

STEPS OF USING TRANSFORMATION MATRIX TO ANALYZE KINEMATICS OF SPECIFIC PLANAR MECHANISM

CÁC BƯỚC DÙNG MA TRẬN CHUYỂN VỊ PHÂN TÍCH ĐỘNG HỌC CƠ CẤU PHẪNG CỤ THỂ

Phan Thanh Nhan

Ho Chi Minh City University of Technology and Education, Vietnam

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ABSTRACT

Currently, when analyzing kinematics of machines and mechanism, most students in universities often use a familiar method (called graphical kinematic analysis) that sketches position, velocity, and diagrams. Graphical kinematic analysis is considered as a simple, intuitive approach but less accurate because values of kinematic quantities are measured from graphical vector diagrams. Moreover, when the input parameters such as length of links, angular velocity of the input link... are changed the calculation process must be repeated from the beginning that consumes much time. Kinematic analysis is a more advanced method thanks to using precise mathematical operations and easy to automate. There are many kinematic analytical methods introduced in some documents in universities but most of them are difficult to use because of complicated application. Using the transformation matrix to analyze kinematics of planar mechanisms is considered as an appropriate, easy-to-use method that can overcome drawbacks of the graphical method. In order to assist students step by step to apply a new method of calculating planar mechanism kinematics, this article will detail the steps of applying the transformation matrix to analyze kinematics of one specific planar mechanism. Finally, the article will show the graphs of location, velocity, and acceleration obtained from a program written in MatLab programming language.

Keywords: Transformation matrix; Analytical kinematics; Planar mechanism; Quick return mechanism; Machine design.

TÓM TẮT

Hiện nay, khi thiết kế tính toán động học cơ cấu máy, sinh viên ở các trường đại học thường sử dụng phương pháp quen thuộc vẽ họa đồ vị trí, vận tốc và gia tốc (còn gọi là phương pháp hình học). Phân tích động học bằng phương pháp hình học có ưu điểm là đơn giản, trực quan nhưng có nhược điểm lớn là công kênh, thiếu chính xác do giá trị các đại lượng động học được đo từ các họa đồ vectơ. Hơn nữa, một khi thay đổi các thông số đầu vào như kích thước các khâu, vận tốc góc khâu dẫn... thì qui trình tính toán phải thực hiện lại từ đầu, tốn công và mất rất nhiều thời gian. Động học giải tích là một phương pháp tiên tiến hơn nhờ dùng các phép toán chính xác và dễ thực hiện tự động. Có nhiều phương pháp động học giải tích được giới thiệu trong các tài liệu ở các trường đại học nhưng hầu hết chúng rất khó dùng vì tính phức tạp khi áp dụng. Dùng ma trận chuyển vị phân tích động học động học cơ cấu phẳng là một phương pháp thích hợp, dễ sử dụng và có thể khắc phục các nhược điểm của phương pháp hình học. Nhằm hỗ trợ sinh viên từng bước áp dụng một phương pháp mới tính toán động học cơ cấu phẳng, bài báo này sẽ trình bày một cách chi tiết các bước ứng dụng ma trận chuyển vị để thực hiện phân tích động học một cơ cấu phẳng cụ thể. Cuối cùng, bài báo sẽ biểu diễn các đồ thị vị trí, vận tốc, gia tốc nhận được từ chương trình được viết từ ngôn ngữ lập trình MatLab.

Từ khóa: Ma trận chuyển vị; Động học giải tích; Cơ cấu phẳng; Máy bào ngang; Thiết kế máy.

1. INTRODUCTION

In planar mechanism, kinematic analysis can be performed either analytically or graphically [1]. Graphical kinematic analysis is considered as a simple, intuitive method but less accurate and very difficult to use by computer-aided methods. Analytical kinematics is based on the matrix called transformation matrix which is established by trigonometric functions in terms of rotation angle of links. Then, for a given value of the position (or orientation) of the input link, the algebraic equations are solved for the position and/or orientation of the remaining links. The first and second time derivative of the algebraic position equations will provide the velocity and acceleration equations for the mechanism. For given values of the velocity and acceleration of the input link, these equations are solved to find the velocity and acceleration of the other links in the system [1].

