APPLICATION OF GENETIC ALGORITHM IN OPTIMIZATION CONTROLLER FOR CART AND POLE SYSTEM

ỨNG DỤNG GIẢI THUẬT DI TRUYỀN TRONG TỐI ƯU HÓA BỘ ĐIỀU KHIỂN CHO HỆ CON LẮC NGƯỢC TRÊN XE

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ABSTRACT

The paper introduces the LQR (liner quadratic regulator) controller optimization method by GA (genetic algorithm) controlling the inverted pendulum on a cart. The paper also presents the model's kinetic equation. Authors build up the LQR controllers to stabilize the model. Q matrix is selected by both experiences and GA. Then, authors compare responses of the system with these LQR controllers through different generations of GA. The result shows that optimized LQR controller through GA responses in a better way via simulation and experiment.

Keywords: *The LQR controller; genetic algorithm; cart and pole system; balance control; inverted pendulum.*

TÓM TẮT

Bài báo giới thiệu phương pháp tối ưu bộ điều khiển LQR bằng giải thuật di truyền (GA) cho mô hình con lắc ngược trên xe. Bài báo trình bày phương trình động học của mô hình. Sau đó, nhóm tác giả xây dựng các bộ điều khiển LQR ổn định mô hình, giữ cho thanh con lắc cân bằng ở vị trí hướng lên. Kết quả điều khiển trong trường hợp ma trận Q lựa chọn bằng kinh nghiệm được so sánh với trong trường hợp ma trận Q được tối ưu bằng GA. Từ đó nhóm tác giả so sánh đáp ứng ngõ ra hệ thống với những bộ điều khiển LQR trên. Kết quả chứng minh bộ điều khiển LQR sau tối ưu bằng GA cho đáp ứng ngõ ra tốt hơn thông qua mô phỏng và trên mô hình thực.

Từ khoá: *Điều khiển LQR; Giải thuật di truyền; hệ xe-con lắc ngược; điều khiển cân bằng; hệ con lắc ngược.*

1. INTRODUCTION

LQR is a common optimizing method with the aim of minimizing quadratic quality. The problem is to find the K matrix of optimal control vector that meets the minimum quality standards. K is calculated by Riccati equation with matrices A, B, Q, R. Matrix A, B are defined by the linearized mathematical equations around equilibrium working point. Q and R are rationally selected based on designer's experiences. The selection of R and Q matrices in LQR is very importance and it straight affects the control effect. The weight matrices were set by experience first and then were adjusted by simulation till obtaining the satisfying output responses. To this process, if the designer known poor about the system, the optimal weight matrices could not be obtained and so the control performance also could not be optimal. Therefore, the problem is how we choose the parameters of the matrix R and Q for the system to respond optimally. The GA is one method that help the designer achieve this task.

GA introduced by Holland in 1975 is a general algorithm for solving optimal problems. GA determines the optimal gains at a reasonable computational cost, and it often does not get trapped in a local optimum. The performance of the GA-tuned controller is investigated using several objective functions and under various operating conditions in comparison to other traditional tuning methods. There have been some studies in optimizing LQR controllers using GA [1], [2]. However, applying GA to optimize LQR controlling the vehicle-pendulum system on simulation and real model is not available. For these reasons, the authors chose the optimal GA method for the LQR controller through simulation and experimental.

The paper consists of five sections. Part 1 introduces the model. Part 2 presents the mathematical equations. Part 3 shows how to use the GA to optimize the LQR parameters. Results on simulation and experimental are presented in part 4. Finally, the conclusion is mentioned in part 5.

2. SYSTEM MODEL OF CART AND POLE

In this study, the model's parameters are shown in Fig. 1. The vehicle moves in the x direction. The inverted pendulum is a straight bar that can rotate around the center of the cart.

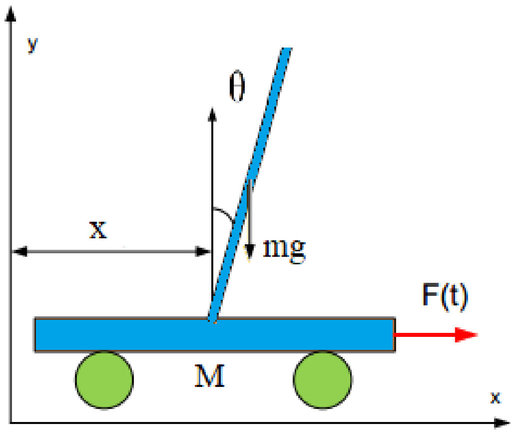

Fig. 1. *Cart and pole system*

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q \quad (1)$$

With $p = \begin{bmatrix} x \\ \theta \end{bmatrix}, Q = \begin{bmatrix} F \\ 0 \end{bmatrix}$

