HIERARCHICAL FUZZY SLIDING MODE CONTROL FOR A CLASS OF SIMO UNDER-ACTUATED SYSTEMS

ĐIỀU KHIỂN TRƯỢT PHÂN CẤP CHO MỘT LỚP CÁC HỆ SIMO UNDER-ACTUATED

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ABSTRACT

In the paper, a hierarchical fuzzy sliding-mode controller (HFSMC) designing method is proposed. This method provides a simple way to achieve asymptotical stability and remove the chattering signal for a class of SIMO under-actuated systems with fuzzy control rules. This class of under-actuated systems is made from several subsystems. Based on this physical structure, the hierarchical structure of the sliding surfaces is designed as follows: at first, the sliding surface of every-subsystem is defined; then, the sliding surface of one sub-system is defined as the first layer sliding surface; the first layer sliding surface is used to construct the second layer sliding surface with the sliding surface of another subsystem. This process continues until the sliding surfaces of the entire subsystems achieve a suitable fuzzy controller which is built due to the final sliding surface to suppress chattering phenomenon of the system. Simulation results show the feasibility of this control method through two typical SIMO under-actuated systems.

Keyword: *Hierarchical fuzzy sliding mode control; under-actuated systems; chattering phenomenon; fuzzy controller; SIMO system.*

TÓM TẮT

Trong bài báo này, một phương pháp điều khiển mờ trượt phân cấp (HFSMC) được đề xuất. Phương pháp này cung cấp một cách đơn giản để đạt được sự ổn định tiệm cận và loại bỏ tín hiệu chattering đối với một loạt các hệ thống phi tuyến SIMO với quy tắc điều khiển mờ. Trong đó, một loạt các hệ thống under-actuated được tạo thành từ một số hệ thống con. Dựa trên cấu trúc vật lý này, cấu trúc phân cấp của các bề mặt trượt được thiết kế như sau: lúc đầu, bề mặt trượt của mỗi hệ thống con được xác định; sau đó, bề mặt trượt hệ thống con thứ nhất được định nghĩa là lớp trượt đầu tiên; lớp trượt đầu tiên được sử dụng để xây dựng lớp trượt thứ hai với bề mặt trượt của một hệ thống con khác. Quá trình này tiếp tục cho đến khi các bề mặt trượt của toàn bộ các hệ thống con được bao gồm một luật điều khiển mờ thích hợp được xây dựng dựa trên lớp trượt cuối cùng để triệt tiêu hiện tượng chattering trong hệ thống. Các kết quả mô phỏng cho thấy tính khả thi của phương pháp điều khiển này thông qua hai hệ thống SIMO under-actuated điển hình.

Từ khóa: *Điều khiển mờ trượt phân cấp; hệ thống under-actuated; hiện tượng chattering; điều khiển mờ; hệ thống SIMO.*

1. INTRODUCTION

The under-actuated systems have fewer control inputs than degrees of freedom and arise in applications, such as robot, industrial machine... The equation of under-actuated systems consists of high nonlinear component and coupler so they are difficult to be controlled [1]. In recent years, there has been growing interest in controlling under-actuated system in theory and practice.

In recent year, sliding mode control (SMC) method has been used widely to design controllers for under-actuated system. SMC is an effective approach to maintain stability of system [2 – 3]. The main advantage of SMC is that the chaos of the under-actuated system was treated by invariant characteristics sliding condition of the system. However, the basic problem still exists in controlling complex systems using SMC controllers, such as: chattering phenomenon. There has been many researches on designing of fuzzy sliding mode control (FSMC) base on sliding mode controller and fuzzy logic controller [4 - 5], [9–10]. The FSMC controller is combination of fuzzy Logic control (FLC) and SMC provide sample method to design controller system. This method still keeps SMC property but chattering phenomenon is reduced. The main advantage of FSMC is reduction of chattering phenomenon. However, in the fuzzy controller which was mentioned, the parameters are not calculated to specific. The controllers in [4–5], [9–10] have not been applied to SIMO under-actuated system with n-subsystems and have not been proved clearly to remove chattering signal.

