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BALANCING CONTROL FOR DOUBLE-LINKED INVERTED PENDULUM ON CART: SIMULATION AND EXPERIMENT

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ABSTRACT

Double-linked Inverted Pendulum, which a Single Input-Multi Output (SIMO), is a highly unstable system and a usual one used for testing and applying control theories. Controlling to stabilize double inverted pendulum is a challenging problem which required a suitable and fast reaction controller. The article presents a solution for stabilizing a double inverted pendulum using a Fuzzy logic controller and a Linear-Quadratic Regulator (LQR) controller. The entire system has been modeled and tested by Matlab/Simulink toolbox. Further, both controllers have been applied to an experimental model in laboratory. Experimental results of stabilizing double inverted pendulum show that with both controllers, the system can be maintained in upright position. However, the LQR controller's quality in reality is better than the Fuzzy logic controller's one.

***Key words**: double-linked Inverted Pendulum; Optimal control (LQR); fuzzy control; inverted pendulum on Cart; SIMO system; under-actuated system; balance control.*

TÓM TẮT

Con lắc ngược hai bậc là hệ thống một vào – nhiều ra, nó có độ bất ổn định cao và thường được dùng nhiều trong việc kiểm tra và vận dụng các giải thuật điều khiển. Vì thế, điều khiển cân bằng hệ con lắc ngược hai bậc tự do là vấn đề khó, đòi hỏi có bộ điều khiển thích hợp và có tốc độ đáp ứng nhanh. Bài báo trình bày phương pháp điều khiển dùng bộ điều khiển mờ và bộ điều khiển tối ưu LQR để cân bằng hệ con lắc ngược hai bậc tự do. Hệ thống và bộ điều khiển được mô phỏng bằng công cụ Matlab/Simulink. Ngoài ra, hai bộ điều khiển trên cũng được áp dụng vào điều khiển mô hình thật nhằm đánh giá chất lượng của bộ điều khiển. Kết quả điều khiển thực tế trên mô hình cho thấy với cả hai bộ điều khiển, con lắc ngược hai bậc thực tế có khả năng duy trì ở vị trí cân bằng hướng lên. Tuy nhiên, độ ổn định của hệ thống do bộ điều khiển LQR tốt hơn của bộ điều khiển mờ.

***Từ khóa**: con lắc ngược kép; điều khiển tối ưu (LQR); điều khiển mờ; con lắc ngược trên xe; hệ một vào nhiều ra; điều khiển cân bằng.*

1. INTRODUCTION

Double-linked inverted pendulum is a SIMO complex, unstable and nonlinear system. It is often used in testing and applying control algorithms. At the upright position, if there is no control signal, only under a weak external force, double-linked pendulum will fall down immediately by the impact of gravity. The controlling of this system consists of keeping the double-linked pendulum upright and moving the cart to the setpoint.

Many algorithms have been successfully applied to the double-linked inverted pendulum, for instance, the controlling of an inverted pendulum using PID [1-3], controllers which rely on fuzzy controller [4], optimal control of double-linked inverted pendulum using LQR controller [5], [6]. However, these algorithms are only in simulation. The application of LQR in experiment requires the system parameters. Double-linked inverted pendulum has been controlled successfully in the world. But, in Viet Nam, there is no such great achievements.

In this paper, after building a mathematical model of system, the authors simulate system using fuzzy controller and LQR. Then, these algorithms are examined by experiments.

The paper conclude of five Sections. Section 1 introduces the paper. Then, Section 2 describes the dynamic equations of double-linked Inverted Pendulum on Cart. Section 3 shows the simulation results in Fuzzy controller and LQR controller. Section 4 listed experimental results under these controllers. Then, conclusion in Section 5 ends the paper.

2. BALANCED THE DOUBLE-LINKED INVERTED PENDULUM

2.1 The model’s kinetic equation

The double-linked inverted pendulum that the authors studied in the paper is the classical pendulum. Homogeneous pendulum rods which are interconnected in a joint and one of them is attached to a cart which allows for movement alongside a single axis. Both pendulums are hold in a vertical position upwards by the force F acting on the cart. The model shown in Fig. 1.

