# PID CONTROLLER DESIGN FOR SECOND-ORDER DELAYED UNSTABLE PROCESS

## THIẾT KẾ BỘ ĐIỀU KHIỂN PID CHO QUÁ TRÌNH KHÔNG ỔN ĐỊNH BẬC HAI CÓ THỜI GIAN TRỄ

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## ABSTRACT

*In this paper, a proportional-integral-derivative (PID) controller cascaded with a second-order lead-lag filter is proposed for unstable second-order process with time delay. On the basis of the internal model control (IMC) principle for the two-degree-of-freedom (2DF) control system, the proposed method can achieve the enhanced performance for both disturbance rejection and set-point tracking problems. Several illustrative examples are included to demonstrate the improved performances of proposed controllers in compared with those of recently published PID-type controllers, since the various controllers are all tuned to have the same degree of robustness by measuring the maximum sensitivity (Ms) value.*

***Keywords*: *Internal Model Control (IMC) principle; Proportional–Integral–Derivative (PID) controller; Two degree of freedom (2DOF) control system; Second-order Lead/lag filter; second-order delayed unstable process (SODUP) model.***

## TÓM TẮT

*Trong bài báo này, bộ điều khiển PID kết hợp với bộ lọc bậc hai được đề xuất cho quá trình không ổn định bậc hai có thời gian trễ. Dựa vào các nguyên tắc thiết kế IMC cho hệ thống điều khiển hai bậc tự do, phương pháp đề xuất có thể nâng cao khả năng thực thi của hệ thống điều khiển trên cả hai phương diện quan trọng trong điều khiển đó là đặc tính đáp ứng khử nhiễu và đáp ứng theo giá trị đặt. Một số ví dụ minh họa được thực hiện nhằm chứng minh sự cải thiện khả năng thực thi của các bộ điều khiển đề xuất khi so sánh với các bộ điều khiển kiểu PID hiện tại, trong khi các bộ điều chỉnh đều được điều chỉnh để các hệ thống điều khiển dùng để so sánh có cùng một mức độ ổn định bền vững.*

***Từ khóa:*** *Nguyên tắc điều khiển IMC; Bộ điều khiển PID; Bộ lọc bậc hai; Hệ thống hai bậc tự do (2DOF); Hệ không ổn định bậc hai có thời gian trễ.*

## 1. INTRODUCTION

In accordance with the literature, it is clear that the design methods of PID controller for unstable processes have recently been attracted much attention of numerous academic and control engineers [1-7]. However, the PID tuning rules for unstable first-order process model are widely introduced in process control field in despite that unstable second-order time delay process model can represents the dynamics of actual process better than the first-order time delay process models.

The IMC-PID controller design has been frequently discussed [8-11] but the design of a simple and effective controller with the perfect improvement of performance has not been fully achieved for a variety of time-delay processes. Some controllers can provide good set-point response but poor disturbance response or inversely. Therefore, the present study is focused on the design of IMC-PID controller cascaded with second-order lead-lag filter for unstable second-order process model for fulfilling various control purposes: simplicity, optimality, analytical form, model-based, and easy to implement in the practice with the excellent performance for both the disturbance rejection and set-point tracking.

To demonstrate the simplicity and effectiveness of the proposed method in compared with many other outstanding design methods, all comparative controllers are all tuned to have the same robustness level according to the maximum sensitivity (*Ms*) value. The simulation results indicate that the proposed method afford the superior PID filter controller for both the disturbance rejection and set-point tracking problems.