Kinetic energy of system:

$$T = T_{pole} + T_{cart} = \frac{1}{2}m(\dot{x}^2 + 2C_1\dot{\theta}\dot{x}\cos\theta + C_1^2\dot{\theta}^2) + \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}M\dot{x}^2 \quad (2)$$

Potential energy of system:

$$P = P_0 + P_1 = mgC_1\cos\theta \quad (3)$$

Lagrangian:

$$L = T - P = \frac{1}{2}m(\dot{x}^2 + 2C_1\dot{\theta}\dot{x}\cos\theta + C_1^2\dot{\theta}^2) + \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}M\dot{x}^2 - mgC_1\cos\theta \quad (4)$$

Lagrangian for motion of cart:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F \quad (5)$$

Lagrangian for rotating motion of pendulum:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = 0 \quad (6)$$

Solve (5) and (6), we have system dynamic equations:

$$\begin{cases} (m+M)\ddot{x} + mC_1\ddot{\theta}\cos\theta - mC_1\dot{\theta}^2\sin\theta = F \\ mC_1\ddot{x}\cos\theta + (J_1 + mC_1^2)\ddot{\theta} - mC_1 g\sin\theta = 0 \end{cases} \quad (7)$$

However, for the convenience of adjusting the motor as well as applying the controller to the real model, the input conversion is the voltage providing for the motor [3].

Modelling motor:

According to the real structure [3], we divide the motor into two parts:

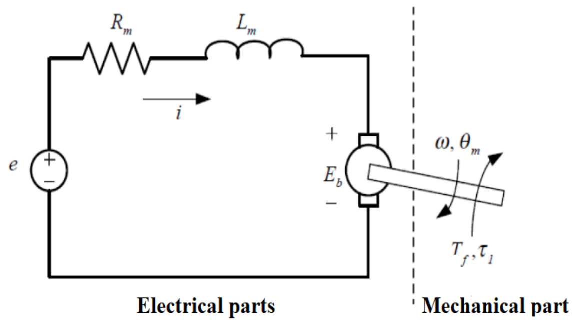

Fig. 2. *The modelling motor*

Electrical parts:

$$e = L_m \frac{di}{dt} + R_m i + K_b \omega \quad (8)$$

Mechanical part:

$$J_m \frac{d\omega}{dt} = K_t i - T_f - C_m \omega - \tau_1 \quad (9)$$

Electrical capacity: $P_e = E_b i$ (W) (10)

Mechanical capacity: $P_m = \tau_m \omega$ (W) (11)

Law preserves mass:

$$P_e = P_m \rightarrow K_b = K_t \quad (12)$$

Solve (8) and (9), Laplace transform:

$$\begin{cases} (L_m s + R_m) I(s) + K_b \Omega(s) = E(s) \\ K_t I(s) - (J_m s + C_m)\Omega(s) = T_f(s) + \tau_1(s) \end{cases} \quad (13)$$

We suppose T_f is constant and $T_f = K_f \operatorname{sgn}(\Omega)$

K_f is constant and $\operatorname{sgn}(\Omega) = \begin{cases} 1 & \Omega > 0 \\ 0 & \Omega = 0 \\ -1 & \Omega < 0 \end{cases}$ (14)

Fig. 2 shows the mathematical model of the motor which is defined by parameters: T_f, L_m, K_b, R_m, J_m. These parameters from Fig. 7 are listed in Table 1:

Table 1: *Parameter of motor*

L_m(H)	0.250868
$K_b\left(V/(rad/sec)\right)$	0.064943
$R_m(\Omega)$	6.835721
$\lvert T_f \rvert$(N.m)	0.010764
$J_m(kg.m^2)$	0.000134
C_m(N.m / (rad / sec))	0.000048

Full-system mathematical model:

Because the electrical speed is much faster than the mechanical speed:

$e \gg L_m \frac{di}{dt} \rightarrow$ can be ignored $L_m \frac{di}{dt}$

$$\rightarrow e = R_m i + K_b \omega \rightarrow i = \frac{e - K_b \omega}{R_m} \quad (15)$$

$$\tau_m = K_t i = \frac{K_t}{R_m}(e - K_b \omega) = \frac{K_t}{R_m} e - \left(\frac{K_t K_b}{R_m}\right)\omega \quad (16)$$

We have: $\dot{x} = \frac{R\omega}{d_l}$ (17)