Table 1. *Symbol*

Symbol	Unit	Description
m_0, m_1, m_2	Kg	Mass of cart, mass of the first pendulum, mass of the second pendulum
l_1, l_2	m	Length of the first and second pendulum
L_1, L_2	m	Distance between center and rotating axis of first and second pendulum
J_1, J_2	Kg m^2	Inertial moment of the first and second pendulum
g	m/s^2	Gravitation acceleration
q_0	m	Cart position
q_1, q_2	rad	The angle of the first and second pendulum
F	N	Force controlling cart
$\dot{q}_0$	m/s	Cart velocity
$\dot{q}_1, \dot{q}_2$	rad/s	Angular velocity of the first and second pendulum
$\ddot{q}_0$	m/s^2	Cart acceleration
$\ddot{q}_1, \ddot{q}_2$	rad/s^2	Acceleration of the first and second pendulum
A, B, C		Matrixes of equation system
K_t	Nm/A	Moment coefficient of motor
K_b	V/(rad/s)	Generating coefficient of motor
R_m	ohm	Ohmic ferrule resistor of motor
C_m	Nm/(rad/s)	Sliding friction coefficient of motor
J_m	Kgm^2	Moment of inertia of motor
t_m	Nm	Inner torque
t_1	Nm	Torque
R	m	Radius of wheel
w	Rad/s	Angular velocity
b_0, b_1, b_2		The friction coefficient of the cart against the rail, the damping constants in the joints of the pendulum

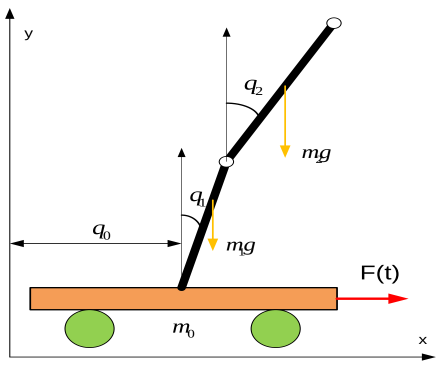

Fig. 1. *Double-linked inverted pendulum on cart*

Set of equations describing the nonlinear characteristic of the system is described by the system of equations (Mandar R. Nalavade, Mangesh J. Bhagat, Vinay V. Patil, 2014) [6] as:

$$\begin{cases} u = z_1\ddot{q}_0 + z_2\cos q_1\ddot{q}_1 + z_3\cos q_2\ddot{q}_2 \\ -z_2\sin q_1\dot{q}_1^2 - z_3\sin q_2\dot{q}_2^2 + b_0\dot{q}_0 \\ 0 = z_2\cos q_1\ddot{q}_0 + z_4\ddot{q}_1 + z_5\cos(q_1-q_2)\ddot{q}_2 \\ +z_5\sin(q_1-q_2)\dot{q}_2^2 - z_2 g\sin q_1 + b_1\dot{q}_1 \\ 0 = z_3\cos q_2\ddot{q}_0 + z_5\cos(q_1-q_2)\ddot{q}_1 + z_6\ddot{q}_2 \\ -z_5\sin(q_1-q_2)\dot{q}_1^2 - z_3 g\sin q_2 + b_2\dot{q}_2 \end{cases} \quad (1)$$

With:

$z_1 = m_0 + m_1 + m_2$; $z_2 = m_1a_1 + m_2A_1$; $z_3 = m_2a_2$;

$z_4 = m_1a_1^2 + m_2A_1^2 + J_1$; $z_5 = m_2A_1a_2$; $z_6 = m_2a_2^2 + J_2$.

Denote that u is the control signal, F is the external force.

The linearized mathematical equations around equilibrium working point:

$q_1 = 0, q_2 = 0$, then we can approximate it: $\sin q_1 = q_1, \sin q_2 = q_2, \cos q_1 = 1, \cos q_2 = 1,$
$\sin(q_1 - q_2) = q_1 - q_2, \cos(q_1 - q_2) = 1.$

Equation system becoming:

$$\begin{cases} F = z_1\ddot{q}_0 + z_2\ddot{q}_1 + z_3\ddot{q}_2 - z_2q_1\dot{q}_1 \\ -z_3q_2\dot{q}_2^2 + b_0\dot{q}_0 \\ 0 = z_2\ddot{q}_0 + z_4\ddot{q}_1 + z_5\ddot{q}_2 + z_5(q_1-q_2)\dot{q}_2^2 \\ -z_2gq_1 + b_1\dot{q}_1 \\ 0 = z_3\ddot{q}_0 + z_5\ddot{q}_1 + z_6\ddot{q}_2 - z_5(q_1-q_2)\dot{q}_1^2 \\ -z_3gq_2 + b_2\dot{q}_2 \end{cases} \quad (2)$$

