DESIGN OF IMC-PID CONTROLLER FOR ENHANCED DISTURBANCE REJECTION OF SOPDT PROCESSES

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ABSTRACT

An analytical method for the proportional-integral-derivative (PID) controller cascaded with a high-order filter is proposed based on the reputable internal model control (IMC) design principles. The proposed method is mainly used to improve the disturbance rejection of the second-order stable processes with time delay. Analytical tuning rules for the proposed PID controller are derived to cope with both the disturbance rejection and set-point tracking problems by using a two degree of freedom (2DOF) control scheme in a simple manner. Several simulation examples are studied for various stable second order plus time delay (SOPDT) processes and the results are fairly compared with those of recently renowned methods. The robustness of the controller is also investigated by inserting perturbation uncertainty in all process parameters simultaneously. The results demonstrate the superior performance of the proposed IMC-PID controller in both nominal and robust cases.

***Keywords*:** *Proportional–Integral–Derivative (PID) controller, High-order filter, Two degree of freedom (2DOF) control scheme, Second order plus dead time (SOPDT).*

TÓM TẮT

Phương pháp phân tích để thiết kế bộ điều khiển PID kết hợp với bộ lọc bậc cao được đề xuất dựa vào các nguyên tắc thiết kế của phương pháp nổi tiếng IMC. Phương pháp đề xuất được sử dụng chủ yếu để cải thiện đặc tính khử nhiễu của hệ ổn định bậc hai có thời gian trễ. Các quy luật điều chỉnh của phương pháp đề xuất được thành lập với mục đích áp dụng cho cả hai hướng thiết kế chính là theo đặc tính khử nhiễu và theo đặc tính giá trị đặt bằng việc sử dụng chiến lược điều khiển hai bậc tự do (2DOF) theo hướng đơn giản. Nhiều nghiên cứu mô phỏng đã được thực hiện cho nhiều quá trình bậc hai có thời gian trễ, trong đó kết quả nghiên cứu được so sánh một cách khách quan với các phương pháp nổi tiếng khác. Tính ổn định bền vững của bộ điều khiển cũng được nghiên cứu bằng việc đồng thời đưa các nhiễu loạn không xác định vào tất cả các biến quá trình. Kết quả mô phỏng chứng minh khả năng thực thi vượt trội của bộ điều khiển IMC-PID được đề xuất trong cả trường hợp hệ thống hoạt động bình thường và bất thường.

Từ khóa: *Bộ điều khiển PID, Bộ lọc bậc cao, Chiến lược điều khiển hai bậc tự do (2DOF), Hệ bậc hai có thời gian trễ (SOPDT).*

1. INTRODUCTION

The PID controller is one of the most common and successful control tools in the process industries because of its simplicity, robustness, powerfulness, as well as optimality. For the industrial problems, the PID controller affords sufficiently well the disturbance rejection and set-point tracking for various control schemes. Therefore, the design area of the PID controller has currently attracted much attention of numerous academic and control engineers.

A number of PID tuning rules have been introduced frequently over the recent decades and one of the best known is possibly the internal model control (IMC) structure contains the internal model of controlled plant, which was spread mainly by Garcia and Morari [1]. The advantages of IMC-PID tuning rules are that the tradeoffs between the closed-loop performance and robustness can be directly obtained by using only one user-defined tuning parameter, which is introduced by Rivera et al. [2], and it is so-call as the closed-loop time constant. However, the main inconvenience of IMC-PID tuning rules has been found out as the possibility of obtaining the sluggish disturbance rejection response, which becomes severely for the process with a small time-delay/time constant ratio (Morari and Zafiriou [3]), despite that the disturbance rejection is much more important than the set-point tracking for many process control applications. To overcome these problems, the two degree of freedom (2DOF) control strategies seem to be one of the best solutions.

For enhanced disturbance rejection, Chen and Seborg [4] demonstrated that the direct synthesis (DS) can adequately achieve the disturbance rejection, wherein the PID controller parameter can be obtained by computing the ideal feedback controller that gives the desired closed-loop response. It is well known that the 2DOF control scheme can be used to improve the disturbance performance for varied time-delay processes. Lee et al.'s method [5] is extended the IMC design approach for the two degree of freedom controller, where two controllers are systematically combined in the way that a first IMC controller for the set-point changes and a second IMC type of controller designed to shape the disturbance response, and the set-point filter specified as the inverse of the IMC controller for disturbance rejection also included to improve the set-point response. Horn et al. [6] found out that the modified IMC filter for deriving the low-order controller (i.e., the PID controller in series with a second-order filter) can provide good disturbance rejection. Obviously, the significant improvement of set-point responses can be obtained by using either modified IMC or SP methods.