In order to assist students step by step to apply a new method of calculating planar mechanism kinematics, this article will detail the steps of applying the transformation matrix to analyze kinematics of one specific planar mechanism. Finally, the article presents the graphs of location, velocity, and acceleration obtained from a program written in MatLab programming language.

2. THEORETICAL BASIS

A mechanism (or linkage) is considered as a collection of the links that are interconnected by kinematic joints forming a single or multiple degree-of-freedom chain. One link is designated the frame because it served as the frame of reference for the motion of all other links [2]. Links are the individual parts which are considered rigid bodies. Theoretically, a true rigid body does not change shape during motion. A joint is a movable connection

between and allows relative motion between the links. The two primary joints are revolute and sliding joints. Linkage can be either open or closed chains (Fig.1,2) [2].

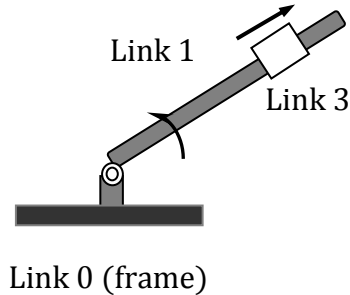


Fig. 1. The open mechanism

In order to establish a general formula for kinematic analysis, it had better to consider kinematics of the i^{th} link in Fig. 3 that is moving in a plane. Point A fixes on the link while point B moves on a straight line. We design two coordinate systems in which one coordinate system named $O_0X_{01}X_{02}$ is fixed to the frame (the fixed coordinate system) and the other with name $O_iX_{i1}X_{i2}$ is fixed to the i^{th} link (the moving coordinate system). Here, A denotes the joint, O_i coincides with A , and O_iX_{i1} coincides with AB . The kinematic input quantities are known as [3]:

$$\begin{cases} x_{A1} = x_{A1}(t) \\ x_{A2} = x_{A2}(t) \\ \theta_i = \theta_i(t) \\ s_i = s_i(t) \end{cases}$$

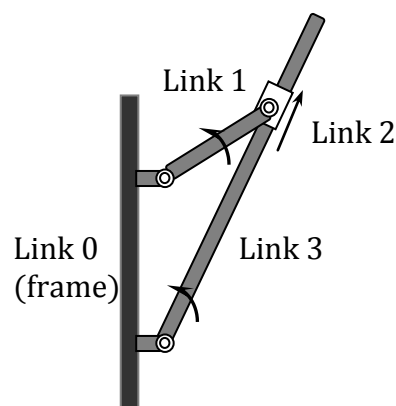


Fig. 2. The closed mechanism

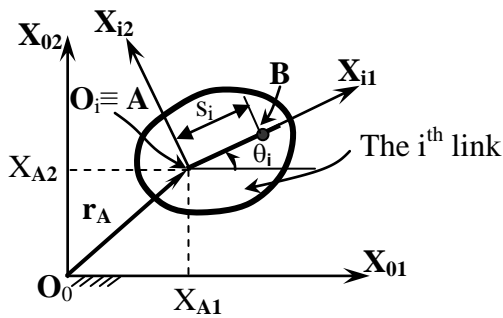


Fig. 3. General kinematic model of planar mechanism

Where $\mathbf{O}_0\mathbf{A} = \mathbf{r}_A = (x_{A1}, x_{A2})$ is vector specified position of point A; (x_{A1}, x_{A2}) is coordinates of point A. The quantity θ_i denotes revolution angle of the link. The quantity s_i denotes the first coordinate of point B and t denotes time. Here, bold characters express vector quantities.

