \frac{1}{|ik^-|} \right) + \Delta V_e F_c V_{m.e} \right) \tag{16}$$

where

$$V_{ok,e}^+ = \frac{J_e}{F_c \Delta V_e ik^+}, \quad V_{ok,e}^- = \frac{J_e}{F_c \Delta V_e ik^-} \tag{17}$$

$$ik^{+} = \sum_{k=1}^{N_{ion+}} c_k^{+} N_k^{+ion} = 1{,}2902 \cdot 10^{-5} \text{ mol/m}^3$$

$$ik^{-} = \sum_{k=1}^{N_{ion-}} c_k^{-} N_k^{-ion} = -1{,}3175 \cdot 10^{-5} \text{ mol/m}^3$$

and where F_c is Faraday constant, F_c = 96484C.mol^{-1}; E_e is electric field intensity in direction of ions motion in an element of mesh; c^+ is positive ion concentration; c^- is negative ion concentration; ΔV_e is element volume; N_k^{+ion} is integer multiple of electron charge for specific positive ion; N_k^{-ion} is integer multiple of electron charge for specific negative ion; q_e^- is whole charge of negative ions in one element; q_e^+ is whole charge of positive ions in one element; N_{ion+} is number of different ion positive charge carriers (elements, compounds); N_{ion-} is number of different negative charge carriers (elements, compounds). Potable water has for instance this composition of ions with mass density m_{io}:

***Table 1**. Mass Density of Ions*

Positive ions	
Na	32.71mg/dm^3
K	1.525mg/dm^3
Mg	43.81mg/dm^3
Ca	157.7mg/dm^3
Negative ions	
F	1.58mg/dm^3
Cl	5.35mg/dm^3
SO_4	13.08mg/dm^3
NO_3	0.54mg/dm^3
HCO_3	762.4mg/dm^3
Neutral substances	
CO_2	4063mg/dm^3
H_2O	1000000mg/dm^3

3. CALCULATION OF MAGNETIC FIELD

In the case of a pipe of large diameter such as used in an LMFBR heat transport system, the pipe wall will exert a relatively small influence on the electromotive force, so that the eddy current i_2 flowing in the plane parallel to the magnetic field can be ignored. Our consideration can thus be limited to i_1 and the resulting magnetic flux generated in the order regions of the applied magnetic field H_m. Therefore the sodium flow can be assumed to be uniform.

The Maxwell equations for a moving conductor are given by

$$\nabla \cdot E = 0 \tag{18}$$

$$\nabla \cdot B = 0 \tag{19}$$

$$\nabla \cdot E = 0 \tag{20}$$

$$\nabla \times B = \mu\left(i + \varepsilon\frac{\partial E}{\partial t}\right) = \mu\sigma E + \mu\sigma V \times B + \mu\varepsilon\frac{\partial E}{\partial t} \tag{21}$$

It has been shown in the preceding section that it suffices to calculate the magnetic field in steady state to determine the response time. Elimination of the time derivatives and use of the vector formulas reduce the Maxwell equations to

$$\nabla^2 B = \mu\sigma\left[B(\nabla \cdot V_0) - (B \cdot \nabla)V_0 + (V_0 \cdot \nabla)B\right] \tag{22}$$

representing the magnetic flux density. The first term on the right-hand side of this equation equals zero from the mass conservation law of fluid, while the second term can be ignored from the assumption of uniform sodium flow. This permits the x-component (direction of applied magnetic field) of the magnetic flux density to be expressed by

$$\nabla^2 B_x - \mu\sigma V_0 \frac{\partial B_x}{\partial z} = 0 \tag{23}$$

The boundary conditions governing the surface of the pipe are assumed to provide for an x-component of the magnetic flux density that is independent of circumferential position:

$$\partial B_x / \partial \varphi = 0 \text{ at } r=a \qquad (24)$$

This means that there exists a uniform magnetic field along the circumference of any given cross section of the pipe.

Since B_x is independent of φ, Eq. (23) can be rewritten in terms of two dimensional (r, z) coordinates:

$$\left(\frac{1}{r}\cdot\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}-\mu\sigma V_0\frac{\partial}{\partial z}\right)B_x=0 \qquad (25)$$

This equation is solved by separation of variables:

$$B_x = e^{qz}\int_0^\infty I_0\left(\sqrt{q^2+\omega^2}\cdot r\right)\cdot\left(C_1\cos\omega z + C_2\sin\omega z\right)d\omega \qquad (26)$$

where I_0 is a modified Bessel function, and $q=\mu\sigma V_0/2$, while C_1 and C_2 are functions of ω and can be determined from the boundary condition.