In order to overcome that disadvantages. This paper proposed "Hierarchical Fuzzy Sliding Mode Control" (HFSMC) which can be applied for SIMO under-actuated systems. This controller is applied to n-subsystems, the parameters are limited to specific and ability to remove chattering signal was proved by clear theory. Firstly, paper introduces hierarchical sliding mode control (HSMC) [6]. Secondly, HSMC is designed to control SIMO under-actuated system. The simulation results show that the proposed controller operates well. Therefore, the HFSMC has better performance than HSMC.

2. HIERARCHICAL SLIDING MODE CONTROL

Consider state space equation of the SIMO under-actuated system with n subsystems as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + b_1 u \\ \dot{x}_3 = x_4 \\ x_4 = f_2 + b_2 u \\ \vdots \\ \dot{x}_{2n-1} = x_{2n} \\ \dot{x}_{2n} = f_n + b_n u \end{cases} \tag{1}$$

Where $X = [x_1, x_2, ..., x_{2n}]^T$ is variable state space; f_i and b_i (i=1, 2, …, n) are nonlinear functions of X; u is control input signal.

Equation (1) show the layers of subsystems. If n = 2, (1) represents pendupot model, single inverted pendulum: If n = 3, it represents double inverted pendulum; If n = 4, It represents triple inverted pendulum. And so on, depending on physical structure, the under-actuated system can be separated into many subsystems. For instance, triple inverted pendulum can be separated into four subsystems: upper pendulum, middle pendulum, lower pendulum and cart. Thus, this system in (1) consists of n-subsystems. The i[th] subsystem includes state variable x_{2i-1} and x_{2i}. Its state space equation is shown as:

$$\begin{cases} \dot{x}_{2i-1} = x_{2i} \\ \dot{x}_{2i} = f_i + b_i u \end{cases} \tag{2}$$

According to [6], the designing process of HSMC is shown as:

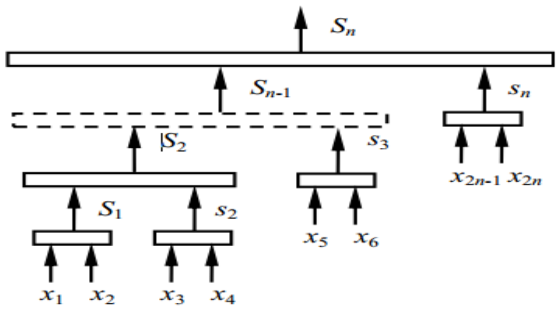

Fig 1. *Hierarchical structure of sliding surface*

The i[th] sliding surface function of system is defined as:

$$s_i = c_i x_{2i-1} + x_{2i} \tag{3}$$

Where c_i is positive constant and limitation of c_i in [6] is $0 < c_i < c_{i0}$

Where,

$$c_{i0} = \left| \lim_{X\to 0} (f_i / x_{2i}) \right| \quad (4)$$

Derivative s_i with respect time of formula (3), then

$$\dot{s}_i = c_i \dot{x}_{2i-1} + \dot{x}_{2i} = c_i x_{2i} + f_i + b_i u \quad (5)$$

When $\dot{s}_i = 0$ of formula (5) the equivalent control input of i[th] subsystem is

$$u_{eqi} = -(c_i x_{2i} + f_i) / b_i \quad (6)$$

According to Fig.1, the i[th] sliding surface is inferred as:

$$S_i = \lambda_{i-1} S_{i-1} + s_i \quad (7)$$

Where $\lambda_{i-1}(\mathrm{i} = 1, 2, \ldots \mathrm{n})$ are constant and $\lambda_0 = S_0 = 0$

With i=n and from [6], HSMC law are inferred as:

$$u_n = \frac{\sum_{r=1}^{n} (\prod_{j=r}^{n} a_j) b_r u_{eqr}}{\sum_{r=1}^{n} (\prod_{j=r}^{n} a_j) b_r} - \frac{k_n S_n + \eta_n \operatorname{sgn} S_n}{\sum_{r=1}^{n} (\prod_{j=r}^{n} a_j) \mathrm{b}_r} \quad (8)$$

From (7) and (8), the structure of HSMC is shown in Fig. 2.

