

IDENTIFYING PARAMETERS FOR CART AND POLE SYSTEM NHẬN DẠNG THÔNG SỐ CHO HỆ CON LẮC NGƯỢC TRÊN XE

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Received 12/12/2015, Peer reviewed 18/2/2016, Accepted for publication 15/3/2016

ABSTRACT

The aim of this paper is to use genetic algorithm to identify parameters of a nonlinear balancing system. In this case, single-linked inverted pendulum was chosen. With identified parameters of nonlinear model, a LQR controller was designed to prove that parameters were identified well. Results which were operated in Matlab/ Simulink environment showed that identifying process was good.

Keywords: *Inverted Pendulum, Optimal control, LQR, Matlab, Simulink, identify, genetic algorithm*

TÓM TẮT
Mục tiêu bài báo là sử dụng giải thuật di truyền để nhận dạng các thông số của một mô hình cân bằng ở dạng phi tuyến. Trong khuôn khổ bài báo này, hệ con lắc ngược một bậc trên xe là đối tượng được lựa chọn. Với các thông số hệ thống được nhận dạng, một bộ điều khiển LQR được thiết kế để chứng minh các thông số đã được nhận diện tốt. Các kết quả thực hiện cho thấy quá trình nhận dạng thông số mô hình là thành công thông qua mô phỏng Matlab/Simulink

Từ khóa: *Con lắc ngược, điều khiển tối ưu, LQR, matlab, Simulink, nhận dạng, giải thuật di truyền.*

1. INTRODUCTION

Inverted pendulum on Cart, called *Cart and Pole* system, is a classical SIMO nonlinear model in control engineering. With just one input is the force on Cart, we have to control both position of Cart and up-position of pendulum. A lot of controlling algorithms were proved to work well with this model [8]. In order to design controller, we have to know exactly both dynamic equations and parameters to calculate controlling signal (in case of LQR, input-output linearization, back stepping, sliding mode control...) or we have to test real model with knowledge of expert to get acceptable controlling parameters (in case of

fuzzy and PID control).

Neuron Network (NN) was presented to identify nonlinear system [5], [6], [3]. But, we cannot identify Cart and Pole system by methods in these papers because we just can take data to training NN when the pendulum swings around the down position and data collected is not appropriate for modelling inverted pendulum around working position (position up). Otherwise, in [11], authors used Neuron Network (NN) to identify nonlinear model of Cart and Pole. But this method requires us to decide a balancing control first before collecting data for identification. In the case, if we do not have the successful controller firstly, it is impossible to collect data for NN training.

One other solution is that we use searching algorithm like genetic algorithm (GA) to identify system. Then, using GA in simulation, we find and optimize controlling parameters. Some authors used GA to identify successfully linear model of system such as [4], [13], [1], [2]. But this way is not appropriate when system is a nonlinear MISO system with no delay because a linear model cannot be completely equivalent to a nonlinear model. Dynamic equation of system can be calculated through physical law but parameters are difficult to be measured.

In this paper, we used genetic algorithm (GA) to identify system parameters. After that, with parameters identified, we designed a LQR controller. The model used in this case was Inverted Pendulum. With the controller designed, the system balanced well. So, the method of using GA to identify nonlinear system parameters was appropriate in controlling a grey-box system.

2. SYSTEM MODEL OF INVERTED PENDULUM

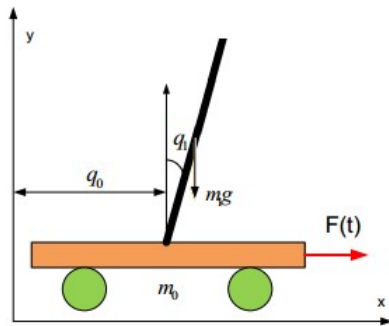


Figure 1. Cart and Pole system

The system in Figure 1 is a cart of which a rigid pole is hinged. The cart is free to move within the bounds of a one-dimensional track. The pole can move in the vertical plane parallel to the track. The controller can apply a force to the cart parallel to the track.