Set as matrix:

$$M(x)\ddot{x} + V(x,\dot{x}) + G(x) = \begin{bmatrix} F & 0 & 0 \end{bmatrix}^T \quad (3)$$

With $M(x) = \begin{bmatrix} z_1 & z_2 & z_3 \\ z_2 & z_4 & z_5 \\ z_3 & z_5 & z_6 \end{bmatrix}$ (4)

$$V(x,\dot{x}) = \begin{bmatrix} b_0\dot{q}_0 & -z_2q_1\dot{q}_1^2 & -z_3q_2\dot{q}_2^2 \\ 0 & b_1\dot{q}_1 & z_5(q_1-q_2)\dot{q}_2^2 \\ 0 & -z_5(q_1-q_2)\dot{q}_1^2 & b_2\dot{q}_2^2 \end{bmatrix} \quad (5)$$

$$G(x) = \begin{bmatrix} 0 & -z_2gq_1 & -z_3gq_2 \end{bmatrix}^T \quad (6)$$

However, for the convenience of adjusting the motor as well as applying the controller to the real model, which has the input conversion as the voltage providing for the motor, structure of motor is described in Fig. 2 below.

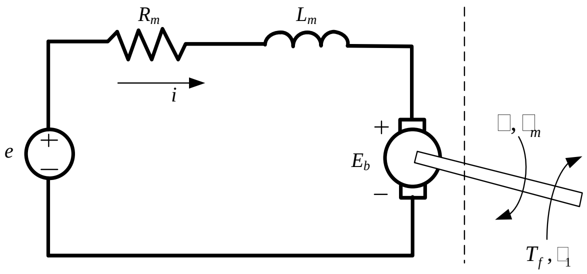

Fig. 2. *The modelling motor*

We have:

$$J_m\frac{d\omega}{dt} = \tau_m - C_m\omega - \tau_1 \quad (7)$$

With: $\tau_m = \dfrac{K_t}{R_m}(e - K_b\omega) = \dfrac{K_t}{R_m}e - \dfrac{K_bK_t}{R_m}\omega$ (8)

We have: $q_0 = r\theta_m \Rightarrow \dot{q}_0 = r\omega$

Substitute into (8): $\tau_m = \dfrac{K_t}{R_m}e - \dfrac{K_bK_t}{R_m r}\dot{q}_0$ (9)

Substitute (9) into (7):

$$\tau_1 = -\frac{J_m}{r}\ddot{q}_0 - \left(\frac{C_m}{r} + \frac{K_bK_t}{R_m r}\right)\dot{q}_0 + \frac{K_t}{R_m}e \quad (10)$$

The force on the cart:

$$F = \frac{\tau_1}{r} = -\frac{J_m}{r^2}\ddot{q}_0 - \left(\frac{C_m}{r^2} + \frac{K_bK_t}{R_m r^2}\right)\dot{q}_0 + \frac{K_t}{R_m r}e \quad (11)$$

$$\Leftrightarrow F = -k_3\ddot{q}_0 - k_2\dot{q}_0 + k_1 e \quad (12)$$

With: $k_1 = \dfrac{K_t}{R_m r}, k_2 = \dfrac{C_m}{r^2} + \dfrac{K_bK_t}{R_m r^2}, k_3 = \dfrac{J_m}{r^2}$

Combining (2), (3) and (12) we have the model's kinetic equations of the system:

$$M_f(x)\ddot{x} + V_f(x,\dot{x}) + G_f(x) = \begin{bmatrix} k_1e & 0 & 0 \end{bmatrix}^T \quad (13)$$