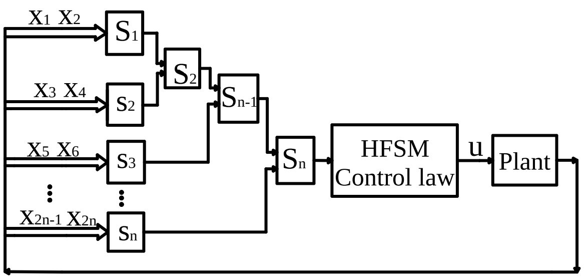

Fig 2. *Structure of HSMC*

3. HIERARCHICAL FUZZY SLIDING MODE CONTROL DESIGN

HFSMC is designed to under-actuated system based on ideas below. In control law of under-actuated is showed in (8) with sign(S_n) function. This is the reason why exists chattering phenomenon. One method of removing the chattering signal is to replace the fixed parameter η_n in equation (8) by a variable value through the fuzzy controller. The value η_n will vary based on the size of sliding surface S_n. When the smaller S_n, the closer variable state is to 0, then η_n decrease to 0 to make sign(S_n) function no longer affect to u_n control signal. We have:

$$\lim_{\eta_n \to 0} \eta_n \operatorname{sgn} S_n = 0 \quad (9)$$

However, if η_n is small at beginning, the control signal u_n will reach to equilibrium position slowly. If η_n is high at beginning, the variable state space will reach to stability point fast but at the stability point, the system oscillates strongly. So, the value of η_n should initially be large to make u_n lead the system to equilibrium position. When the system at equilibrium position, η_n is as small as possible. In order to change value of η_n based on the value of sliding surface S_n. We will calculate η_n through a fuzzy controller. The fuzzy controller inputs are sliding surface S_n value. Structure of HFSMC is shown in Fig. 3:

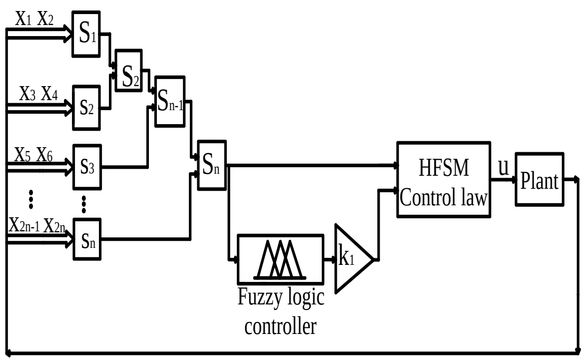

Fig 3. *Structure of HFSMC for under-actuated system*

Fuzzy rule in “Fuzzy logic controller” block is show in Table 1 as:

Table 1. *The rule of Fuzzy block*

STT	S_n	η_n
1	A	A
2	B	B
3	C	C
4	D	D
5	E	E
6	F	F
7	G	G

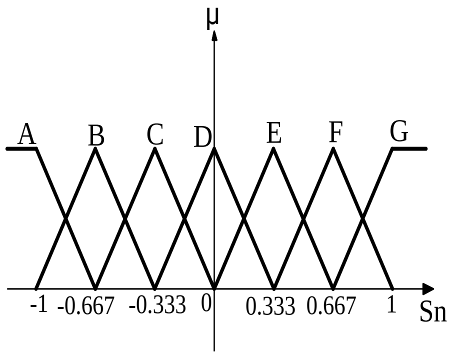

Fig 4. *Membership functions of input fuzzy blocks*

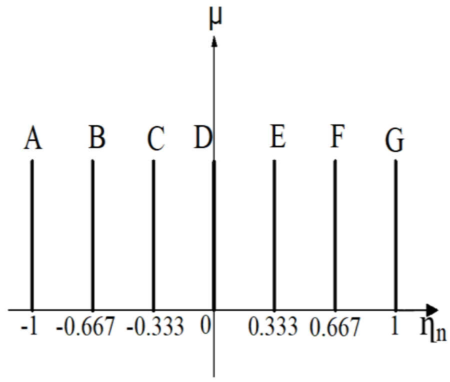

Fig 5. *Membership functions of output fuzzy block*

The membership functions in Fig. 4 and Fig. 5 are standard function. In order to adjust fuzzy controller parameters, it is necessary to choose k1 parameter in after processing block is showen in figure 3. The parameter k1 determines ability to remove chattering signal in the system.

The control parameter k1 need to be chosen by searching algorithm such as: genetic algorithm or try-error-test.

4. STABILITY ANALYSIS AND ABILITY TO REMOVE CHATTERING PHENOMENON OF HFSMC

Two theorems will be proved in this section. The theorem 1 analyzes the asymptotic stability of all sliding layers. The theorem 2 analyzes ability to remove chattering signal of HFSMC controller.

Theorem 1: Consider layers of under-actuated (1) system. If control law is chosen as (8) and i[th] sliding surfaces are identified as (7) then S_i is asymptotic stabilized.

Proof: The Lyapunov function of i[th] layer is chosen as:

$$V_i(t) = S_i^2 / 2 \tag{10}$$

By consider stability of i[th] layer, as [6] we have:

$$\dot{S}_i = -k_i S_i - \eta_i \operatorname{sgn} S_i \tag{11}$$

Derivative $V_i(t)$ with respect time of formula (10), from (11) we have:

$$\dot{V}_i = S_i.\dot{S}_i = S_i(-k_i S_i - \eta_i \operatorname{sgn} S_i)$$

$$= -k_i S_i^2 - \eta_i |S_i| \tag{12}$$

Derivative two sides of formula (12), we obtain

$$\int_0^t \dot{V}_i d\tau = \int_0^t (-k_i S_i^2 - \eta_i |S_i|) d\tau \tag{13}$$

With:

$$V_i(0) = V_i(t) + \int_0^t (k_i S_i^2 + \eta_i |S_i|) d\tau$$

$$\geq \int_0^t (k_i S_i^2 + \eta_i |S_i|) d\tau \tag{14}$$

Therefore, yieds

$$\lim_{t\to\infty} \int_0^t (k_i S_i^2 + \eta_i |S_i|) d\tau \leq V_i(0) < \infty \tag{15}$$

According to Barbalat lemma, there exits

$$\lim_{t\to\infty} (k_i S_i^2 + \eta_i |S_i|) d\tau \leq V_i(0) < \infty \tag{16}$$

From (16), it is meaning $\lim_{t\to\infty} S_i = 0$ then the i[th] sliding surface of S_i is asymptotical stability.

***Theorem 2*: ** Consider an under-actuated system (1), if control law is chosen as (8) and replaced η_n parameter fixed of formula (8) by a variable based on the magnitude of the sliding surface S_n . Through fuzzy controller, the chattering signal in system will be removed.

Proof: From the formula (8) the main factor produce chattering phenomenon in system is $\eta_n \operatorname{sgn} S_n$ function. In order to overcome this phenomenon, the fuzzy block was added in controller to remove sign function:

Sliding surface S_n is fuzzificated as Fig. 4.

The fuzzy rules are shown on Table 1:

R^1 : If S_n is A Then η_n^1 = A

R^2 : If S_n is B Then η_n^2 = B

R^3 : If S_n is C Then η_n^3 = C

R^4 : If S_n is D Then η_n^4 = D

R^5 : If S_n is E Then η_n^5 = E

R^6 : If S_n is F Then η_n^6 = F

R^7 : If S_n is G Then η_n^7 = G

By using center of gravity defuzzificating method, parameter η_n is determined:

$$\eta_n = \frac{\sum_{i=1}^{7} \beta_i \eta_n^i}{\sum_{i=1}^{7} \beta_i} \quad (17)$$

