ANALYSIS OF THE DERFORMTION IN INDUCTION TRIANGLE HEATING USING LAMINATED PLATE THEORY

PHÂN TÍCH SỰ BIẾN DẠNG QUÁ TRÌNH ĐỐT NÓNG CẢM ỨNG TỪ DẠNG TAM GIÁC DỰA VÀO LÝ THUYẾT TẤM PHẲNG LỚP

Nguyen Truong Thinh[1], Pham Van Dieu[2]
[1]Ho Chi Minh City University of Technology and Education
[2]Gia lai Vocational College

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ABSTRACT

Induction triangle heating for plate structures often results in produce various curved thick plate with some types of concave in shipyard industry. The problem of deformation in around an induction heating are being of major concern in shipyard industry. The induction heating induced deformation formulas for induction triangle heating, composed process parameters such as heat input, size of induction heating, velocity of inductor and plate's thickness, are developed analytically by using the approximation of several rectangular cuboidal inclusions with eigenstrain in an infinite laminated theory. The source of deformation in induction process is the plastic strains which are caused by non-uniform temperature gradient. The distributions of plastic strain corresponding to eigenstrain are assumed by FEM solutions. In this paper, the formulas for plate deformation as transverse and longitudinal shrinkages as well as vertical deflection produced by induction triangle heating are formed based on eigenstrain concept using laminated plate theory to consider some cuboidal inclusions with eigeinstrain. The residual deformation that was due to thermal process was depends on the magnitude and region of plastic strains at heating affected zone. Comparison of the calculated results with experimental and finite element method (FEM) data shows the accuracy and validity of the proposal method.

Keywords: *Induction heating, Triangle heating, Laminated plate theory, Shipyard, Plate deformation, Residual stress.*

TÓM TẮT

Đốt nóng bằng cảm ứng từ dạng tam giác cho các tấm thép thường được sử dụng trong việc biến dạng các tấm thép dày để tạo ra các đường cong khác nhau trong công nghiệp đóng tàu thủy. Và hiện nay biến dạng cho các tấm thép trong công nghiệp đóng tàu hầu hết là sử dụng dạng gia nhiệt cảm ứng từ. Các công thức tính toán cho sự biến dạng do đốt nóng bằng cảm ứng từ dạng tam giác cùng với các tham số quá trình như nhiệt lượng đưa vào, kích thước đầu gia nhiệt, vận tốc đầu gia nhiệt và chiều dày tấm thép đã được triển khai bằng phương pháp phân tích với các phần tử biến dạng riêng (eigenstrain) dạng lớp dạng khối theo lý thuyết tấm phẳng lớp vô hạn. Nguyên nhân gây biến dạng của quá trình đốt nóng bằng cảm ứng từ là do trường nhiệt độ không đồng đều tạo ra biến dạng đàn hồ trong quá trình đốt nóng. Sự phân bố của biến dạng đàn hồi này theo giá trị biến dạng riêng cũng được phân tích dựa trên phương pháp phần tử hữu hạn (FEM). Trong bài báo này các công thức tính toán biến dạng của tấm thép theo hướng dài và dọc tấm thép cũng như hướng thẳng đứng cũng *được trình bày dựa trên khái niệm biến dạng riêng. Ứng suất dư tạo ra do trường nhiệt độ không đồng đều phụ thuộc vào biên độ và biến dạng đàn hồi ở vùng ảnh hưởng nhiệt (HAZ). Ngoài ra, trong bài báo quá trình tính toán phân tích cũng được so sánh với kết quả thí nghiệm và phân thích bằng phương pháp phần tử hữu hạn cho thấy các công thức đưa ra đảm bảo độ chính xác và có thể sử dụng cho việc phân tích biến dạng của quá trình đốt nóng bằng cảm ứng từ.*

Từ khóa: *Đốt nóng bằng cảm ứng từ, Đốt nóng dạng tam giác, Lý thuyết tấm phẳng lớp, đóng tàu, Biến dạng tấm thép, Ứng suất dư.*

1. INTRODUCTION

Induction heating deformation is a common and important problem in shipyard industry. The line heating process is an effective and economical method for forming flat metal plates into three-dimensional shapes in constructing of ships, trains, airplanes, and in rapid prototyping of complex curved objects [1]. Two types of heat sources can be usually used in the line heating process: oxyacetylene torch and induction coil. Compared with the heat source of an oxyacetylene torch, that of the induction coil (inductor) has the following advantages as the power and its distribution are easier to control and reproduce. Besides, the induction heating system can be more easily integrated with a robotic system for automation than the flame heating system. The line heating process utilizes generally line heating or triangle heating as the heating path on a plate according to the desired shape of a plate in forming a ship hull. The triangle induction heating is a process of heating steel plate with the triangular pattern by an electromagnetic induction inductor moving along a trajectory to generate a plastic region which has also a triangular shape. In this process, the inductor generally combines 2 movements: linear and angular. As the inductor revolves around an axis to form a circle, it moves simultaneously along one of the three types of pattern as parallel, branch, and zigzag as in **Figure 1**.