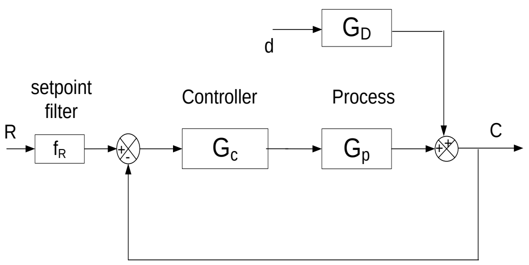



***Fig. 1.*** *Feedback control structure of proposed method*

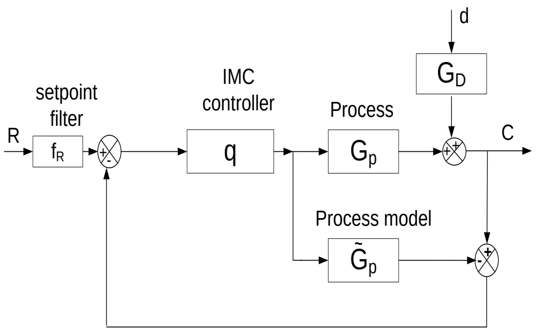



***Fig. 2.*** *IMC structure*

## 2. DESIRED CLOSED-LOOP RESPONSE AND IDEAL CONTROLLER

The general open-loop transfer function of second-order process model is given by:

$$\mathrm{G_P}(s)=\mathrm{G_D}(s)=\frac{\mathrm{K}e^{-\theta s}}{(\tau_1 s\pm1)(\tau_2 s\pm1)} \quad (1)$$

where K, θ, $\tau_1$ and $\tau_2$ are process gain, time delay, and time constant, respectively.

According to the 2DOF design structure, the sensitivity and complementary sensitivity functions, which are often called as the set-point and disturbance rejection, are easily found as the following transfer function:

$$\frac{\mathrm{y(s)}}{\mathrm{r(s)}}=\frac{\mathrm{G_P(s)}\mathrm{G}_C\mathrm{(s)}}{1+\mathrm{G_P(s)}\mathrm{G}_C\mathrm{(s)}}\mathrm{f_r(s)} \quad (2)$$

$$\frac{\mathrm{y(s)}}{\mathrm{d(s)}}=\frac{\mathrm{G}_D\mathrm{(s)}}{1+\mathrm{G_P(s)}\mathrm{G}_C\mathrm{(s)}} \quad (3)$$

The closed-loop transfer functions for the desired set-point and disturbance responses are respectively simplified based on IMC control scheme as shown in Fig. 2:

$$\frac{\mathrm{y(s)}}{\mathrm{r(s)}}=\frac{\mathrm{p_A}(s)\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{(\lambda s+1)^n} \quad (4)$$

$$\frac{\mathrm{y(s)}}{\mathrm{d(s)}}=\left[1-\frac{\mathrm{p_A}(s)\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{(\lambda s+1)^n}\right]\mathrm{G_d}(s) \quad (5)$$

where the notation is summarized as follows: $\tilde{\mathrm{G}}_\mathrm{P}(s),\mathrm{G_c}(s),\mathrm{q(s)}$ , and $\mathrm{f_r}(s)$ denote the the process model, the equivalent feedback controller, the IMC controller, and the set-point filter, respectively. $\mathrm{y}(s)$, $\mathrm{r}(s)$, $\mathrm{d}(s)$ and $\mathrm{u}(s)$ correspond to the controlled output, set-point input, disturbance input, and manipulated variables.

It is important to note that: In accordance with the IMC parameterization [8, 9], the process model $\tilde{\mathrm{G}}_\mathrm{P}(s)$ is factored into two parts:

$$\tilde{\mathrm{G}}_\mathrm{P}(s)=\mathrm{p_m}(s)\mathrm{p_A}(s) \quad (6)$$

where $\mathrm{p_m}(s)$ is the portion of the model inverted by the controller (minimum phase), $\mathrm{p_A}(s)$ is the portion of the model not inverted by the controller (it is the non-minimum phase that may be included the dead time and/or right half plane zeros and chosen to be all-pass), and the requirement that $\mathrm{p_A}(0)=1$ is necessary for the controlled variable to track its set-point.

The IMC controller $q(s)$ is simply designed as:

$$q(s) = p_m^{-1}(s) f(s) \quad (7)$$

The parameters of $f(s)$ should be determined for an optimal compromise between robustness and performance, typically one-parameter filters with unity steady-state gain is used by the following form:

$$f(s) = \frac{\sum_{i=1}^{m} \beta_i s^i + 1}{(\lambda s + 1)^n} \quad (8)$$

where $\lambda$ is an adjustable parameter, which is directly related to the closed-loop time constant. The integer *n* is selected to be large enough for the IMC controller proper. The parameter $\beta_i$ is determined to cancel the poles near zero in $G_d(s)$.