The position, velocity and acceleration of point B can be written in compact formulae as following [4]:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{T}_i \cdot \mathbf{u}_i \quad (1)$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_i \cdot \mathbf{I} \cdot \mathbf{T}_i \cdot \mathbf{u}_i + \mathbf{T}_i \cdot \dot{\mathbf{u}}_i \quad (2)$$

$$\mathbf{a}_B = \mathbf{a}_A + (\varepsilon_i \cdot \mathbf{I} - \omega_i^2 \cdot \mathbf{E}) \cdot \mathbf{T}_i \cdot \mathbf{u}_i + 2\omega_i \cdot \mathbf{I} \cdot \mathbf{T}_i \cdot \dot{\mathbf{u}}_i + \mathbf{T}_i \cdot \ddot{\mathbf{u}}_i \quad (3)$$

Where $\mathbf{r}_B = \begin{Bmatrix} x_{B1} \\ x_{B2} \end{Bmatrix}$ is the vector specified position of point B in the fixed coordinate system.

$\mathbf{r}_A = \begin{Bmatrix} x_{A1} \\ x_{A2} \end{Bmatrix}$ is the vector specified position of point A in the fixed coordinate system.

$\mathbf{u}_i = \mathbf{AB} = \begin{Bmatrix} s_i \\ 0 \end{Bmatrix}$ is the vector specified position of point B in the moving coordinate system.

$\mathbf{T}_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix}$ is the transformation matrix of the i^{th} link compare to the fixed coordinate system [4].

$\mathbf{v}_B = \begin{Bmatrix} v_{B1} \\ v_{B2} \end{Bmatrix}$ is the velocity of point B in the fixed coordinate system (absolute velocity).

$\mathbf{v}_A = \begin{Bmatrix} v_{A1} \\ v_{A2} \end{Bmatrix}$ is the velocity of point A in the fixed coordinate system (absolute velocity).

$\dot{\mathbf{u}}_i = \begin{Bmatrix} \dot{s}_i \\ 0 \end{Bmatrix}$ is the velocity of point B in the moving coordinate system (relative velocity).

$\omega_i = \dot{\theta}_i$ is the angular velocity of the i^{th} link

$\mathbf{I} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is the constant matrix.

$\mathbf{a}_A, \mathbf{a}_B$ are the absolute acceleration of point A and B.

$\mathbf{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the unit matrix.

$\dot{\omega}_i = \varepsilon_i$ is angular acceleration of the i^{th} link.

$\ddot{\mathbf{u}}_i = \begin{Bmatrix} \ddot{s}_i \\ 0 \end{Bmatrix}$ is relative acceleration of point B in the moving coordinate system.

The third component on a right side of the equation (3) is known as *Coriolic* acceleration [3].

In a special case, if point B is fixed to the i^{th} link, it means $s_i = \text{const}$ leading to $\dot{\mathbf{u}}_i = \ddot{\mathbf{u}}_i = 0$.

Equations (2), (3) can be rewritten as

$$\mathbf{v}_B = \mathbf{v}_A + \omega_i \cdot \mathbf{I} \cdot \mathbf{T}_i \cdot \mathbf{u}_i \quad (4)$$

$$\mathbf{a}_B = \mathbf{a}_A + (\varepsilon_i \cdot \mathbf{I} - \omega_i^2 \cdot \mathbf{E}) \cdot \mathbf{T}_i \cdot \mathbf{u}_i \quad (5)$$

3. STEPS FOR APPLICATION

Step 1: Numbering 1, 2, 3, ..., n for the links in mechanism (only numbering the rotation links)

Step 2: Sketch the fixed coordinate system ($X_{01}X_{02}$) attached to the fixed link and the rotation coordinate system ($X_{i1}X_{i2}$) attached the i^{th} rotation link in mechanism at the revolute joints. Denote θ_i ($i = 1..n$) are considered revolute angles of the rotation

coordinate systems compare to the fixed coordinate system.

Step 3: Draw diagrams of joint-link-joint from the input link to others, then write position, velocity, acceleration equations to describe position, velocity and acceleration of joints in mechanism (using equation (1) to (5)).

Step 4: Establish compatible condition for position, velocity and acceleration so as to obtain two algebraic equations containing two unknowns.

Step 5: Write program to calculate kinematic unknowns as functions of position, velocity and acceleration of the input link.

4. EXAMPLE OF ANALYSIS OF SPECIFIC MECHANISM KINEMATICS

Quick return mechanism with shape machine is chosen to illustrate application of formulae from (1) to (5).