Line heating and triangle heating are some important production processes that are widely used to produce various curved thick plate for shipyard industry [2]. In the triangle heating, which is usually applied to obtain the concave type of plate, the width and depth of a heated region change almost linearly along the heating path. The triangle heating method with induction heating equipment has been concerned in the forming process of steel plate in shipyard to form the bow and stern plates of ship's hulls, which is most labor-consuming job[3]. However, the method needs to be verified in terms of effectiveness and efficiency. For the purpose of this, the method should be first analyzed with a mathematical model, which can offer the relationship between the deformation and the heating parameters. Thermal strains and stresses are generated by non-uniform temperature gradient, which is produced in base metal by heating, and cooling cycles occurred in induction heating process. When thermal stresses exceed elastic limit, residual stresses and distortions are appeared [3].

There are usually three ways to determine the deformation of heating process like as: 1) Experimental formulas, 2) Thermal elastic-plastic FEM method, and 3) Method based eigenstrains. The experimental, numerical and analytical methods were proposed for prediction of residual distortions due to induction heating process. Experimental method is only fit for simple shape structure and heating process. In numerical method, induction induced thermal-elastic-plastic processing is analyzed in computer by using the finite element method. In analytical method, the residual distortion is calculated elastically by using the eigenstrain corresponding plastic strain as an initial strain. There is a large time and cost consuming to estimate the deformations of induction triangle heated plate by using experimental and numerical methods.

In this paper, induction induced deformation formulas, composed the induction parameters such as heat input, thickness and travel speed, is developed analytically by the use of combining several cuboidal inclusions approximating triangle eigeinstrain region in an infinite laminated plate theory. The deformations are dominated by distributions of the plastic trains in induction heating. We assume that plastic strains, driving forces to make the deformations, are produced in critical heating region. The plastic strains and its region corresponded to eigeinstrains and approximated sizes of cuboidal inclusion.

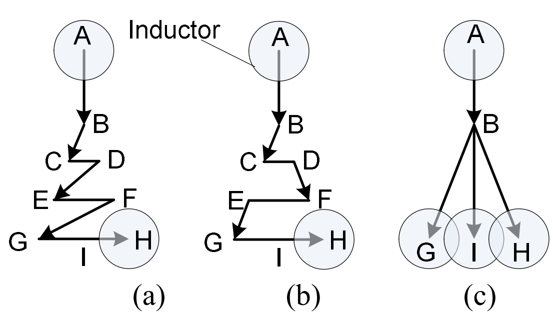

Fig. 1. *Trajectory of inductor: (a) type of parallel, (b) type of zigzag and (c) type of branch.*

2. TRIANGLE INDUCTION HEATING PROCESS

An alternating voltage applied to induction coil results in an alternating current in the coil circuit. Alternating coil current will produce in its surrounding a time-variable magnetic field that has the same frequency as the coil current. This magnetic field induces eddy current in the steel plate located near the coil. These induced currents have same frequency as the coil current, whereas, their direction is opposite to coil current. The induced current generates electric-resistance heat by the Joule effect in the plate and this can be used to bend the plate [4].

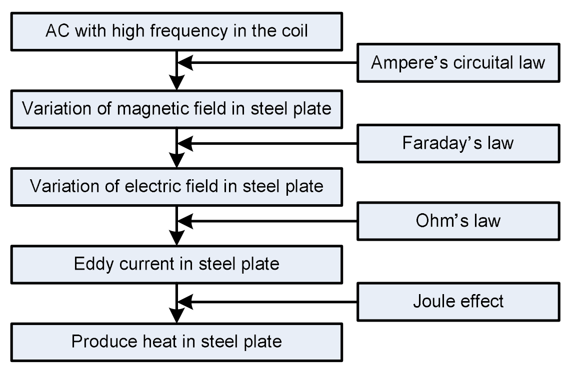

Fig. 2. *Steps of heat generation and their related laws in induction heating of steel plate.*

The basic electromagnetic phenomena of induction heating are quite well discussed at several textbooks. In the modeling of the induction heating of a steel plate, the electromagnetic phenomenon should be close examined. The procedure of calculating heat in induction heating process is shown with a flowchart in **Figure 2**, where each problem step and its related governing law are presented [5-6].

Where E is the electric field intensity and J_e is the Eddy current density. Heat source is expressed as:

$$q = \frac{\left[\mathrm{Re}\left(J_e\right)\right]}{\sigma} \quad (1)$$

Where q is the heat source. For the special case where the source current density is assumed to be time harmonic, the heat input for the average time can be calculated as:

$$\bar{q} = \frac{1}{\tau}\int_0^{\tau} q dt \quad (2)$$

The heat input for the average time can be written as

$$\bar{q} = \frac{1}{2}\omega^2 \sigma A^* A \quad (3)$$

Where $\bar{q}$ is the heat input for the average time and * is complex conjugate.