Consequently, several PID controller tuning rules have been frequently proposed in the literature. However, due to the difficulty of the time-delay process, the design of a simple and effective controller with the perfect improvement of performance has not been fully achieved for a variety of time-delay processes. The proposed method is aimed to design of the PID controller cascaded with a higher order compensator to fulfill the various control purposes, such as simple tuning rules, analytical form, model-based, and easy to implement in the practice with the excellent performance for both the regulatory and servo problems, by utilizing the clever reduction technique to convert the ideal feedback controller to the low order PID controller in a simple manner.

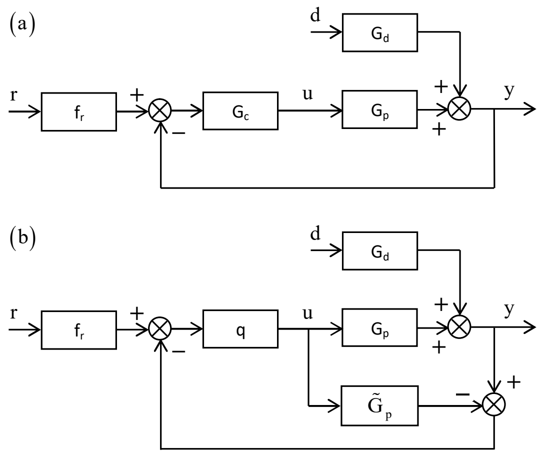

Figure 1. *Block diagram of feedback control strategies. (a) Classical feedback control. (b) Internal model control*

Simulation studies have been fairly considered to demonstrate the simplicity and effectiveness of the proposed method in compared with numerous prominent design methods by using the same robustness level (i.e., maximum sensitivity *Ms* value) for all comparative controllers. The simulation results confirm that the proposed method can afford the superiority of performance for both the disturbance rejection and set-point tracking.

2. PROPOSED METHOD

2.1. Design of generalized controller

Consider an open-loop single-input, single-output (SISO) process with general transfer function for the second order stable/unstable and delay, which is represented as follows:

$$G_P(s) = \frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \tag{1}$$

where K denotes the process gain. τ_1 and τ_2 correspond to real poles in process model, whereas θ represents the delay.

For the feedback control system, the equivalent feedback controller $G_c(s)$ is implemented for the process $G_P(s)$. From the standard block diagram of the feedback control strategies as shown in Fig.1, where $q(s)$, and $f_r(s)$ denote the IMC controller and the set-point filter, respectively. $y(s), r(s), d(s)$, and $u(s)$ correspond to the controlled output, set-point input, disturbance input, and manipulated variables. If there is no model error (i.e., $G_P(s) = \tilde{G}_P(s)$), then the set-point and disturbance responses in the IMC control structure can be simplified as:

$$\begin{aligned} y(s) &= G_P(s)q(s)f_r(s)r(s) \\ &+\left[1-\tilde{G}_P(s)q(s)\right]G_d(s)d(s) \end{aligned} \tag{1}$$

According to the IMC parameterization [8], the process model $\tilde{G}_P(s)$ is factored into two parts:

$$\tilde{G}_P(s) = p_m(s)p_A(s) \tag{2}$$

where $p_m(s)$ and $p_A(s)$ denote the portion of the model inverted by the controller (minimum phase) and the portion of the model not inverted by the controller (it is the non-minimum phase that may be included the dead time and/or right half plane zeros and chosen to be all-pass), respectively. The requirement that $p_A(0) = 1$ is necessary for the controlled variable to track its set-point.

The IMC controller can be designed as:

$$q(s) = p_m^{-1}(s)f(s) \tag{3}$$

where the IMC filter $f(s)$ is chosen to fulfilled the condition by following form:

$$f(s) = \frac{\sum_{i=1}^{v} \beta_i s^i + 1}{(\lambda s+1)^n} \tag{4}$$

where λ is an adjustable parameter for the tradeoffs between the performance and robustness. The integer n is selected to be large enough for the IMC controller proper. The parameter β_i is determined to cancel the poles near zero in $G_d(s)$.

$$1-G_P(s)q(s)\Big|_{s=z_{d1},z_{d2},\dots,z_{dm}} = \left|1-\frac{p_A(s)\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{(\lambda s+1)^n}\right|_{s=z_{d1},z_{d2},\dots,z_{dm}} = 0 \qquad (5)$$