Example [5]: Mechanism of the crank and slotted lever type with shaping machine is shown in Fig. 4. The dimensions of the various links are as follows :

$O_1O_2 = 800 \text{ mm}$; $O_1B = 300 \text{ mm}$; $O_2D = 1300 \text{ mm}$; $DR = 400 \text{ mm}$;

The crank O_1B rotates constant at $n = 40 \text{ r.p.m.}$ in the counter clockwise direction.

Draw graphs of position, velocity and acceleration of point R (or of the cutting tool) in a term of the crank angle.

Solution:

The crank O_1B , the link O_2D and the link DR are numbered as 1, 2, 3

Sketch the fixed coordinate system (in red color) and the rotation coordinate systems attached links 1, 2, and 3 (in black color). θ_1 , θ_2 , θ_3 are considered revolute

angles of the rotation coordinate systems compare to the fixed coordinate system. (Fig. 5)

Denote: $O_1B = l$; $O_2D = b$; $DR = c$; $O_1O_2 = d$; $O_2B = s$

Establish equation to determine position of point R

$$O_1 \xrightarrow{\text{Link 1}} B: \mathbf{r}_B = \mathbf{r}_{O_1} + \mathbf{T}_1 \cdot \mathbf{u}_1$$

$$O_2 \xrightarrow{\text{Link 2}} B: \mathbf{r}_B = \mathbf{r}_{O_2} + \mathbf{T}_2 \cdot \mathbf{u}_2$$

$$\text{Deduce: } \mathbf{r}_{O_1} + \mathbf{T}_1 \cdot \mathbf{u}_1 = \mathbf{r}_{O_2} + \mathbf{T}_2 \cdot \mathbf{u}_2$$

where

$$\mathbf{r}_{O_1} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}; \mathbf{r}_{O_2} = \begin{Bmatrix} 0 \\ -d \end{Bmatrix};$$

$$\mathbf{T}_1 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{bmatrix}; \mathbf{u}_1 = \begin{Bmatrix} l \\ 0 \end{Bmatrix};$$

$$\mathbf{T}_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 \end{bmatrix}; \mathbf{u}_2 = \begin{Bmatrix} s \\ 0 \end{Bmatrix}$$