To determine the region where the temperature reaches above the critical point and the plastic strains are produced, the transient heat flow analysis is performed with a numerical method. Transient temperature distribution in the steel plate during induction heating could be obtained by heat flow analysis. The size of heat affected zone (HAZ) heated above the critical point of 723°C is considered to be the region where the plastic strains are produced.

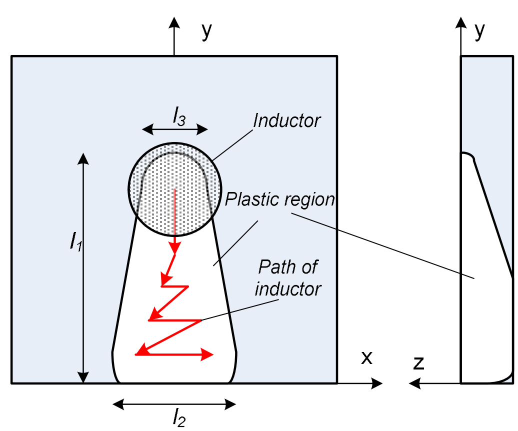

***Fig. 3.** Shape of plastic region in triangle induction heating.*

3. FORMULATION OF DEFORMATION IN INDUCTION TRIANGLE HEATING

3.3.2 Deformation of plate containing inclusion with eigenstrains

In this section, the problem of an infinite laminate with an eigenstrain in cuboidal zones. The laminate is composed of isotropic linear elastic materials. The eigenstrain varies through the thickness of the laminate. The problem is formulad by using classical laminated plate theory in which the displacement fields in the laminated plate are expressed in terms of the in-plane displacement on the main plane and the transverse, longitudinal shrinkages displacements. To consider the deformation of a plate, the plate is assumed to be composed of thin layers of isotropic linear elastic materials. The vertical, transverse and longitudinal deformations of the layered plate is analyzed by using an infinite laminated plate theory to consider a cuboidal inclusions with eigenstrains which are corresponded to plastic strains resulted from the induction heating. The cross section of the layered plate is shown schematically in **Figure 3**. The plate has a constant thickness of h. Eigenstrain of ε^* is prescribed in a cuboidal inclusion which has a height of d_i and a constant cross section of $2d_i\, b_i$.

Consider a deformation of an infinite plate composed of thin layers of isotropic linear elastic materials. According to classical laminated plate theory, the displacements at any point of a plate are written as

$$u_i = u_i^0 - x_3 w_{,i}\left(x_1, x_2\right), \quad \left(i = 1, 2\right) \quad (4)$$

$$u_3 = w\left(x_1, x_2\right)$$

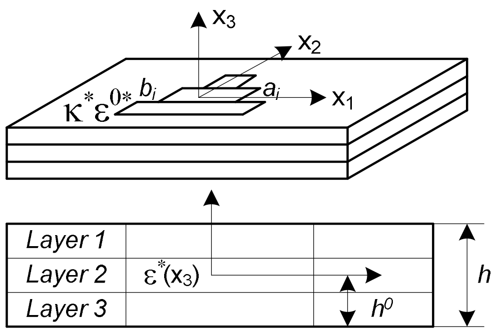

Fig. 4. *Plate containing inclusion with eigenstrain.*

Where u_i and u_3 are the in-plane and traverse displacements, respectively, u_i^0 is the in-plane displacement on the main plane which will defined exactly later, and the subscript comma (,) denotes a partial derivate with respective to the in-plane Cartesian coordinates, x_1 and x_2.

Employing a method based on the influence functions, which is an extension of the Maysel's relation to laminate problems, it can be shown that integral type solutions for displacements are expressed as:

$$u_i^0(x_1,x_2) = \int_\Omega \tilde{N}_{kl}^{[i]}(\xi_1,\xi_2;x_1,x_2)\varepsilon_{kl}^{0*}dA \tag{5}$$

$$w(x_1,x_2) = \int_\Omega \tilde{M}_{kl}^{[3]}(\xi_1,\xi_2;x_1,x_2)\kappa_{kl}^{*}dA \tag{6}$$

We consider a homogeneous infinite plate containing an inclusion in which an eigenstrain $\varepsilon_{kl}^*(x_3)$ is prescribed. The generalized eigenstrains $\varepsilon_{kl}^{0*}(x_3)$ and $\kappa_{kl}^*(x_3)$ are thus uniform in the subregion Ω as shown in **Figure 4**. Since the plate and inclusion are homogeneous, the expression for out-of-plane displacement field be expressed as follows [4].