Substitute (17) into (16)

$$\rightarrow \tau_m = \frac{K_t}{R_m} e - \frac{K_b K_t}{R_m R_c} \dot{x} \quad (18)$$

Substitute (18) into (11) (as skipping T_f)

$$\tau_1 = -\frac{J_m d_l}{R_c} \ddot{x} - d_l \left(\frac{C_m}{R_c} + \frac{K_b K_t}{R_m R_c}\right)\dot{x} + \frac{K_t}{R_m} e \quad (19)$$

The force on the cast:

$$F = \frac{d_l \tau_1}{R_c} = \frac{d_l}{R_c}\left[\frac{K_t}{R_m} e - d_l \left(\frac{K_b K_t}{R_m R_c} + \frac{C_m}{R_c}\right)\dot{x} - \frac{J_m d_l}{R_c}\ddot{x}\right] \quad (20)$$

With: $k_1 = \frac{d_l K_t}{R_m R_c}$ (21)

$$k_2 = \frac{d_l^2 K_t K_b}{R_m^2 R_c} + \frac{d_l^2 C_m}{R_c^2} \quad (22)$$

$$k_3 = \frac{d_l^2 J_m}{R_m^2} \quad (23)$$

$$\rightarrow F = k_1 e - k_2 \dot{x} - k_3 \ddot{x} \quad (24)$$

Combining (13), (20), (24), we have the kinetic equations of the cart:

$$M_f(q)\ddot{q} + V_{mf}(q,\dot{q})\dot{q} + G_f(q) = \begin{bmatrix} k_1 \\ e \end{bmatrix} \quad (25)$$

Where: $\begin{cases} M_f(q) = \begin{bmatrix} m + M + k_3 & mC_1 \cos\theta \\ mC_1 \cos\theta & J_1 + mC_1^2 \end{bmatrix} \\ V_{mf} = \begin{bmatrix} k_2 & -mC_1 \dot{\theta}^2 \sin\theta \\ 0 & 0 \end{bmatrix} \\ G_f = \begin{bmatrix} 0 \\ -mC_1 g \sin\theta \end{bmatrix} \end{cases}$ (26)

After identification, the motor parameters as in Fig. 7 are shown in Table 1. Then, system parameters are shown in Table 2:

Table 2: *System parameters*

	Parameter	Description	Unit	Value
1	M	Mass of cart	Kg	0.3
2	M	Mass of pendulum	Kg	0.11
3	C_1	Length of pendulum	m	0.32
4	G	Gravitation acceleration	m/s^2	9.81
5	J_1	Inertial moment of pendulum	kgm^2	0.0038
6	R_c	Radius of wheel	m	0.024
7	d_1	The clutch gear ratio		1
8	τ_1	Torque	Nm	
9	ω	Motor velocity	Rad/s	
10	τ_m	Inner torque	Nm	
11	F	Force controlling cart	N	

3. OPTIMIZING THE LQR CONTROLLER USING GA

3.1 GA METHOD

In this case, GA used is off-line. Parameters for GA program are listed as below:

- Size of population: N = 100
- Linear Ranking Selection: η=0.2
- Decimal coding
- Two-point crossover
- Crossover parameter: 0.8
- Mutation parameter: 0.2

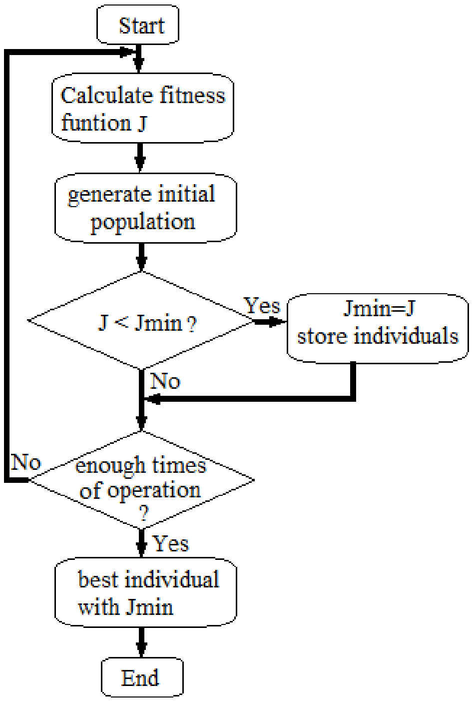

Fig. 3. *Flow chart of GA Searching process [8]*

Choose fitness function:

$$J = \sum_{i=1}^{n}[e_1(i)\dot{e}_1(i)+e_2(i)\dot{e}_2(i)] \quad (27)$$

With $e_1=x$, $e_2=\dot{x}$ and n as number of samples in one time of simulation, value of J depends on the error of the cart position, the pendulum angle. In this case, if operating in 100s, with sample-time as 0.01s, we have n = 10001 sample.