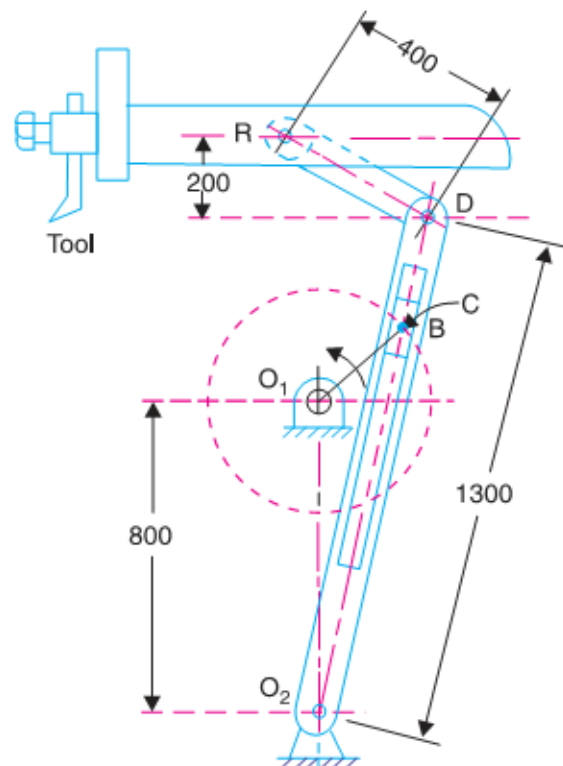


Fig. 4. The quick return mechanism with shaping machine

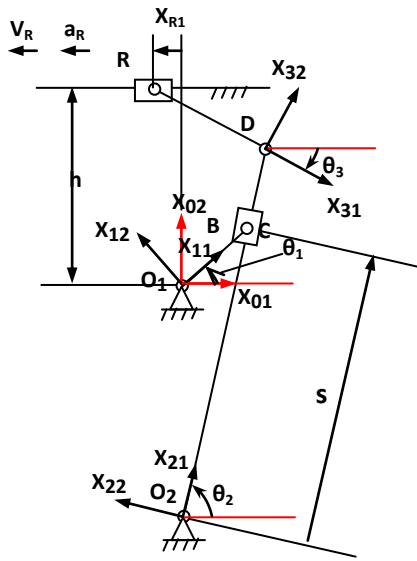


Fig. 5. Sketch the coordinate systems attached links

Substituting and combining above equations, we get equation systems:

$$\begin{cases} s \cdot \cos \theta_2 = l \cdot \cos \theta_1 \\ -d + s \cdot \sin \theta_2 = l \cdot \sin \theta_1 \end{cases}$$

Solving two algebraic equations to get two unknowns s and θ_2 be functions of given quantities as d, l, θ_1

$$s = \sqrt{(l \cdot \cos \theta_1)^2 + (d + l \cdot \sin \theta_1)^2} \quad (6);$$

$$\theta_2 = \cos^{-1}((l/s) \cdot \cos \theta_1) \quad (7)$$

$$O_2 \xrightarrow{\text{Link 2}} D: \mathbf{r}_D = \mathbf{r}_{O_2} + \mathbf{T}_2 \cdot \mathbf{u}_2$$

$$D \xrightarrow{\text{Link 3}} R: \mathbf{r}_R = \mathbf{r}_D + \mathbf{T}_3 \cdot \mathbf{u}_3$$

$$\text{Deduce: } \mathbf{r}_R = \mathbf{r}_{O_2} + \mathbf{T}_2 \cdot \mathbf{u}_2 + \mathbf{T}_3 \cdot \mathbf{u}_3$$

where

$$\mathbf{T}_3 = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 \\ \sin \theta_3 & \cos \theta_3 \end{bmatrix}; \mathbf{u}_3 = \begin{Bmatrix} -c \\ 0 \end{Bmatrix}$$

Here, $\mathbf{u}_2 = \begin{Bmatrix} b \\ 0 \end{Bmatrix}$, vector specifies position of point D in the rotation coordinate system.

$\mathbf{r}_R = \begin{Bmatrix} X_{R1} \\ h \end{Bmatrix}$, vector specifies position of point R in the fixed coordinate system.

Substituting and combining above equations, we get equation systems:

$$\begin{cases} X_{R1} = b \cos \theta_2 - c \cos \theta_3 \\ h = -d + b \sin \theta_2 - c \sin \theta_3 \end{cases}$$

Solving two algebraic equations to get two unknowns X_{R1} and θ_3 as functions of given quantities as d, h, b, c, θ_2

$$\theta_3 = \sin^{-1}((b \sin \theta_2 - h - d)/c) \quad (8);$$

$$X_{R1} = b \cos \theta_2 - c \cos \theta_3 \quad (9)$$

Establish equation to determine velocity of point R

$$O_1 \xrightarrow{\text{Link 1}} B: \mathbf{v}_B = \mathbf{v}_{O_1} + \omega_1 \cdot \mathbf{I} \cdot \mathbf{T}_1 \cdot \mathbf{u}_1$$

$$O_2 \xrightarrow{\text{Link 2}} B: \mathbf{v}_B = \mathbf{v}_{O_2} + \omega_2 \cdot \mathbf{I} \cdot \mathbf{T}_2 \cdot \mathbf{u}_2 + \mathbf{T}_2 \cdot \dot{\mathbf{u}}_2$$

$$\text{Deduce: } \mathbf{v}_{O_1} + \omega_1 \cdot \mathbf{I} \cdot \mathbf{T}_1 \cdot \mathbf{u}_1 = \mathbf{v}_{O_2} + \omega_2 \cdot \mathbf{I} \cdot \mathbf{T}_2 \cdot \mathbf{u}_2 + \mathbf{T}_2 \cdot \dot{\mathbf{u}}_2$$

$$\text{where } \mathbf{v}_{O_1} = \mathbf{v}_{O_2} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}; \omega_1 = \pi n / 30;$$

$$\mathbf{I} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}; \dot{\mathbf{u}}_3 = \begin{Bmatrix} \dot{s} \\ 0 \end{Bmatrix}$$