$$u_i^0 = \frac{1}{8}\left\{\begin{array}{l} 4\nu^A H_{,i}\delta_{kl} \\ +\left(3-\nu^A\right)\left(H_{,l}\delta_{ik}+H_{,k}\delta_{il}\right) \\ -\left(1+\nu^A\right)\left(H_{ik,l}+H_{il,k}\right) \end{array}\right\}\varepsilon_{kl}^{0*} \tag{7}$$

$$\begin{aligned} u_i &= u_i^0 - x_3 w_{,i}(x_1,x_2) \\ &= \frac{1}{8}\left\{\begin{array}{l} 4\nu^A H_{,i}\delta_{kl} \\ +\left(3-\nu^A\right)\left(H_{,l}\delta_{ik}+H_{,k}\delta_{il}\right) \\ -\left(1+\nu^A\right)\left(H_{ik,l}+H_{il,k}\right) \end{array}\right\}\varepsilon_{kl}^{0*} \\ &\quad - x_3 w_{,i}(x_1,x_2) - x_3 w_{,i}(x_1,x_2) \end{aligned} \tag{8}$$

$$u_3 = -\frac{1}{4\pi}\left\{\begin{array}{l} \left(1+\nu^D\right)H\delta_{kl} \\ +\left(1-\nu^D\right)H_{kl} \end{array}\right\}\kappa_{kl}^{*} \tag{9}$$

Here $\nu^A = A_{12}/A_{11}, \nu^D = D_{12}/D_{11}$, and the integrals H and H_{kl} are defined respectively by

$$u_i^0 = \frac{1}{8}\left\{\begin{array}{l} 4\nu^A H_{,i}\delta_{kl} \\ +\left(3-\nu^A\right)\left(H_{,l}\delta_{ik}+H_{,k}\delta_{il}\right) \\ -\left(1+\nu^A\right)\left(H_{ik,l}+H_{il,k}\right) \end{array}\right\}\varepsilon_{kl}^{0*} \tag{10}$$

To be easy to calculate, we determine the shrinkages of plate in main plane, thus value of x_3 along vertical axis is zero. We have the displacements of plate calculated with below formulas like as:

$$u_1^0 = \frac{1}{8}\left\{\begin{array}{l} 4\nu^A H_{,1}\delta_{kl} \\ +\left(3-\nu^A\right)\left(H_{,l}\delta_{1k}+H_{,k}\delta_{il}\right) \\ -\left(1+\nu^A\right)\left(H_{1k,l}+H_{1l,k}\right) \end{array}\right\}\varepsilon_{kl}^{0*} \tag{11}$$

$$u_2^0 = \frac{1}{8}\left\{\begin{array}{l} 4\nu^A H_{,2}\delta_{kl} \\ +\left(3-\nu^A\right)\left(H_{,l}\delta_{2k}+H_{,k}\delta_{2l}\right) \\ -\left(1+\nu^A\right)\left(H_{ik,l}+H_{2l,k}\right) \end{array}\right\}\varepsilon_{kl}^{0*} \tag{12}$$

$$u_3 = -\frac{1}{4\pi}\left\{\begin{matrix}\left(1+\nu^D\right)H\delta_{kl}\\ +\left(1-\nu^D\right)H_{kl}\end{matrix}\right\}\kappa_{kl}^* \qquad (13)$$

Where $\nu^D = D_{12}/D_{11}$. H and H_{kl} are the introduced integrals, which are given in a study of Beom and Kim [5]. When the rectangular inclusion with a dilatational eigenstrain as a special case is then considered, the main plane eigenstrain ε_{ij}^{0*} and the eigencurvature κ_{ij}^* have the following forms: $\varepsilon_{ij}^{0*} = \varepsilon^{0*}\delta_{ij}$ and $\kappa_{ij}^* = \kappa^*\delta_{ij}$. Using the relation of $H_{ik,k} = H_{,i}$, the deformation can be modified as follows for the typical case of thermal eigenstrain in an isotropic material. We obtain:

$$\begin{aligned} u_1^0 &= \frac{1+\nu^A}{2}\varepsilon^{0*}a_1\tilde{u}_1^0 \\ &= \frac{1+\nu^A}{2\pi}\varepsilon^{0*}H_{,x_1}\left(x_1,x_2\right) \end{aligned} \qquad (14)$$

$$\begin{aligned} u_2^0 &= \frac{1+\nu^A}{2}\varepsilon^{0*}a_1\tilde{u}_2^0 \\ &= \frac{1+\nu^A}{2\pi}\varepsilon^{0*}H_{,x_2}\left(x_1,x_2\right) \end{aligned} \qquad (15)$$

$$\begin{aligned} u_3 &= -\frac{1+\nu^D}{2}\kappa^*\tilde{\kappa}_{ij} \\ &= -\frac{1+\nu^D}{2\pi a_1^2}\left[H\left(x_1,x_2\right)-H\left(0,0\right)\right]\kappa^* \end{aligned} \qquad (16)$$

When the plastic strain is uniform through the thickness direction, the laminated plate is consisted of two laminae and one of the laminae contains the inclusion with the uniform eigenstrains, and the thickness of the top lamina is equal to that of the inclusion, eigencurvature is then determined as follows.