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q_p \quad \text{with } q = \begin{bmatrix} q_0 \\ q_1 \end{bmatrix} \quad (1)$$

Kinetic energy of system:

$$T = T_0 + T_1 = \frac{1}{2} m_0 \dot{q}_0^2 + \frac{1}{2} J_1 \dot{q}_1^2 + \frac{1}{2} m_1 (\dot{q}_0 + \dot{q}_1 l_1 \cos q_1)^2 \quad (2)$$

Potential energy of system:

$$P = P_0 + P_1 = m_1 g l_1 \cos q_1 \quad (3)$$

Lagrangian:

$$L = T - P = \frac{1}{2} m_0 \dot{q}_0^2 + \frac{1}{2} J_1 \dot{q}_1^2 + \frac{1}{2} m_1 (\dot{q}_0 + \dot{q}_1 l_1 \cos q_1)^2 - m_1 g l_1 \cos q_1 \quad (4)$$

Lagrangian for motion of cart:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_0} \right) - \frac{\partial L}{\partial q_0} = F - b_0 \dot{q}_0 \quad (5)$$

Lagrangian for rotating motion of pendulum:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_1} \right) - \frac{\partial L}{\partial q_1} = -b_1 \dot{q}_1 \quad (6)$$

Solve (5) and (6), we have system dynamic equations

$$\begin{cases} (m_0 + m_1) \ddot{q}_0 + m_1 l_1 (\ddot{q}_1 \cos q_1 - \dot{q}_1 \sin q_1) = F - b_0 \dot{q}_0 \\ J_1 \ddot{q}_1 + m_1 l_1^2 (\ddot{q}_1 \cos^2 q_1 - 2 \dot{q}_1^2 \sin q_1 \cos q_1) + \\ + m_1 l_1 (\ddot{q}_0 \cos q_1 - \dot{q}_0 \dot{q}_1 \sin q_1) + \\ + m_1 \dot{q}_1^2 l_1^2 \cos q_1 \sin q_1 \\ + m_1 \dot{q}_0 \dot{q}_1 l_1 \sin q_1 - m_1 g l_1 \sin q_1 = -b_1 \dot{q}_1 \end{cases} \quad (7)$$

So, if we need is to identify system parameters as m_0 ; m_1 ; l_1 ; J_1 ; b_0 ; b_1 .

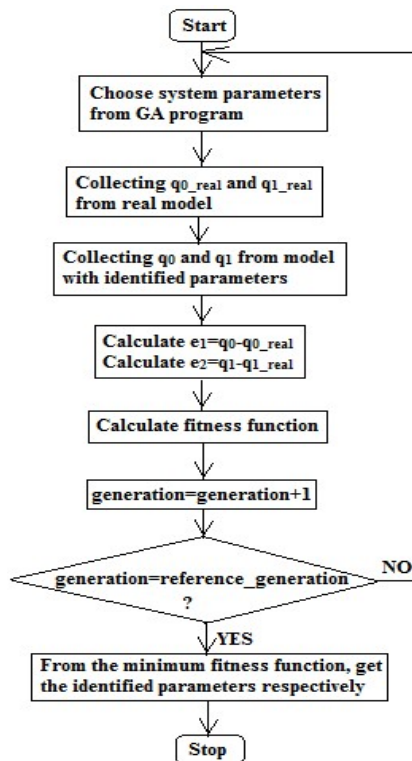
We choose parameters of real model was listed in Table 2 below:

Table 1. Real System parameters

Parameter	Unit	Definition	
m_0	Kg	Mass of cart	0.033
m_1	Kg	Mass of Pendulum	1.999
L_1	m	Length of First pendulum	0.2
l_1	m	Distance between center and rotating axis of pendulum	0.115
J_1	kgm^2	Inertial moment of pendulum	0.023
g	m/s^2	Gravitation acceleration	9.81
F	N	Force on Cart	
q_0	m	Position of Cart	
q_1	Rad	Angle of Pendulum	
b_0	kg/s	Viscous Coefficient of Cart	0.0001
b_1	Nms	Viscous Coefficient of Rotating Axis of pendulum	0.0001

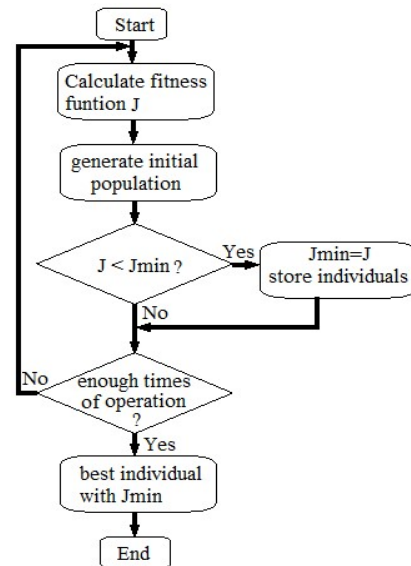
3. GENETIC ALGORITHM

In this case, GA used is off-line.