Where β_i is the probability fucntion of the rule-i :

$$\begin{aligned} &\beta_1 = \mu_A(\mathrm{S}_n) && \beta_2 = \mu_B(\mathrm{S}_n) \\ &\beta_3 = \mu_C(\mathrm{S}_n) && \beta_4 = \mu_D(\mathrm{S}_n) \\ &\beta_5 = \mu_E(\mathrm{S}_n) && \beta_6 = \mu_F(\mathrm{S}_n) \\ &\beta_7 = \mu_G(\mathrm{S}_n) \end{aligned} \quad (18)$$

Arccoding to (18), from (17), we obtain:

$$\lim_{S_n\to 0} \eta_n = \lim_{S_n\to 0} \frac{\sum_{i=1}^{7} \beta_i \eta_n^i}{\sum_{i=1}^{7} \beta_i} = 0 \quad (19)$$

From (19) we deduce:

$$\lim_{S_n\to 0} \eta_n \operatorname{sgn} S_n = 0 \quad (20)$$

Arccoding to Theorem 1 we have:

$$\lim_{t\to\infty} S_n = 0 \quad (21)$$

From (20) and (21), we deduce:

$$\lim_{t\to\infty} \eta_n \operatorname{sgn} S_n = 0 \quad (22)$$

Arccoding to (22), when t is asymptotic to ∞ then $\eta_n \operatorname{sgn} S_n$ function will be removed completely in control law (8). So, at stabilized position, chattering signal was removed in HFSMC.

5. SIMULATION RESULTS

Pendubot and double inverted pendulum are typical two under-actuated systems, they are usually used to verify feasibility for new control method. Their mathematical equation is shown in (1). In this section, the control method presented will be applied to enhance control of pendubot and double inverted pendulum system. The simulation results indicate that this control method is feasible.

5.1 Pendubot

The pendubot which is shown on Fig. 6 consists of two subsystems: link 1 (signed number 1) with a transmission and link 2 (signed number 2) with no transmission. The control objective is to control link 1, link 2 to balanced and stable at deserted positon.

The parameters and variables on Fig. 2 are determined as: θ_1 is angle between link 1 and horizontal axis, θ_2 is angle between link 2 and link 1. m_i, l_i and l_{ci} mass, length and distance to center of link I (where i=1, 2); τ_1 is control moment. When n=2 then the formula (1) is space state equation of pendubot system which is shown below:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + b_1 u \\ x_3 = x_4 \\ \dot{x}_4 = f_2 + b_2 u \end{cases} \quad (23)$$

Where $x_1 = \theta_1 - \pi/2$ is angle between link 1 and vertical axis, $x_3 = \theta_2$ is angle link 2 and link 1; x_4 is angle velocity of link 2, $u = \tau_1$ is input control signal. The expression f_1, f_2, b_1 and b_2 are indicated in [7]. In order to compare HSMC controller and HFSMC controller, the parameters of pendubot system is chosen according to [7] and [8]:

q_1 = 0.0308 kg.m^2, q_2 = 0.0106 kg.m^2, q_3 = 0.0308 kg.m^2, q_4 = 0.2086 kg.m^2, q_5 = 0.0630 kg.m^2, g = 9.81 m.s^{-2}.

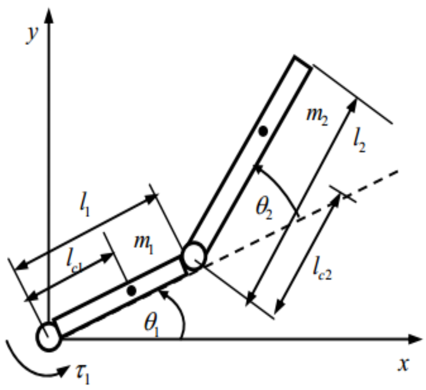

Fig 6. *Pendubot system structure*

Arccoding to (4), limiting-line of c_1 and c_2 is calculated as:

$$\begin{cases} c_{10} = g\left|(q_3 q_5 - q_2 q_4)/(q_1 q_2 - q_3^2)\right| = 66.97 \\ c_{20} = g\left|\left[q_5(q_1 + q_3) - q_4(q_2 + q_3)\right]/(q_1 q_2 - q_3^2)\right| \\ \quad = 68.68 \end{cases}$$