The ideal feedback controller $G_c(s)$ that yields the desired loop responses given by Eq. (4) perfectly is constituted by:

$$G_c(s) = \frac{q(s)}{1 - \tilde{G}_P(s) q(s)} = \frac{p_m^{-1}(s)\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda s + 1)^n - p_A(s)\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)} \quad (9a)$$

The IMC-PID tuning rules can be obtained by comparing Eq. (9) with the following PID controller:

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\left(\frac{1 + cs + ds^2}{1 + as + bs^2}\right) = \left(\frac{K_c}{s\tau_I}\right)\left(1 + \tau_I s + \tau_I \tau_D s^2\right)\left(\frac{1 + cs + ds^2}{1 + as + bs^2}\right) \quad (9b)$$

## 3. IMC-PID TUNING RULES FOR SODUP MODELS

### 3.1. SODUP model with one unstable pole

The transfer function of the process model can be given as:

$$G_P(s) = G_D(s) = \frac{Ke^{-\theta s}}{(\tau_1 s - 1)(\tau_2 s + 1)} \quad (10)$$

The IMC filter structure is also chosen as:

$$f(s) = \left(\beta_2 s^2 + \beta_1 s + 1\right) / (\lambda s + 1)^4 \quad (11)$$

Then, the IMC controller is obtained by:

$$q(s) = (\tau_1 s - 1)(\tau_2 s + 1)\left(\beta_2 s^2 + \beta_1 s + 1\right) / K(\lambda s + 1)^4 \quad (12)$$

The ideal feedback controller can be obtained by using Eq. (9a) and the tuning rules are listed as follows:

$$K_c = \frac{-\beta_1}{K(4\lambda + \theta - \beta_1)} \quad (13)$$

$$\tau_I = \beta_1 \quad (14)$$

$$\tau_D = \frac{\beta_2}{\beta_1} \quad (15)$$

$$a = \frac{\left(6\lambda^2 - \frac{\theta^2}{10} - \beta_2 + \frac{8\lambda\theta}{5} + \frac{3\theta\beta_1}{5}\right)}{(4\lambda + \theta - \beta_1)} + (\tau_1 - \tau_2) \quad (16)$$

$$b = \frac{\left(4\lambda^3 + \frac{12\lambda^2\theta}{5} + \frac{\lambda\theta^2}{5} + \frac{\theta^3}{60} - \frac{3\theta^2\beta_1}{20} + \frac{3\theta\beta_2}{5}\right)}{(4\lambda + \theta - \beta_1)} + \tau_1\tau_2 + a(\tau_1 - \tau_2) \quad (17)$$

$$c = \frac{2\theta}{5} \quad (18)$$

$$d = \frac{\theta^2}{20} \quad (19)$$

$$\beta_1 = \frac{\tau^2\left[\left(1 + \frac{\lambda}{\tau}\right)^4 e^{\theta/\tau} - 1\right] - \tau_2^2\left[\left(1 - \frac{\lambda}{\tau_2}\right)^4 e^{-\theta/\tau_2} - 1\right]}{(\tau_2 - \tau_1)} \quad (20)$$

$$\beta_2 = \tau_2^2\left[\left(1 - \frac{\lambda}{\tau_2}\right)^4 e^{-\theta/\tau_2} - 1\right] + \beta_1\tau_2 \quad (21)$$

### 3.2. SODUP model with two unstable poles

The transfer function of the process model can be given as:

$$G_P(s) = G_D(s) = \frac{Ke^{-\theta s}}{(\tau_1 s - 1)(\tau_2 s - 1)} \quad (22)$$

For this process model, the IMC filter, the IMC controller can be chosen as Eq. (11) and Eq. (12). The resulting PID controller tuning rules can be calculated by using above design principle (i.e., from Eq. (13) to Eq. (21) for the other kinds of SODUP in terms of changing the signs of $\tau_1$ and $\tau_2$.