The result of running the GA program converged after about 50 generations. The value of J is shown in Fig. 4:

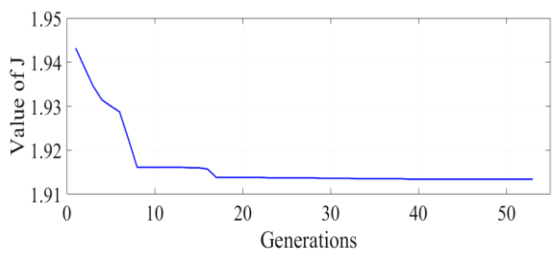

Fig. 4. *The change of J over generations*

Fig. 4 shows the value of the fitness function of the best chromosome in evolution from one generation to the next, approaching the minimum.

Matrix Q is chosen as:

$$Q_1 = \begin{bmatrix} 10^5 & 0 & 0 & 0 \\ 0 & 10^{-1} & 0 & 0 \\ 0 & 0 & 10^5 & 0 \\ 0 & 0 & 0 & 10^{-1} \end{bmatrix} \quad (28)$$

We get the matrix (29) and (30) from searching by GA. Matrix Q has the smallest J fitness function of the initial 100 parents:

$$Q_2 = \begin{bmatrix} 46292 & 0 & 0 & 0 \\ 0 & 91.2486 & 0 & 0 \\ 0 & 0 & 93919 & 0 \\ 0 & 0 & 0 & 50.8533 \end{bmatrix} \quad (29)$$

The matrix Q has the smallest J fitness function after 200 generations:

$$Q_3 = \begin{bmatrix} 361502 & 0 & 0 & 0 \\ 0 & 86 & 0 & 0 \\ 0 & 0 & 420533 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix} \quad (30)$$

3.2 Building the LQR controller

From [4], [5], we can calculate a LQR controller from identified system parameters. The robustness of LQR control was proved in [6]. From the model, we can calculate matrix A and B to linearize around working-point to get linear equation of system [8]:

$$\begin{cases} \dot{x} = Ax + B \times F \\ y = Cx \end{cases} \quad (31)$$

Conducting to discrete A, B with sample-time is 0.01s, we have A_d, B_d:

$$A_d = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2.0604 & -1.4441 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 4.8291 & 26.3768 & 0 \end{bmatrix} \quad (32)$$

$$B_d = \begin{bmatrix} 0 \\ 0.7063 \\ 0 \\ -4.1820 \end{bmatrix} \quad (33)$$

Choose matrix R:

$$R = 1 \quad (34)$$

With symetric positive definite matrixes Q and R. By calculting $K_d = \text{dlqr}(A, B, Q, R)$ from Matlab, we have:

K_{d1} = [-242.5648 -134.5639 -283.6083 -33.8703]
with A_d, B_d, Q_1, R. (35)

K_{d2} = [-159.4334 -95.2601 -257.2087 -28.0662]
with A_d, B_d, Q_2, R. (36)

K_{d3} = [-292.3100 -165.6592 -370.9837 -47.9265]
with A_d, B_d, Q_3, R. (37)

(35) is LQR_1 controller.

(36) is LQR_2 controller.

(37) is LQR_3 controller.

4. SIMULATION RESULTS AND EXPERIMENTAL RESULTS

4.1 Simulation results

Running simulation program with the initial values: $x = 0.01$ (m); $\dot{x} = 0$ (m/s); $\theta = \frac{\pi}{9}$ (rad); $\dot{\theta} = 0.02$ (rad/s).

The simulation of system is shown in Fig. 5 and 6 below:

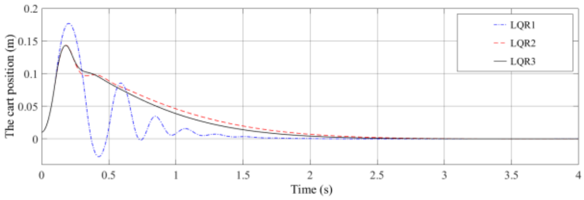

Fig. 5. *The cart position*

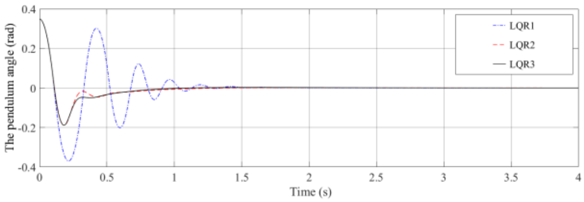

Fig. 6. *The pendulum angle*

The value of fitness function J:

$J = 4.3965$ with LQR_1.