$\dot{s} = v_{C/O_2D}$ is velocity of the slider C on link O_2D

Substituting and combining above equations, we get equation systems:

$$\begin{cases} -\omega_2 \cdot s \cdot \sin \theta_2 + \dot{s} \cdot \cos \theta_2 = -\omega_1 \cdot l \cdot \sin \theta_1 \\ \omega_2 \cdot s \cdot \cos \theta_2 + \dot{s} \cdot \sin \theta_2 = \omega_1 \cdot l \cdot \cos \theta_1 \end{cases}$$

Solving two algebraic equations to get two unknowns \dot{s} and ω_2 be functions of given quantities as $s, l, \omega_1, \theta_1, \theta_2$

$$v_{C/O_2D} = \dot{s} = \omega_1 \cdot l \cdot \sin(\theta_2 - \theta_1) \quad (10)$$

$$\omega_2 = (\omega_1 \cdot l \cdot \cos \theta_1 - \dot{s} \cdot \sin \theta_2) / (s \cdot \cos \theta_2) \quad (11)$$

$$O_2 \xrightarrow{\text{Link 2}} D: \mathbf{v}_D = \mathbf{v}_{O_2} + \omega_2 \cdot \mathbf{I} \cdot \mathbf{T}_2 \cdot \mathbf{u}_2$$

$$D \xrightarrow{\text{Link 3}} R: \mathbf{v}_R = \mathbf{v}_D + \omega_3 \cdot \mathbf{I} \cdot \mathbf{T}_3 \cdot \mathbf{u}_3$$

Deduce:

$$\mathbf{v}_R = \mathbf{v}_{O_2} + \omega_2 \cdot \mathbf{I} \cdot \mathbf{T}_2 \cdot \mathbf{u}_2 + \omega_3 \cdot \mathbf{I} \cdot \mathbf{T}_3 \cdot \mathbf{u}_3$$

Here, $\mathbf{u}_2 = \begin{Bmatrix} b \\ 0 \end{Bmatrix}$, vector specifies position of point D in the rotation coordinate system.

$\mathbf{v}_R = \begin{Bmatrix} v_R \\ 0 \end{Bmatrix}$, vector specifies velocity of point R in the fixed coordinate system.

Replacement and combination of above equations, we get equation systems:

$$\begin{cases} v_R = -\omega_2 b \sin \theta_2 + \omega_3 c \sin \theta_3 \\ 0 = \omega_2 b \cos \theta_2 - \omega_3 c \cos \theta_3 \end{cases}$$

Solving two algebraic equations to get two unknowns v_R and ω_3 be functions of given quantities as $b, c, \omega_2, \theta_2, \theta_3$

$$\omega_3 = (b/c) \cdot (\cos \theta_2 / \cos \theta_3) \cdot \omega_2 \quad (12)$$

$$v_R = -\omega_2 b \sin \theta_2 + \omega_3 c \sin \theta_3 \quad (13)$$

Establish equation to determine acceleration of point R

$O_1 \xrightarrow{\text{Link 1}} B$:

$$\mathbf{a}_B = \mathbf{a}_{O_1} + (\varepsilon_1 \cdot \mathbf{I} - \omega_1^2 \cdot \mathbf{E}) \cdot \mathbf{T}_1 \cdot \mathbf{u}_1$$

$O_2 \xrightarrow{\text{Link 2}} B$:

$$\mathbf{a}_B = \mathbf{a}_{O_2} + (\varepsilon_2 \cdot \mathbf{I} - \omega_2^2 \cdot \mathbf{E}) \cdot \mathbf{T}_2 \cdot \mathbf{u}_2 + 2\omega_2 \cdot \mathbf{I} \cdot \mathbf{T}_2 \cdot \dot{\mathbf{u}}_2 + \mathbf{T}_2 \cdot \ddot{\mathbf{u}}_2$$