## 4. SIMULATION STUDY

In order to have a fair comparison, some performance indices are considered in this part.

To evaluate the closed-loop performance, the response that exceeds the ultimate value following a step change in the disturbance or the set-point, the required control effort, and the robustness of the control system the IAE criterion, overshoot (OV) measuring, total variation (TV), and the peak value of the sensitivity function (Ms) [8-11] are used here.

***Table 1.*** *PID controller parameters and performance indices for example 1*

| | Proposed | Shamsuzzoha | Lee et al. |
|---|---|---|---|
| $K_C$ | 8.582 | 5.179 | 3.194 |
| $\tau_I$ | 4.404 | 7.009 | 15.284 |
| $\tau_D$ | 1.203 | 1.472 | 1.975 |
| $\lambda$ | 0.738 | 1.179 | 3.635 |
| *Ms* | 1.95 | 1.95 | 1.95 |
| Set-point tracking | | | |
| **IAE** | 2.626 | 3.880 | 8.217 |
| **OV** | 0.008 | 0.031 | 0.000 |
| **TV** | 19.958 | 63.35 | 2.439 |
| Disturbance rejection | | | |
| **IAE** | 0.515 | 1.376 | 4.786 |
| **OV** | 0.150 | 0.213 | 0.412 |
| **TV** | 3.693 | 19.559 | 2.552 |
| Proposed: $a=0.0630$, $b=0.047389$, $c=0.3756$, $d=0.04409$, $\gamma=0.3$, $f_r(s)=(1.3213s+1)/(5.2985s^2+4.4042s+1)$. | | | |
| Shamsuzoha: $^{b}a=0.0321$, $b=0$, $c=0.4695$, $d=0$ $\gamma=0.3$, $f_r(s)=(2.1026s+1)/(10.3163s^2+7.0086s+1)$. | | | |
| Lee et al.: $f_r(s)=1/(12.9939s+1)$. | | | |

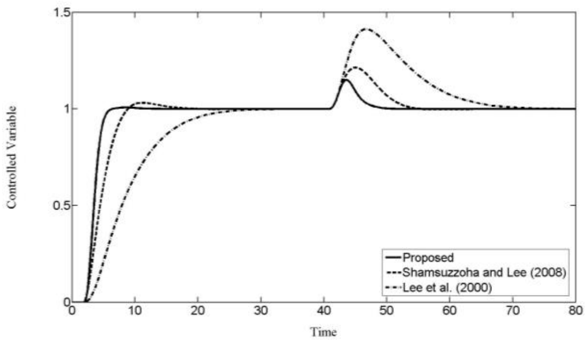
### 4.1. Example 1

The following unstable process is studied by many authors [4-7], which has the open-loop transfer function as follows:

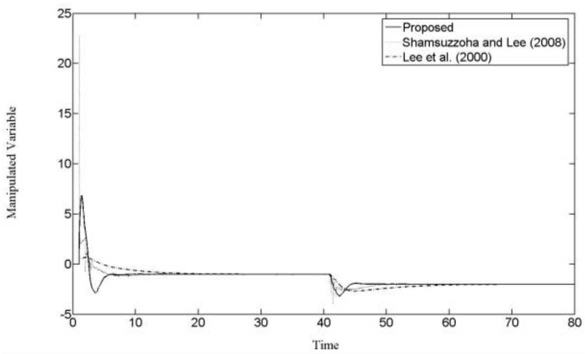
$$G_p(s)=G_d(s)=\frac{e^{-0.939s}}{(5s-1)(2.07s+1)} \tag{23}$$

The proposed, Shamsuzoha and Lee [7], together with Lee et al. [4] design methods were utilized to design the PID controllers as shown in Table 1. All the tuning methods were adjusted to have the same robustness level as MS = 1.95.