$$\kappa^* = \frac{C_{11}-C_{12}}{D_{11}+D_{12}}\int_{h^{(2)}-h^0}^{h^{(3)}+h^{(2)}-h^0}\varepsilon^* x_3 dx_3 \qquad (17)$$

Where, extensional stiffness tensors C_{11} and C_{12} are given by $C_{11} = E/\left(1-\nu_0^2\right)$ and $C_{12} = \nu E/\left(1-\nu^2\right)$, bending stiffness tensor D_{ij} is $\int_{-h^0}^{h^0} C_{ij}x_3^2 dx_3 \left(ij = 11, 12\right)$, and eigencurvature is defined as $\kappa_{11}^* = \kappa_{22}^* = \kappa^*$.

The deformations of steel plate in induction heating can be calculated by substituting the magnitude and size of the plastic strains into Eq. (17). The width and depth of the rectangular inclusion of the plastic region are presented for the plate in induction heating as shown **Figure 4** where the plate is assumed to be consisted of two laminas and the inclusion is located in one of them.

When above equations, we can define the vertical deformations in terms of material properties, plate thickness, and heat input as follows at different shapes of inclusions of trapezoid and ellipse for the steel plate in induction heating, respectively.

$$\begin{aligned} u_3 = &-\frac{3\left(1+\nu\right)\left(b_{1i}+b_{2i}\right)}{32\pi b_{1i}\left(h^0\right)^2}\times \\ &\left[\alpha T_c - \sigma_{yl}\left(\frac{1}{Kb_{1i}}+\frac{1-\nu}{E}\right)\right]\times \\ &\left(h^0 - \frac{\left(b_{1i}+b_{2i}\right)h}{4b_{1i}}\right)\times \\ &\left[H\left(x_1,x_2\right)-H\left(0,0\right)\right] \end{aligned} \qquad (18)$$

$$\begin{aligned} u_3 = &-\frac{3\pi\left(1+\nu\right)h_i}{8\pi\left(h^0\right)^3}\times \\ &\left[\alpha T_c - \sigma_{yl}\left(\frac{1}{Kb_{1i}}+\frac{1-\nu}{E}\right)\right]\times \\ &\left(h^0 - \frac{\pi h_i}{4}\right)\left[H\left(x_1,x_2\right)-H\left(0,0\right)\right] \end{aligned} \qquad (19)$$

The eigenstrain on main plane can be

expresses as Eq. (3). Thus the shrinkages of deformation are displacements in-plane of plate Cartesian coordinates, x_1 and x_2. We can determine with below equations:

$$u_1^0 = \frac{1+\nu^A}{2\pi}\left[\alpha T_c - \sigma_{yl}\left(\frac{1}{aK} + \frac{1-\nu_1}{E_1}\right)\right] \times H_{,x_1}(x_1, x_2) \quad (20)$$

$$u_2^0 = \frac{1+\nu^A}{2\pi}\left[\alpha T_c - \sigma_{yl}\left(\frac{1}{aK} + \frac{1-\nu_1}{E_1}\right)\right] \times H_{,x_2}(x_1, x_2) \quad (21)$$

The angular distortion θ can be then derived as follows.

$$\theta = \frac{u}{l} \quad (22)$$

Where *l* is distance from the center line of triangle heating and *u* is displacement of a point.

4. RESULTS AND DISCUSSIONS

4.1. Analytical Results

Deformation of a plate in the induction heating with triangle method, which is of great important for the forming of plate, can be calculated by combining the FEM solutions to determine the shapes of the plastic region. To verify the solution for the triangle heating method, a simulation is performed with a steel plate which has 1000 *mm* in length, 1000 *mm* in width, and 30 *mm* in thickness. Based on the simulations in previous section, in this case, plastic region as approximating the triangle heating is performed with a trapezoidal heating shape of 350 *mm* in height, 198.2 *mm* and 92.6 *mm* in base lines on the upper surface. The induction heating part is located at the center of one edge on the surface. When the triangle heating is carried out from the starting point to the end point over a steel plate, the eigenstrain region can be modeled simply for both width and depth of the region to be increased linearly along its heating line. The temperature distribution is first calculated by the FEM model, and then the size of the eigenstrain region is computed as described in the previous section. For mild steel, Young's modulus and yield stress become very small at a temperature above 723^0C. Due to the restraint of the surrounding material during induction heating process, not only the area with temperature above 723^0C will become plastic but the region with a lower temperature is also expected to become plastic. Thus, the critical temperature is chosen to be 600^0C. The simplified model for the eigenstrain region used in this study is shown in **Figure 5** with the plane and section views, where the region is divided into 35 discrete line-heating segments to simplify the computations with the developed analytic solution derived from the laminated plate theory. The width and depth of the region are previously determined by the results of the heat flow analysis.