With $M_f(x) = \begin{bmatrix} z_1 + k_3 & z_2 & z_3 \\ z_2 & z_4 & z_5 \\ z_3 & z_5 & z_6 \end{bmatrix}$ (14)

$$V_f(x,\dot{x}) = \begin{bmatrix} b_0\dot{q}_0 + k_2 & -z_2 q_1 \dot{q}_1^2 & -z_3 q_2 \dot{q}_2^2 \\ 0 & b_1\dot{q}_1 & z_5(q_1 - q_2)\dot{q}_2^2 \\ 0 & -z_5(q_1 - q_2)\dot{q}_1^2 & b_2\dot{q}_2 \end{bmatrix} \quad (15)$$

$$G_f(x) = \begin{bmatrix} 0 & -z_2 g q_1 & -z_3 g q_2 \end{bmatrix}^T \quad (16)$$

3. SIMULATION RESULTS

3.1 Fuzzy logic controller

The system parameters are shown in table:

***Table 2.** The system parameters*

Parameter	Value	Unit
m_0	0.35	Kg
m_1	0.133	Kg
m_2	0.025	Kg
L_1	0.2	m
L_2	0.23	m
l_1	0.115	m
l_2	0.17	m
J_1	0.0017	Kg m^2
J_2	0.059	Kg m^2
g	9.81	m/s^2
K_t	5.3e-3	Nm/A
K_b	5.3e-3	V/(rad/s)
R_m	2.7	ohm
C_m	5e-4	Nm/(rad/s)
J_m	0.049e-4	Kgm^2
b_0	0.05	
b_1	0.001	
b_2	0.001	

Double-linked inverted pendulum is a SIMO system. Input is the voltage supplying to the motor. Output consists of the cart position, the first pendulum angle and the second pendulum angle. Authors suggest using Fuzzy controller to stabilize the system. Controller diagram is shown in Fig. 3.

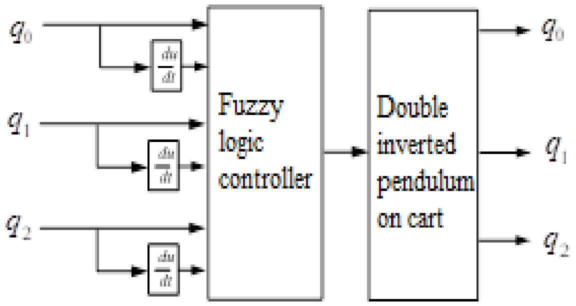

***Fig. 3.** Diagram of Fuzzy logic controller*

The selected fuzzy controller includes 6 inputs: $(q_0, q_0\_dot, q_1, q_1\_dot, q_2, q_2\_dot)$. Therefore, the number of rules using in fuzzy controller is numerous. If each input has n membership functions, the rule number will be n^6. In this paper, the authors simplify it by selecting each input which has 2 membership functions. The membership functions of standardized input as follows:

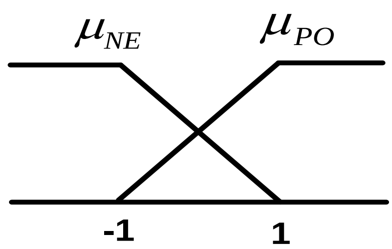

***Fig. 4.** The membership functions of standardized input*

The number of rule using in fuzzy logic controller is 64, and Sugeno-type fuzzy inference is selected. Some of used rules are listed below:

***Table 3.** The number of rule*

q_0	$\dot{q}_0$	q_1	$\dot{q}_1$	q_2	$\dot{q}_2$	U
NE	NE	NE	NE	NE	NE	0.5
NE	NE	NE	NE	NE	PO	-0.3
…	…	…	…	…	…	…
PO	PO	PO	PO	PO	NE	0.3
PO	PO	PO	PO	PO	PO	-0.5

The initial values of $\begin{bmatrix} q_0 & \dot{q}_0 & q_1 & \dot{q}_1 & q_2 & \dot{q}_2 \end{bmatrix}$ are: $\begin{bmatrix} 0.1 & -0.1 & -0.1 & 0.1 & 0.1 & -0.1 \end{bmatrix}$.