The resulting output responses to unit step changes in the set-point ($t$ = 0) and disturbance ($t$ = 40) are shown in Fig. 3. It is apparent from the table and figures that the proposed PID controller provides the best performance among all three PID controllers for both the disturbance rejection and set-point tracking.




(a)




(b)

***Fig. 3.*** *Simulation results of PID controllers for the SODUP with one unstable pole. (a) Control variables, (b) Manipulated variables*

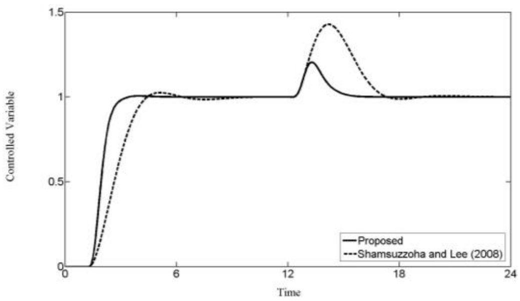
### 4.2. Example 2

The SODUP with two unstable poles is considered here. The open-loop transfer function is given by [4, 6,7]:

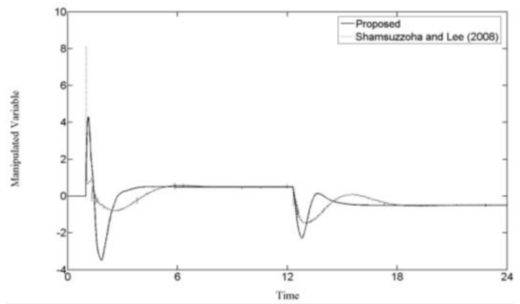
$$G_p(s) = G_d(s) = \frac{2e^{-0.939s}}{(3s-1)(s-1)} \tag{23}$$

The above unstable process model was extensively studied by several authors. Shamsuzzoha and Lee [7] has demonstrated the superiority of the method over many well-known methods. Therefore, the proposed method is compared with those of Shamsuzzoha and Lee [7]. For both comparative design methods, the adjustable parameters $\lambda$ were selected to obtain the same degree of robustness by *Ms* value.

Figure 4 shows the set-point and disturbance responses afforded by all of the comparative methods, which demonstrate the clear advantage of the proposed method over Shamsuzzoha and Lee [7]. Besides, the controller characteristics summarized in Table 2 also confirmed the significant improvement of performance for the proposed method.




(a)




(b)

***Fig. 4***. *Simulation results of PID controllers for the SODUP with two unstable pole. (a) Control variables, (b) Manipulated variables*

***Table 2.*** *PID controller parameters and performance indices for example 2*

| | Proposed | Shamsuzzoha |
|---|---|---|
| $K_C$ | 4.927 | 1.687 |
| $\tau_I$ | 1.32 | 1.8145 |
| $\tau_D$ | 1.061 | 2.382 |
| $\lambda$ | 0.2885 | 0.5131 |
| *Ms* | 2.30 | 2.30 |
| Set-point tracking | | |
| **IAE** | 1.006 | 1.006 |
| **OV** | 0.0059 | 0.025 |
| **TV** | 18.416 | 42.033 |
| Disturbance rejection | | |
| **IAE** | 0.272 | 1.079 |
| **OV** | 0.2173 | 0.4293 |
| **TV** | 8.8079 | 12.6335 |
| Proposed: $a = 0.01233$, $b = 0.04815$, $c = 0.12$, $d = 0.0045$, $\gamma = 0.35$, $f_r(s) = (0.462s+1)/(1.40s^2+1.32s+1)$. | | |
| Shamsuzoha: $a = 0.00931$, $b = 0$, $c = 0.15$, $d = 0$ $\gamma = 0.35$, $f_r(s) = (0.54436s+1)/(4.261s^2+1.8145s+1)$. | | |

### 5. CONCLUSIONS

The IMC-PID controllers cascaded with second-order lead/lag filter were introduced for SODUP models on the basis of IMC control theory.

It is indicated from the simulation results that the proposed method can be effectively applied to design the feedback control system for the second-order unstable processes in order to achieve the fast and well balanced output responses with small IAE values.

### ACKNOWLEGEMENT

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