In the simulation of this triangle heating case, the triangle heating model with shape of trapezoid is illustrated as shown in **Figure 5.** The magnitudes of the eigenstrains can be calculated and the out-of-plane deformation can be obtained with Equations (17-18). The magnitude of the deformation at each calculating point at various y positions along the direction *x* is obtained. The deformed shape of the plate heated with the condition of this case is shown 3-dimensionally in **Figure 6**.

Obvious change of deformation is

observed around the heating line, and it increases rapidly at last intervals of the line. Most part of the plate except the heated area showed to be moved with same amount of linear displacements along the y positions due to the locally concentrated deformation of the heated area. The result shows that sharp deformations are presented around the ending point of the heating, and that most part of the plate except around the heating line behaves linear displacement.

To verify the analytical solution, an induction heating experiment is also performed with a mild steel plate which has the size of 500 *mm* in length, 400 *mm* in width, and 30 *mm* in thickness as shown in **Figure 7**(a).

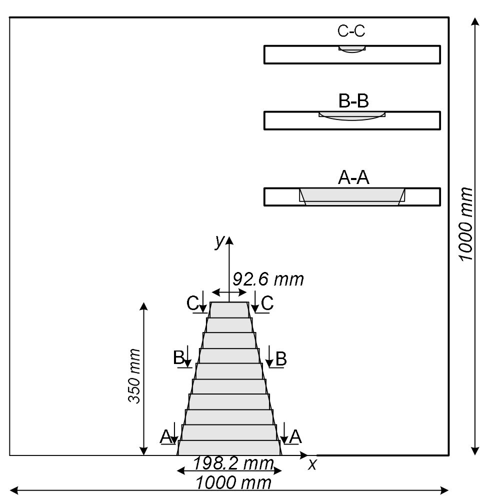

Fig. 5. *Triangle heating model in deformation analysis.*

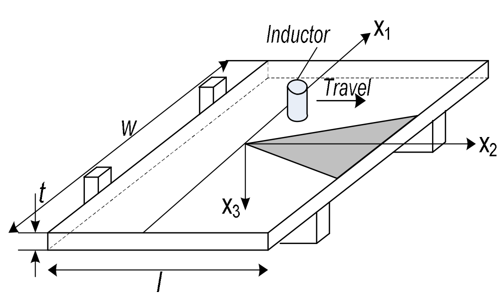

Fig. 6. *Schematic diagram of experiment and coordinate system*

Size of heating triangle of experiment is equal to that in the analytical case. In the experiment, the pattern of triangle heating method is that the heat source of inductor is moving to follow a pattern of zigzag as shown in **Figure 7**(b). From the analytical and experimental results in advance, it was revealed that that the HAZ distributions with three different types of triangle heating are different one another. In this study, the zigzag pattern is chosen for the linear movement of the inductor because it produces a uniform shape of HAZ. The heating conditions for the triangle heating are as shown in **Table 1**. The inductor has an external diameter of 50 *mm* and its trajectory consists of 2 movements with the moving patterns of an orbit of circle and a zigzag. The diameter of the orbit is 100 *mm* as shown in **Figure 7**(b). The rotation velocity of the inductor is 0.25 *rad/s* and linear velocity of the inductor 10 *mm/s*.

The depths of HAZ (Heat Affected Zone) regions with the temperature of above 600^0C are presented at y = 0 *mm*, 50 *mm*, 100 *mm*, 150 *mm*, 200 *mm*, 250 *mm*, and 300 *mm*. The depths of HAZ in direction *y* at plane x = 0 mm in simulation is shown in **Figure 7**(b). When the dimensions of HAZ are compared with shapes of eigenstrain region, it is proved that the shapes of eigenstrain obtained are similar to those of HAZ with the temperature of above 600^0C. These results are used for the proposed analysis.

Table 1: Triangle induction heating conditions

Material	Mild steel
Dimensions of plate	400(mm) x 500(mm) x 30(mm)

Size of triangle heating	198.2(mm) x 92.6(mm) x 350(mm)
Input power	40kW, 16kHz
Pattern of inductor's path	Zigzag
Angular velocity of inductor	0.25 radian/sec
Linear velocity of inductor	10 mm/sec
Critical temperature	≥ 600°C