The simulation of system is shown in Fig. 5 and 6 below:

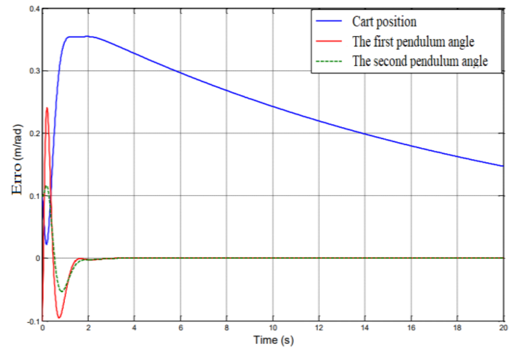

Fig. 5. *The response of output system with Fuzzy logic control*

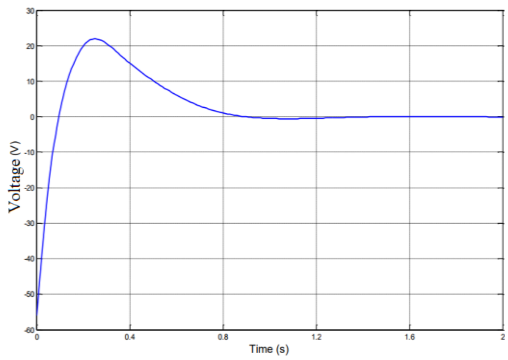

Fig. 6. *The input voltage for motor with Fuzzy logic control*

3.2 LQR Controller

The continuous-time state-space description of the linearized inverted pendulum system has the form:

$\dot{X} = AX + BU$ and $Y = CX + DU$

With $X = \begin{bmatrix} q_0 & \dot{q}_0 & q_1 & \dot{q}_1 & q_2 & \dot{q}_2 \end{bmatrix}^T$

A cost functional defined as:

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt \qquad (17)$$

Q and R is positive semi-definite matrix.

The feedback control law that minimizes the value of the cost is:

$$u(t) = -R^{-1}B^T P(x)x(t) = -Kx(t) \qquad (18)$$

And P is found by solving the continuous time Riccati differential equation:

$$PA + A^T P - PBR^{-1}P + Q = 0 \qquad (19)$$

Thence, we infer the value of K.

From the model, we can calculate matrix A, B, C, D by Matlab:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -7.1264 & -2.3342 & 0.0117 & 0.0025 & -0.0001 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 37.4813 & 67.6738 & -0.3399 & -5.8622 & 0.1406 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.9323 & -28.2368 & 0.1418 & 27.6806 & -0.6639 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0.3505 & 0 & -1.8434 & 0 & 0.0459 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \; D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Chosen: $Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, and $R = 1$

By calculating $\mathrm{K} = \mathrm{dlqr(A, B, Q, R)}$ from Matlab, we have:

$$K = \begin{bmatrix} 1 & 0.8897 & -272.3202 & -11.5566 & 423.9344 & 76.0792 \end{bmatrix}$$

The initial values of $\begin{bmatrix} q_0 & \dot{q}_0 & q_1 & \dot{q}_1 & q_2 & \dot{q}_2 \end{bmatrix}$ are: $\begin{bmatrix} 0.1 & -0.1 & -0.1 & 0.1 & 0.1 & -0.1 \end{bmatrix}$.

The simulation of system is shown in Fig. 7 and 8 below:

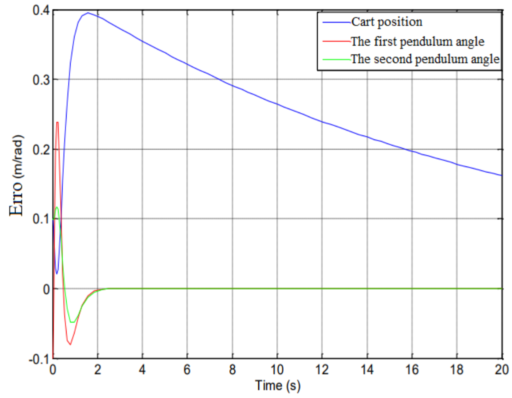

Fig. 7. The response of output system with LQR control

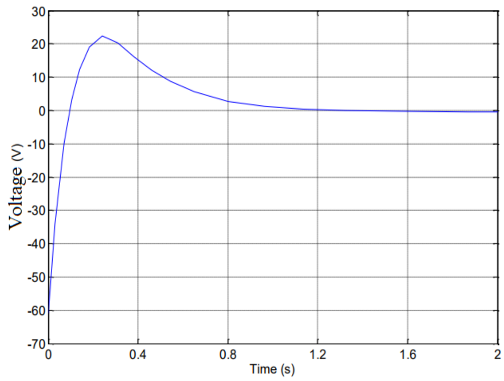

Fig. 8. The input voltage for motor with LQR control

From the simulation results, double-linked pendulum has stabilized at the upward position after 2 seconds with both the fuzzy controller and the LQR controller. However, cart position takes time to get back to the setpoint.