The parameters of pendubot system are chosen as: c_1 =5.807, c_2 =7.346, a_1=1.826, k_2= 3.687 and η_2 =1.427. The initial status vector is $\theta_0 = \left[\frac{\pi}{2} + 0.1, 0.1, -0.1, -0.2\right]^T$

The reference status vector $\theta_d = [0,0,0,0]^T$.

The parameters of HFSMC controller for pendubot system are chosen the same as with HSMC controller. However, HFSMC controller has more one parameter chosen k_1=0.01.

The Figures 7, 8, 9, 10 compare simulation results between HSMC controller and HSFMC controller for pendubot system. It indicates the angles of link 1 and link 2 of both HSMC controller and HFSMC controller moves to stable position in about 2 seconds. The overshoot of link 1, 2 under HFSMC controller is smaller than under HSMC controller (in Fig. 7, 8). Eventhough settling time of HFSMC is longer than HSMC (in Fig. 7, 8), the chattering in control signal is decreased remarkably (in Fig. 9, 10)

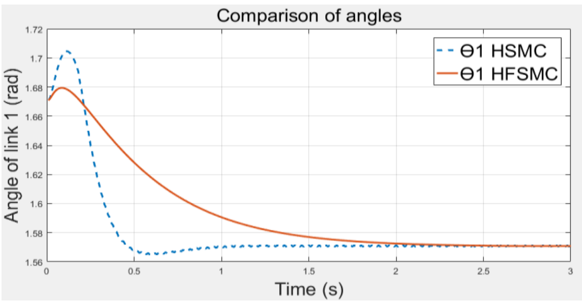

Fig 7. *The angle link 1 of pendubot using HSMC controller and HFSMC controller*

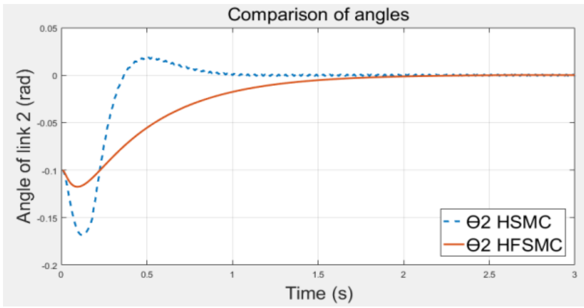

Fig 8. *The angle link 2 of pendubot using HSMC controller and HFSMC controller*

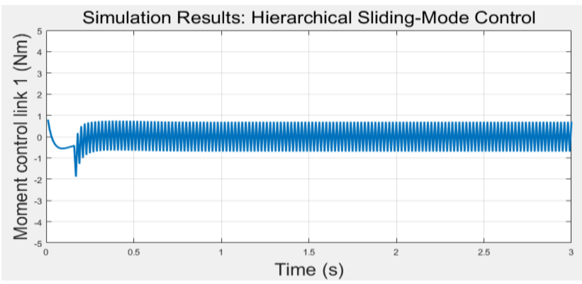

Fig 9. *Momen impact to link 1 of pendubot using HSMC controller*

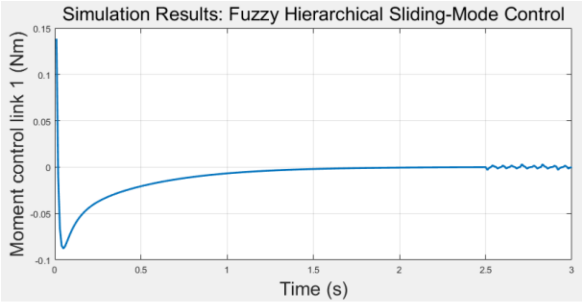

Fig 10. *Momen impact to link 1 of pendubot using HFSMC controller*

5.2 Double inverted pendulum system

Double inverted pendulum system consists of two single inverted pendulums on a cart showed in Fig. 11. This system is composed of three subsystems: upper inverted pendulum, lower inverted pendulum and cart. Its control objective is to stay steady to balance double inverted pendulum and take the cart to its original position [9].