4. CALCULATED RESULTS

A computer program was developed which performs the numerical integration expressed by (26). The results of computation are shown in Fig. 5. A diameter 0.6m is adopted for the pipe, on which a magnetic field is applied, which is uniform over a distance of 0.9m, as shown by the solid line in Fig. 5, thus representing an aspect ratio of 1.5. The value, which is taken for the conductivity of sodium, is 4.52×10^{-6}℧/m (mho/m) which is the actual value for 400°C. The calculated distribution of the magnetic flux density along the pipe axis is indicated for several sodium velocities by the curves drown in Fig. 5. The gradual shifting seen of these curves clearly illustrates how entrained deviation increases with rising sodium velocity.

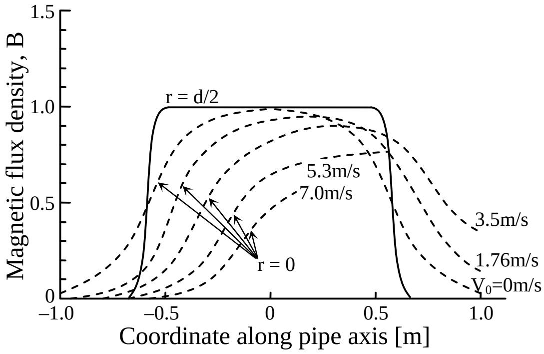


Figure 5.*Entrainment of magnetic field by sodium flow*

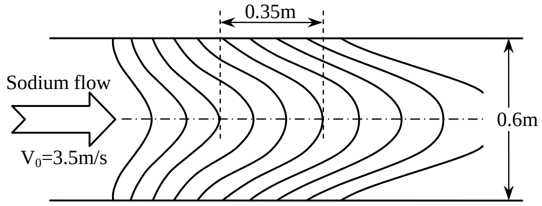


Figure 6.*Example of calculated configuration of magnetic force lines distorted by sodium flow*

The deviation of magnetic force lines by the action of sodium flow is shown for the case of 3.5m/s flow velocity in Fig. 6, which reveals that the distance entrained attain 0.35m at the electrodes. From (2), this value of L corresponds to a response time of 0.1s.

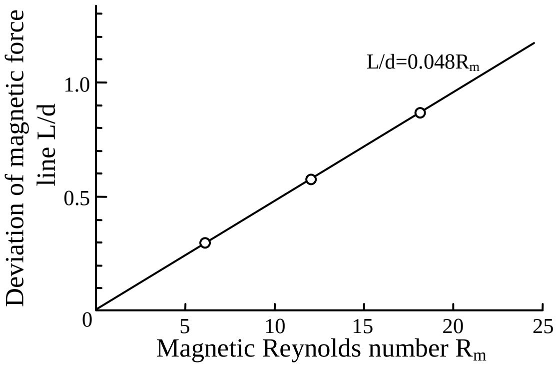


Figure 7.*Calculated relation between magnetic Reynolds number and deviation of magnetic force line*

The distance of entrained deviation, calculated in terms of the dimensionless quantity L/d, is plotted in Fig. 7 for three values of sodium flowrate expressed in dimensionless magnetic Reynolds number $R_m=\mu\sigma V_0 d$.

The plots are seen to fall on a straight line expressed by

$$L/d = 0.048R_m = 0.048\mu\sigma V_0 d \quad (27)$$

and recalling (1) the EMF response time

$$\tau = L/V_0 = 0.048\mu\sigma d^2 \quad (28)$$

5. CONCLUSIONS

A model is proposed for evaluating the response time of EMF, based on the principle of liquid MHD effect that the delay registered in the output signal from the EMF is due to the displacement of magnetic flux which intersects the electrodes connected to lead wires, upon change occurring in the sodium flow. The response time of EMF is shown to be proportional to the electric conductivity of the fluid and to the square of the pipe diameter. Sample calculation performed with this model is found to give an EMF response time of 0.1s for the case of sodium at 400°C passing through a pipe of 0.6m diameter.

EMF using MHD effect with such fast response times can measure liquids that flow for relatively short periods of time, such as in batch and fill operations. EMF that sense velocity and level can measure the flow of liquids in partially filled pipes, such as interceptor sewers and storm water culverts.

EMF liners and electrodes can be constructed of materials that do not contaminate the liquid. Therefore, these generators can be applied when liquid contamination is an issue, such as in sanitary applications.

Straight run requirements are relatively short, so EMF technology can be applied where limited straight run is available. In addition, magnetic flowmeter technology has no Reynolds number constraints, so it can be applied where the liquid exhibits high or varying viscosity.

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