4. EXPERIMENTAL RESULTS

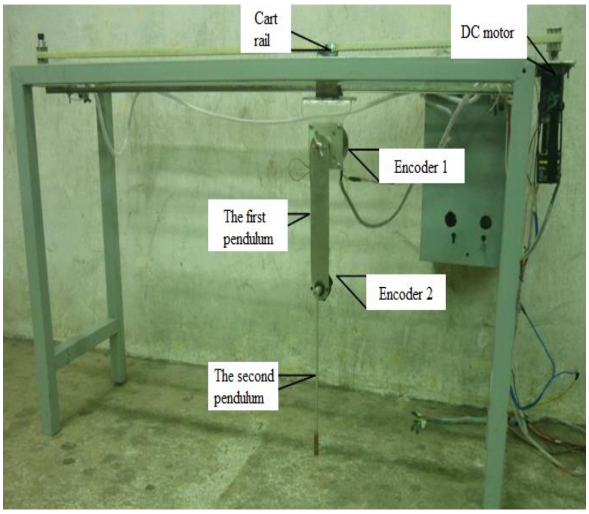

Fig. 9. Double-linked inverted pendulum on cart model in practice

The real model is shown in Fig. 9. The cart is free to move within the bounds of a one-dimensional track. The first pendulum joint with cart, the other ends of the first pendulum joint with the second pendulum. Both of them can move in the vertical plane parallel to the track.

4.1 Fuzzy logic controller

With the built fuzzy controller, authors take experiment upon model. The response is shown as in Fig. 10, 11, 12, 13. The double-linked pendulum has been remained in the upward-condition. However, the fluctuation still has existed, and balancing-time still has not been optimized.

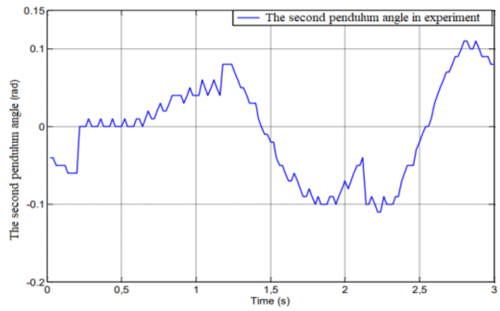

Fig. 10. The second pendulum angle with Fuzzy logic control

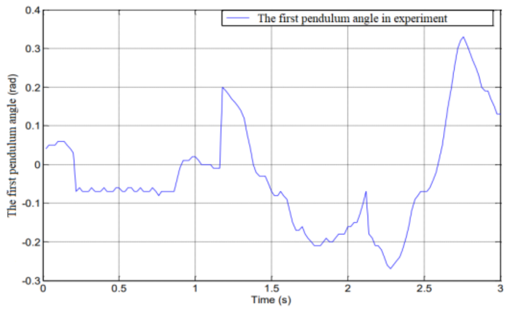

Fig. 11. The first pendulum angle with Fuzzy logic control

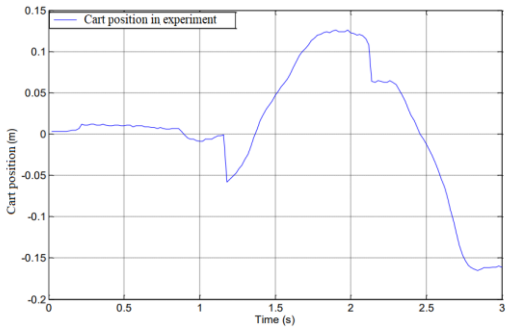

Fig. 12. *The cart position with Fuzzy logic control*

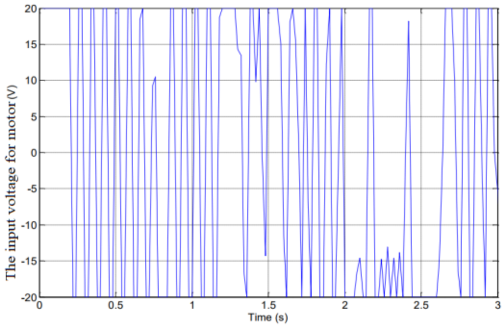

Fig. 13. *The input voltage for motor with Fuzzy logic control*

4.2 Using LQR controller

Conducting to discrete A, B with sample-time is 0.01s, we have A_d; B_d. Also with the Q and R matrix in the simulation, we use the Matlab program called 'dlqr' to find the K matrix as:

$$K_d = [0.8820 \quad -1.6091 \quad -253.8944 \quad -11.4004 \quad 384.0136 \quad 68.8291]$$