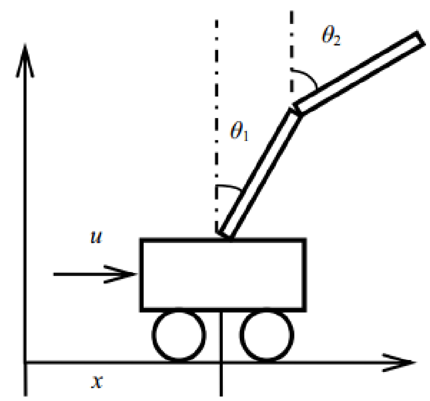

Fig 11. *Double inverted pendulum structure*

The signs in Fig. 11 are determined as: θ_1 is angle of upper pendulum and vertical axis, θ_2 is angle of lower pendulum and vertical axis, x is cart position. u is control force. When n=3 in (1), the inverted pendulum state space equation is determined as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + b_1 u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2 + b_2 u \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3 + b_3 u \end{cases} \quad (24)$$

Where $x_1 = \theta_1$; $x_3 = \theta_3$; $x_5 = x$; x_2 is angle velocity of lower pendulum, x_4 is angle velocity of upper pendulum, x_6 is angle velocity of cart, u is control signal, f_i and b_i ($i = 1,2,3$) is determined in [10].

In order to compare between HSMC controller and HFSMC controller, the parameters are chosen as [9]: Cart mass: M=1kg, lower pendulum mass: $m_1 = 1kg$, upper pendulum mass: $m_2 = 1kg$, upper pendulum length $l_1 = 0.1m$ and lower pendulum length: $l_2 = 0.1m$, gravitational acceleration $g = 9.81ms^{-2}$.

Arccoding to (4), limiting-line of c_1, c_2, c_3 are calculated as:

$$\begin{cases} c_{10} = g\left|\dfrac{A^2(B/3 - m_2l_2/4)}{(m_2/4 - A/3)(B^2 - AC) - m_2(B - Al_1)^2/4}\right| \\ = 294.39 \\ c_{20} = g\left|\dfrac{A^2(C - Bl_1)/2}{l_2\left[(m_2/4 - A/3)(B^2 - AC) - m_2(B - Al_1)^2/4\right]}\right| \\ = 98.31 \\ c_{30} = g\left|\dfrac{AB(B/3 - m_2l_1/4) + A(Cm_2 - Bm_2l_1)/2}{(m_2/4 - A/3)(B^2 - AC) - m_2(B - Al_1)^2/4}\right| \\ = 11.44 \end{cases}$$

Where $A = M + m_1 + m_2, B = m_1l_1/2 + m_2l_1$ and $C = m_1l_1^2/3 + m_2l_2^2$.

The HSMC controller parameters are chosen as:

c_1=7.3170, c_2 =3.8760, c_3 =1.9560, a_1 =0.8190, a_2 =0.3170, k_3 =3.5020, η_2 =8.6910.

Initial state vector:

$$X_0 = [-0.1, 0, 0.1, 0, 0.1, 0]^T$$

The parameters of HFSMC controller for double inverted pendulum system are chosen the same as HSMC controller. However, HFSMC controller have more one parameter chosen k1=0.01.

In Fig, 12, 13, 14, the responese of system under HFSMC in smaller than under HSMC. Morover, settling time is the same in both cases. Under HFSMC, chattering phenomenon of force in cart is removed completely compared to HSMC controller (Fig. 15, 16).