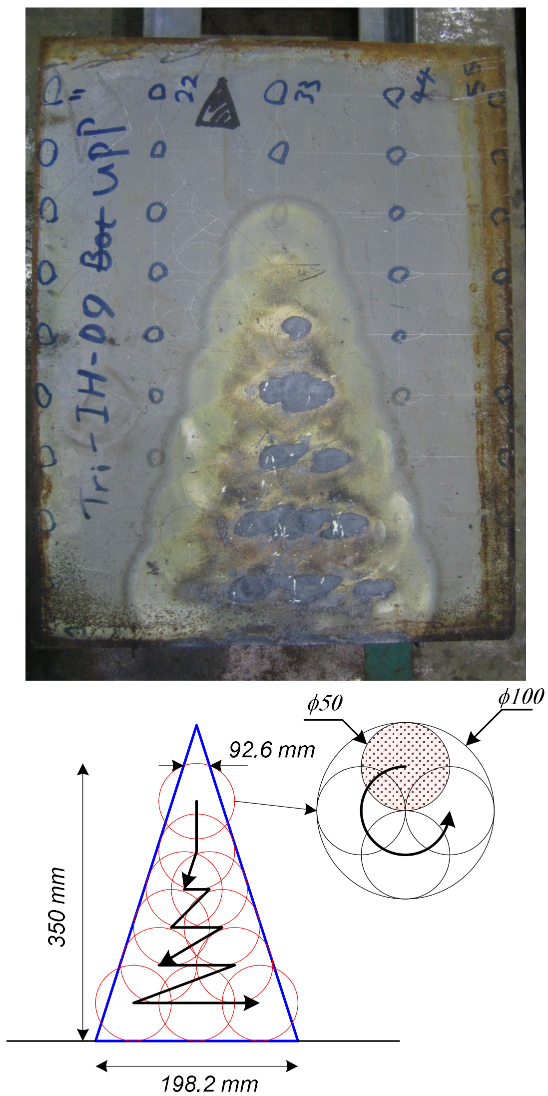

***Fig. 7.** Result of triangle induction heating: (a) heated specimen and (b) trajectory of inductor.*

To confirm the credibility of the analysis for the triangle heating process, the angular distortions of experiment and analysis along direction x are compared. **Table 2** presents the results of the deformations for angular distortions of analysis and experiment at various y positions along the direction x as well as errors between them. It can be observed from the results that values of errors can be allowable for the analytical model in previous section to be used to calculate deformations of plate in triangle heating process.

Table 2: Results of experiment and analysis

y	Angular distortion θ (rad)		% Error
	experiment	analysis	
10	0.0080	0.0090	12.525
25	0.0081	0.0097	19.753
50	0.0091	0.0104	14.286
100	0.0097	0.0107	10.309
150	0.0096	0.0103	7.292
200	0.0089	0.0095	6.742
250	0.0083	0.0085	3.012
300	0.0072	0.0075	4.167
350	0.0068	0.0064	5.1479
400	0.0043	0.0052	20.930
450	0.0039	0.0042	7.692

The angular distortions resulted from the analytical model and the experiment is graphically shown at the same time in **Fig. 8**, which shows a good match between the analytical and experimental results. From the comparison, it is also obvious that, although there are some errors of angular distortion, the developed analytic solution of Equations. (67-68) can help determine bending deformations in triangle heating process and out-of-plane deformation in triangle heating of steel plate can be efficiently predicted by the simplified solution (**Fig. 9**). Besides, two heated triangles were experienced with two different positions and experiment results were accuracy and validity of the proposal method (**Fig. 10**). Despite recent efforts in the development of predictive stress and deformation methodologies for induction heating using the analysis method, the success has been achieved for practical problems. The temperature differential in the heat affected area creates a non-uniform distribution of heat in the workpiece. As the temperature increases, the yield strength decreases, the coefficient of thermal expansion increases, the thermal conductivity decreases and the specific heat increases. In addition, induction heating causes changes in the physical phases and metallurgical structures in the weld. To anticipate the weld stresses and deformation from a straightforward analysis of heat is difficult.

5. CONCLUSIONS

To efficiently predict deformation of steel plate during triangle heating in induction heating process, an analytical model was developed using the laminated plated theory and the heat-flux and heat flow analyses, and the following conclusions are derived.

1) The heat flux and heat flow models are developed with the numerical method to predict the size of the heat affected zone in induction heating process.

2) The laminated plate theory can be quite well associated with the disk model of plastic region and the inclusions of the eigenstrain

3) An analytic solution to predict the out-of-plane distortion of a steel plate can be derived by the plate theory.

4) The sizes of the plastic region of a heated plate according to the heating parameters are formulated with simplified equations to be used in the deformation analysis.

5) The analytic solution was applied in prediction of deformation for steel forming with triangle heating method to verify the efficiency and the effectiveness of the developed model.

6) Induction heating experiment revealed that the analytic solution could predict quite well deformation of steel plate in triangle heating.