**Figure 2.** Block diagram of GA program

Parameters for GA program are listed as below:

- Size of population: N=20
- Linear Ranking Selection: $\eta = 0.2$
- Decimal coding
- Two-point crossover
- Crossover parameter: 0.8
- Mutation parameter: 0.2

**Figure 3.** Flow chart of GA Searching process

Choose fitness function:

$$J = \sum_{i=1}^n [e_1(i)]^2 + \sum_{i=1}^n [e_2(i)]^2 \quad (8)$$

With $e_1 = q_{0_real} - q_0$, $e_2 = q_{1_real} - q_1$ and n is number of samples in one time of simulation. If the identified model is as similar to the real model, fitness function J will be small.

In this case, we operate Simulink program of simulation system in 10s, with sample-time is 0.01s. So, we have $n=1001$ sample.

4. SIMULATION

Impact a pulse-shaped Force on the Cart as in Figure 4.

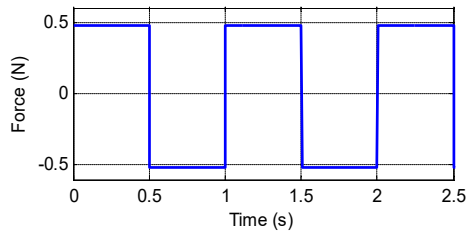


Figure 4. Force on Cart for Identifying Experiment

With Force on Cart as in Figure 4, cart moves gradually toward one direction as in Figure 7 and pendulum swing as Figure 8. With these data, we can use GA program to identify parameters as in Figure 2 and Figure 3.

The data of identification is listed as in Table 2 below and output of identified model is respective showed in Figure 7 and Figure 8, compared with initial data.

Table 2. Comparison of parameters of real model and identified model after 7000 generation.

	Real Parameters (Column 1)	Identified Paramaters (Column 2)
m0	0.35 kg	0.033 kg
m1	0.133 kg	1.999 kg
l1	0.115 m	0.152 m
L1	0.2 m	0.378 m
J1	0.0023	0.046
b0	0.0001	0.0054
b1	0.0001	0.0026
Fitness J	0	550.15

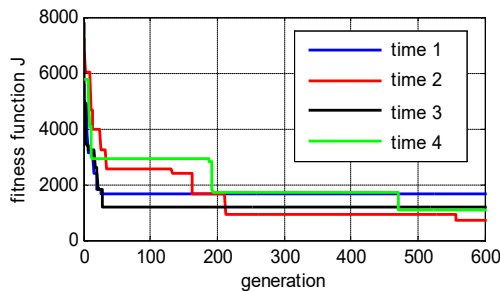


Figure 5: Changing of fitness functions J in 600 generation

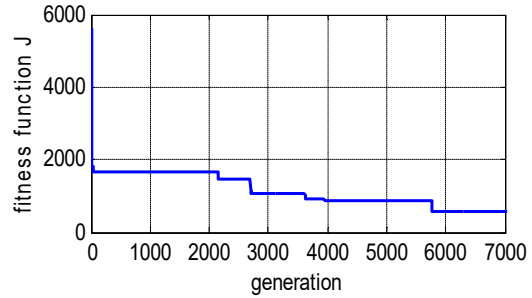


Figure 6. Changing of fitness function J in 7000 generations

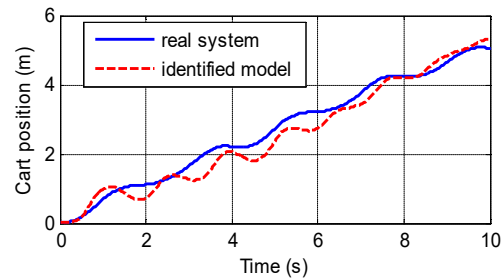


Figure 7. Comparison of output 1 (Cart Position) between real model and identified model.