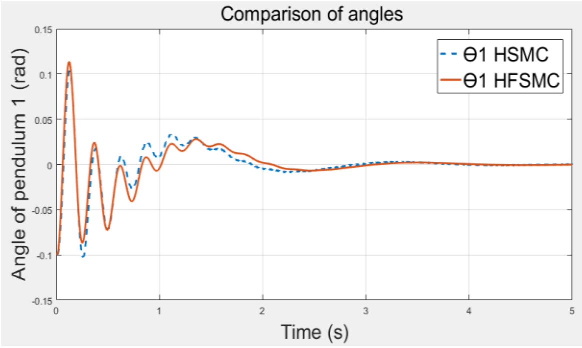

Fig 12. *Upper pendulum angle using HSMC controller and HFSMC controller*

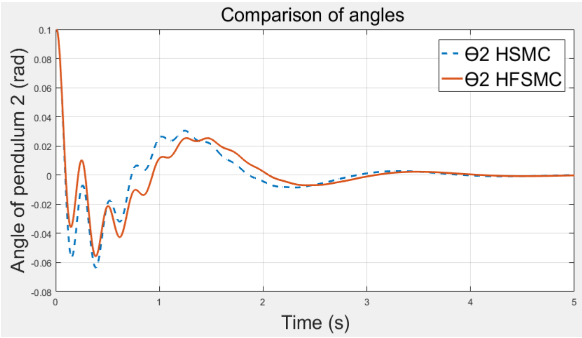

Fig 13. *Lower pendulum angle using HSMC controller and HFSMC controller*

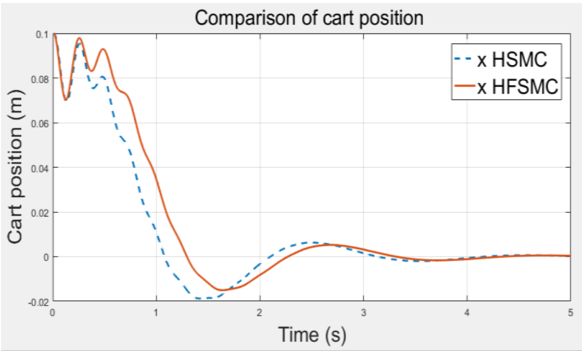

Fig 14*. Cart position of double inverted pendulum using HSMC controller and HFSMC controller*

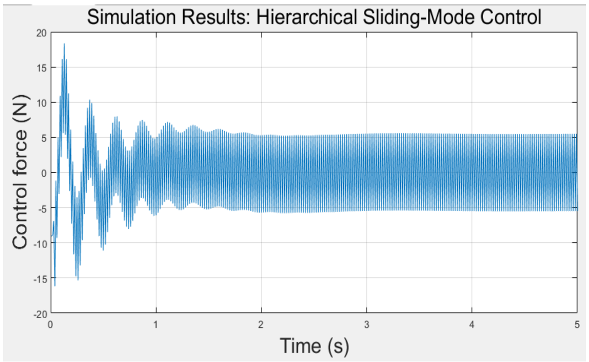

Fig 15. *Control force applying to double inverted pendulum using HSMC controller*

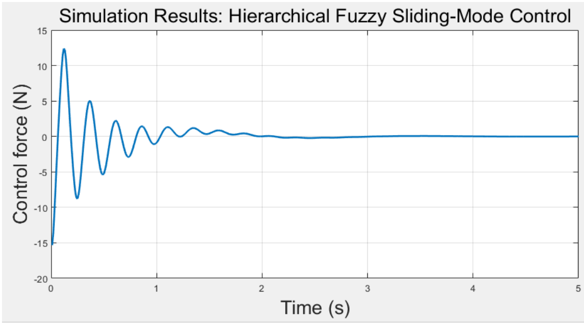

Fig 16. *Control force applying to double inverted pendulum using HFSMC controller*

6. CONCLUSION

This article introduces the hierarchical fuzzy sliding controller and how to successfully build a hierarchical fuzzy sliding controller for under-actuated SIMO systems. Theory and simulation results show that the HFSMC in pendubot and double pendulum system have completely removed chattering phenomenon compared to HSMC.

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