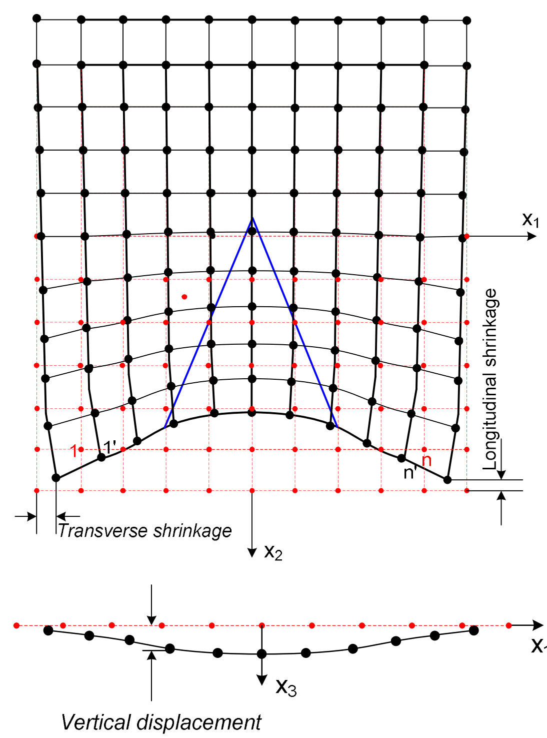

Fig 13. *Movements of node after induction triangle heating calculated by laminated plate theory.*

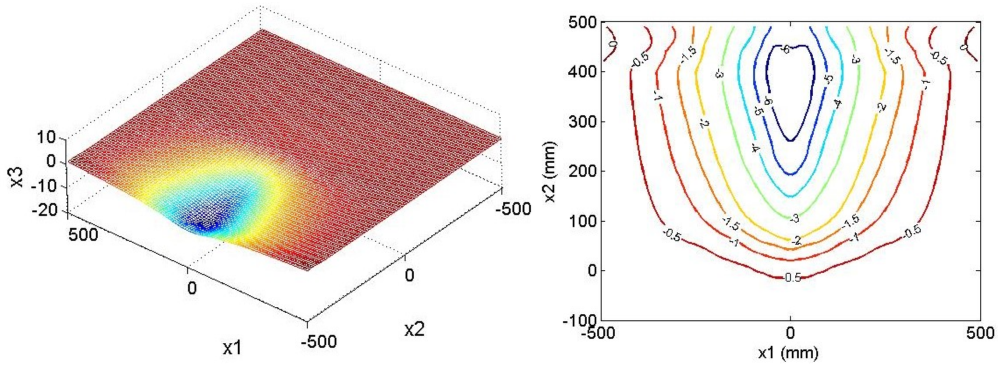

Fig. 9. *Triangle heating experiment results.*

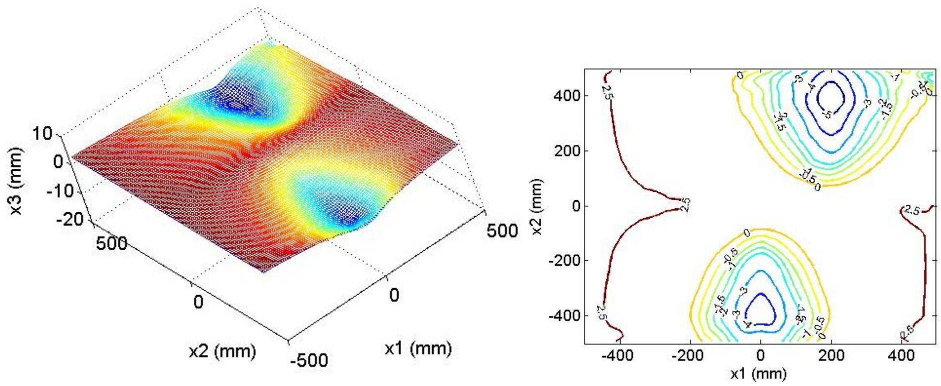

Fig. 10. *Two heated triangles experiments with two different positions.*

REFERENCES

[1] Valery Rudnev, Don Loveless, Raymond, Micah Black. Handbook of Induction Heating. Marcel Deckker 2003.

[2] Y. Favennec, V. Labbe, F. Bay. Induction heating processes optimization a general optimal control approach. Journal of Computational Physics 2003; 187: 68-94.

[3] Alexandre Masserey, Jacques Rappaz, Roland Rozsnyo, Rachid Touzani, “Power formulation for the optimal control of an industrial induction heating process thixoforming”, International Journal of Applied Electromagnetics and Mechanics 19, pp. 51-56, 2004.

[4] Chang Doo Jang, Tea Hoon Kim, Dea Eun Ko, Thomas Lamb, Yun sok Ha, “Prediction of heat plate deformation due to triangle heating using the inherent strain method”, Journal of Marine Science and Technology, 2005.

[5] G. Yu, R.J. Anderson, T. Maekawa, N.M. Patrikalakis, “Efficient simulation of shell forming by line heating”, International Journal of Mechanical Sciences 43, pp. 2349-2370, 2001.

[6] Y. Favennec, V. Labbe, F. Bay, “Induction heating processes optimization a general optimal control approach”, Journal of Computational Physics 187, pp 68-94, 2003.

Corresponding author:
Assoc. Prof. Dr Nguyen Truong Thinh
Faculty of Mechanical Engineering

Email: thinhnt@hcmute.edu.vn