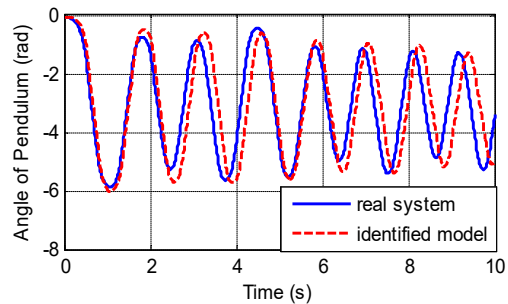


Figure 8. Comparison of output 2 (angle of pendulum) between real model and identified model

From [7], [10], we can calculate a LQR controller from identified system parameters. The robustness of LQR control was proved [12]. From the model, we can calculate matrix A and B to linearize around working-point to get linear equation of system:

$$\begin{cases} \dot{x} = Ax + B \times F \\ y = Cx \end{cases} \quad (9)$$

With symmetric positive definite matrixes Q and R, we can calculate $K = lqr(A, B, Q, R)$ from Matlab.

With $R = 1$ and $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (10)

Matrix K can be calculated to be:

- With real parameters of system in column 1 of the Table 2:

$$K = [-1.0000 \quad -2.5199 \quad -45.9901 \quad -6.1926]$$

- With identified parameters in column 2 of the Table 2:

$$K = [-1.0000 \quad -1.7552 \quad -14.9432 \quad -2.5558]$$

Both results of K are used to control the same model with parameters as in Column 1 of Table 2. The diagram of controller and system is as in Figure 9:

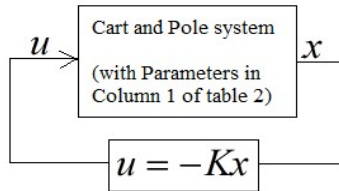


Figure 9. Diagram of LQR controller

Comparisons are shown in Figure 10 and Figure 11 below:

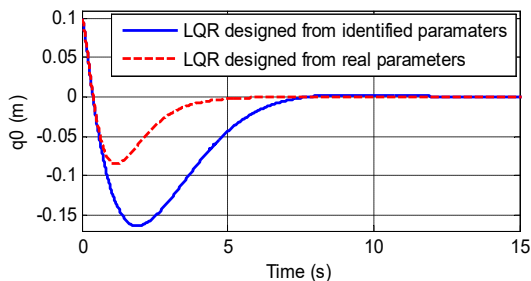


Figure 10. Comparison of Cart position of real model with different matrix K

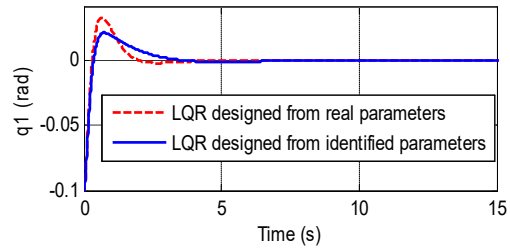


Figure 11. Comparison of Angle of pendulum of real model with different matrix

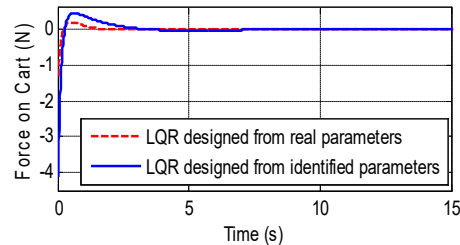


Figure 12. Comparison of Force on Cart of real model with different matrix K

From Figure 10 and Figure 11, we can see that LQR controller designed from identified parameters operate well even though worse than LQR controller designed from exact parameters.

We can identify better parameters by searching with more generations in GA program or taking longer time in collecting data for identification. But these actions may make GA run longer but result is not improved so much

5. CONCLUSION

In this paper, parameters are identified by GA program from a collecting data from operation of model. Then, LQR controller designed from these parameters stabilized successfully that model. So, from this case, we can apply Genetic Algorithm methods in identifying parameters of other systems in order to design controller respectively.

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