With the library supporting embedded programming for STM32F4 microcontroller in the Matlab-Simulink environment, the authors experiment upon the model shown in Fig. 9. The output response as follow:

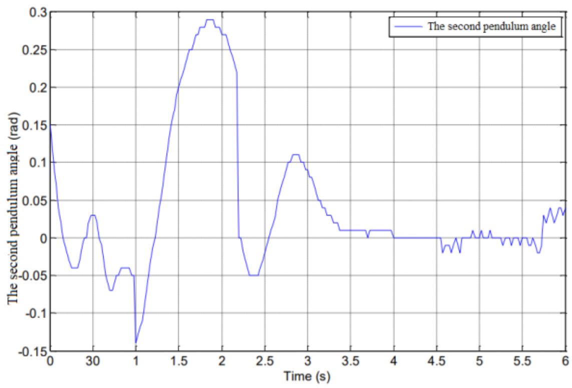

Fig. 14. *The second pendulum angle with LQR control*

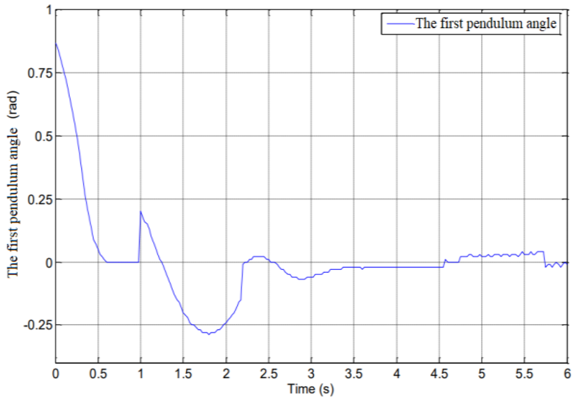

Fig. 15. *The first pendulum angle with LQR control*

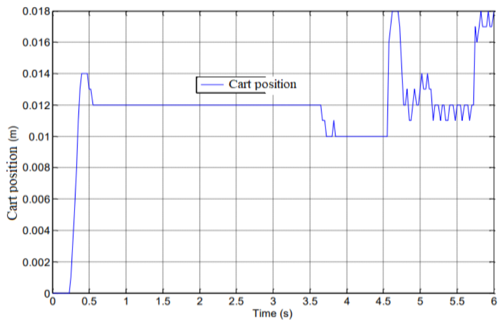

Fig. 16. *The cart position with LQR control*

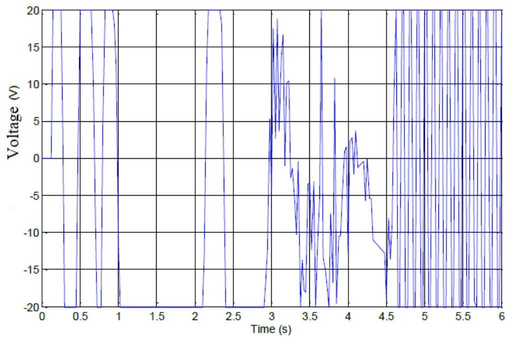

Fig. 17. *The input voltage for motor with LQR control*

From experimental results, double-linked inverted pendulum can be kept at upright position. LQR gives more effective results than Fuzzy controller. However, the pendulum angles have not been optimal.

5. CONCLUSION

This paper presents the construction of a double-linked inverted pendulum balance controller using Fuzzy and LQR controller. Both controllers show good simulation results. By experiment, double-linked pendulum is balanced in limited time. The

LQR controller gives better control results than the Fuzzy logic controller. Despite being well-controlled, the results have not been optimized, in a consequence, the controller optimization need to be executed.

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