Deduce:

$$\begin{aligned} \mathbf{a}_{O_1} + (\varepsilon_1 \cdot \mathbf{I} - \omega_1^2 \cdot \mathbf{E}) \cdot \mathbf{T}_1 \cdot \mathbf{u}_1 \\ = \mathbf{a}_{O_2} \\ + (\varepsilon_2 \cdot \mathbf{I} - \omega_2^2 \cdot \mathbf{E}) \cdot \mathbf{T}_2 \cdot \mathbf{u}_2 \\ + 2\omega_2 \cdot \mathbf{I} \cdot \mathbf{T}_2 \cdot \dot{\mathbf{u}}_2 + \mathbf{T}_2 \cdot \ddot{\mathbf{u}}_2 \end{aligned}$$

where $\mathbf{a}_{O_1} = \mathbf{a}_{O_2} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$; $\varepsilon_1 = \dot{\omega}_1 = 0$;

$$\mathbf{E} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \ddot{\mathbf{u}}_2 = \begin{Bmatrix} \ddot{s} \\ 0 \end{Bmatrix}$$

$\ddot{s} = a_{C/O_2D}$ is acceleration of the slider C on link O_2D

Substituting and combining above equations, we get the equation systems.

Solving two algebraic equations to get two unknowns \ddot{s} and ε_2 be functions of given quantities as $l, s, \dot{s}, \omega_1, \omega_2, \theta_1, \theta_2$

$$a_{C/O_2D} = \ddot{s} = -\omega_1^2 l \cos(\theta_1 - \theta_2) + \omega_2^2 s \quad (14)$$

$$\varepsilon_2 = (\omega_1^2 l \cos \theta_1 - \omega_2^2 s \cos \theta_2 - 2\omega_2 \dot{s} \sin \theta_2 + \ddot{s} \cos \theta_2) / (s \sin \theta_2) \quad (15)$$

$O_2 \xrightarrow{\text{Link 2}} D$:

$$\mathbf{a}_D = \mathbf{a}_{O_2} + (\varepsilon_2 \cdot \mathbf{I} - \omega_2^2 \cdot \mathbf{E}) \cdot \mathbf{T}_2 \cdot \mathbf{u}_2$$

$D \xrightarrow{\text{Link 3}} R$:

$$\mathbf{a}_R = \mathbf{a}_D + (\varepsilon_3 \cdot \mathbf{I} - \omega_3^2 \cdot \mathbf{E}) \cdot \mathbf{T}_3 \cdot \mathbf{u}_3$$

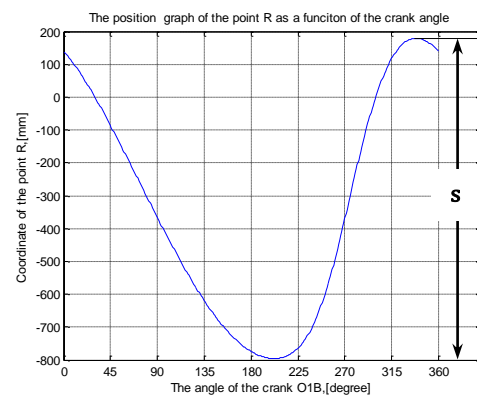


Fig. 6. The position graph of point R as a function of the crank angle

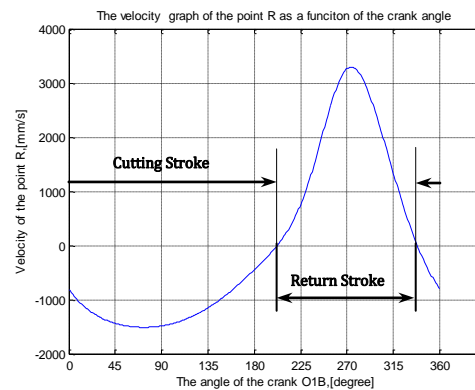


Fig. 7. The velocity graph of point R as a function of the crank angle

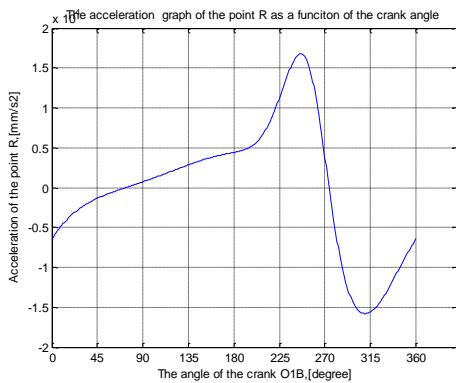


Fig. 8. The acceleration graph of point R as a function of the crank angle

Deduce:

$$\mathbf{a}_R = \mathbf{a}_{O2} + (\varepsilon_2 \cdot \mathbf{I} - \omega_2^2 \cdot \mathbf{E}) \cdot \mathbf{T}_2 \cdot \mathbf{u}_2 + (\varepsilon_3 \cdot \mathbf{I} - \omega_3^2 \cdot \mathbf{E}) \cdot \mathbf{T}_3 \cdot \mathbf{u}_3$$

Here, $\mathbf{u}_2 = \begin{Bmatrix} b \\ 0 \end{Bmatrix}$, vector specifies position of point D in the rotation coordinate system.

$\mathbf{a}_R = \begin{Bmatrix} a_R \\ 0 \end{Bmatrix}$, vector specifies velocity of point R in the fixed coordinate system.

Finally, we get two unknowns ε_3 and a_R be functions of given quantities as $b, c, \varepsilon_2, \omega_2, \omega_3, \theta_2, \theta_3$

$$\varepsilon_3 = (\varepsilon_2 b \cos \theta_2 - \omega_2^2 b \sin \theta_2 + \omega_3^2 c \sin \theta_3) / (c \cos \theta_3) \quad (16)$$

$$a_R = -\omega_2^2 b \cos \theta_2 - \varepsilon_2 b \sin \theta_2 + \omega_3^2 c \cos \theta_3 + \varepsilon_3 c \sin \theta_3 \quad (17)$$

One program of code written in MatLab programming language has been developed based on equations from (6) to (17).

Run the program and show graphs of position, velocity and acceleration of point R in terms of the crank angle. These graph are displayed in Fig. 6, 7, and 8.

From the graphs, we can determine some kinematic parameters as followings:

- The stroke of the cutting tool: $s = 985 \text{ mm}$
- Maximum speed of the cutting tool in the cutting stroke: 1511 mm/s
- Maximum speed of the cutting tool in the return stroke: 3294 mm/s
- Maximum acceleration of the cutting tool in the cutting stroke: 16763 mm/s^2

5. CONCLUSION

The article presents theoretical basis of the transformation matrix and kinematical equations to solve kinematic problems of planar mechanisms. The paper also shows the steps of applying the transformation matrix to analyze kinematics of one specific planar mechanism (crank and slotted lever quick return motion mechanism is selected to illustrate). Results obtained from a program written in Matlab language is also introduced to students. Obviously, in a comparison to graphical method, the application of the transformation matrix is valuable to automatic synthesis of kinematics and kinetics for any mechanisms. It is possible that this paper will support the new calculation for students in Vietnam universities in kinematical analysis of mechanisms.

REFERENCE

- [1] Ahmed A. Shabana – *Dynamics of Multibody Systems* – Second Edition – Cambridge University Press 2000
- [2] David H. Myszka– *Machine and Mechanism Applied Kinematics Analysis* – Four Edition – Prentice Hall Press 2012
- [3] R. C. Hibbeler – *Engineering Mechanics-Dynamics* – 13th Edition – Prentice Hall Press 2014

- [4] Phan Thanh Nhan –*Using transformation matrix to analyze planar mechanism kinematics* – Tạp chí Khoa học Giáo dục Kỹ thuật - Số 36 (6/2016)
- [5] R. S.Khurmi, J. K. Gupta –*Theory of Mechines* – Eurasia Publishing House 2005

Corresponding author:

Phan Thanh Nhan

Ho Chi Minh City University of Technology and Education

Email: nhanpt@hcmute.edu.vn