$J = 1.9128$ with LQR_2.

$J = 1.8812$ with LQR_3.

From Fig. 5 and 6, the response of output system after being optimized is more stable and well transient period. Overshoot is decreased, setting error to approach to 0 - through the gradual decrease of J value.

4.2 Experimental results

Experimental inverted pendulum on cart model is shown in Fig. 7.

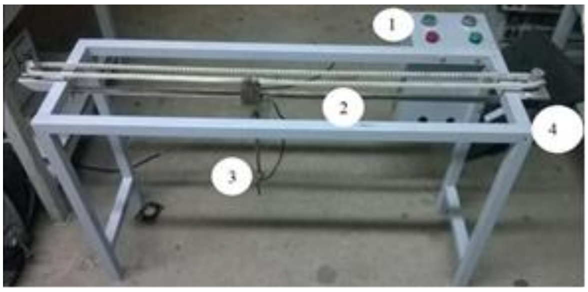

Fig. 7. *Inverted pendulum on cart model in practice*

1: Control panel; 2: Rail for the movement of cart in x axis; 3: Pendulum.; 4: 24VDC motor.

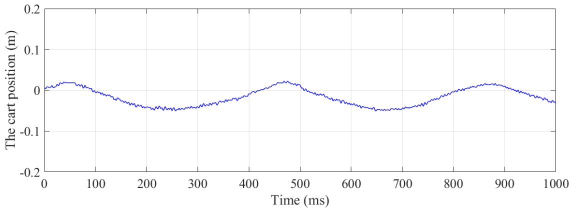

Fig. 8. *The cart position with LQR1 controller*

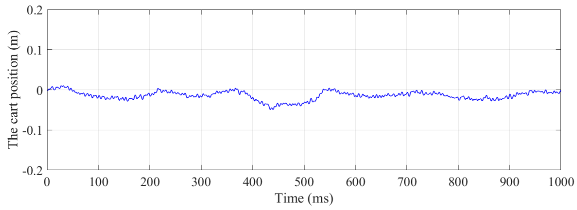

Fig. 9. *The cart position with LQR2 controller*

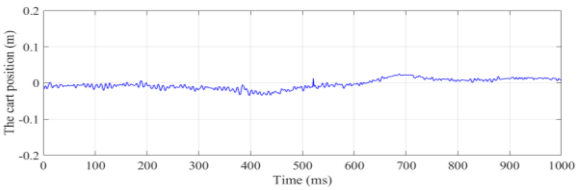

Fig. 10. *The cart position with LQR3 controller*

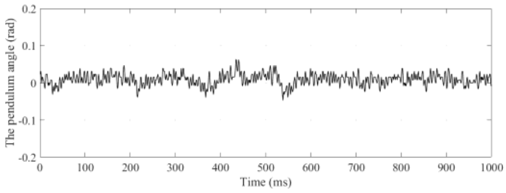

Fig. 11. *The pendulum angle with LQR1 controller*

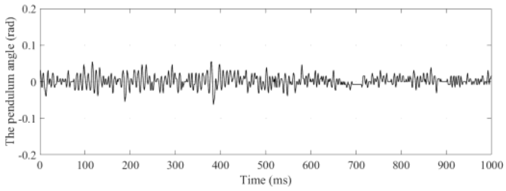

Fig. 12. *The pendulum angle with LQR2 controller*

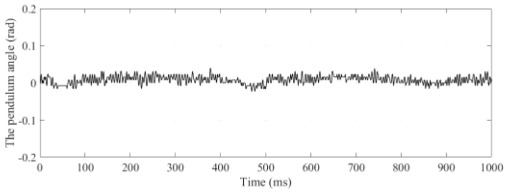

Fig. 13. *The pendulum angle with LQR3 controller*

With the real output response, it is found that the following optimal LQR controller (LQR_2, LQR_3) still provide better output response of position (Figures 8, 9, 10) and the pendulum angle (Figures 11, 12, 13) than the LQR controller chosen by the authors (LQR_1).

5. CONCLUSION

The output response on simulations and real models shows that system is stable around the working point. The optimized LQR controller gives better responses, reduces overshoot significantly and increases the system stability. The optimization method of LQR controller using GA is demonstrated to perform well in both simulation and real model.

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