Substituting Eq. 4 into Eq. 3, the IMC controller is obtained by:

$$q(s)=p_m^{-1}(s)\frac{\left(\sum_{i=1}^{v}\beta_i s^i+1\right)}{(\lambda s+1)^n} \qquad (6)$$

The ideal feedback controller $G_c(s)$ that yields the desired loop responses can be constituted by:

$$G_c(s)=\frac{q(s)}{1-\tilde{G}_P(s)q(s)} \qquad (7)$$

Thus, the ideal feedback controller for achieving the desired loop response can be easily found by:

$$G_c(s)=\frac{p_m^{-1}(s)\left(\sum_{i=1}^{v}\beta_i s^i+1\right)}{(\lambda s+1)^n-p_A(s)\left(\sum_{i=1}^{v}\beta_i s^i+1\right)} \qquad (8)$$

The resulting ideal controller given by Eq. 8 does not have a standard PID-type controller form, even it is physically realizable. Therefore, this ideal feedback controller should be converted into the suitable PID-type controller form more closely by using the clever approximation technique. In this paper, we utilize the low-order Padé approximation in the different manner with previous design methods in terms of the most closely controller approximates the ideal feedback controller.

2.2. IMC-PID tuning rules for SOPDT model

The stable SOPDT model is usually considered to design the PID controller in the process industries. The process transfer function is given by Eq. 1.

For the 2DOF control structure, the IMC filter is reasonably selected as:

$$f(s)=\frac{\left(\beta_2 s^2+\beta_1 s+1\right)}{(\lambda s+1)^4} \qquad (9)$$

Hence, the IMC controller is obtained by

$$q(s)=(\tau_1 s+1)(\tau_2 s+1)\left(\beta_2 s^2+\beta_1 s+1\right)\big/K(\lambda s+1)^4 \qquad (10)$$

Accordingly, the ideal feedback controller is found as:

$$G_c(s)=\frac{(\tau_1 s+1)(\tau_2 s+1)\left(\beta_2 s^2+\beta_1 s+1\right)}{K\left[(\lambda s+1)^4-e^{-\theta s}\left(\beta_2 s^2+\beta_1 s+1\right)\right]} \qquad (11)$$

It should be noted that the values of β_1 and β_2 are calculated to cancel out the poles at τ_1 and τ_2 by solving the equation:

$$1-\left(\beta_2 s^2+\beta_1 s+1\right)e^{-\theta s}/(\lambda s+1)^4\Big|_{s=-1/\tau_1,-1/\tau_2}=0 \qquad (12)$$

Hence,

$$\beta_1=\frac{\tau_1^2\left[\left(1-\frac{\lambda}{\tau_1}\right)^4 e^{-\theta/\tau_1}-1\right]-\tau_2^2\left[\left(1-\frac{\lambda}{\tau_2}\right)^4 e^{-\theta/\tau_2}-1\right]}{(\tau_1-\tau_2)} \qquad (13)$$

$$\beta_2=\tau_2^2\left[\left(1-\frac{\lambda}{\tau_2}\right)^4 e^{-\theta/\tau_2}-1\right]+\beta_1\tau_2 \qquad (14)$$

The delay term $e^{-\theta s}$ in the denominator can be reasonably approximated by 2/1 Padé expansion as following:

$$e^{-\theta s} = \frac{\left(1 - \frac{2\theta s}{3} + \frac{\theta^2 s^2}{6}\right)}{\left(1 + \frac{\theta s}{3}\right)} \quad (15)$$

Substituting Eq. 15 into Eq. 11, rearranging it, and comparing with the PID controller in cascaded with the high order compensator given by Eq. 12:

$$\begin{aligned} G_c(s) &= K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1 + cs + ds^2}{1 + as + bs^2} \\ &= \frac{1}{s}\left(\frac{K_c}{\tau_I}\right)\left(1 + \tau_I s + \tau_I \tau_D s^2\right)\left(\frac{1 + cs + ds^2}{1 + as + bs^2}\right) \end{aligned} \quad (16)$$

Finally, the analytical tuning rules of the proportional, integral, and derivative terms of the proposed PID controller can be compactly obtained as:

$$K_c = \frac{\beta_1}{K\left(4\lambda + \theta - \beta_1\right)} \quad (17)$$

$$\tau_I = \beta_1 \quad (18)$$

$$\tau_D = \frac{\beta_2}{\beta_1} \quad (19)$$

$$a = \frac{\left(6\lambda^2 - \frac{\theta^2}{10} - \beta_2 + \frac{8\lambda\theta}{5} + \frac{3\theta\beta_1}{5}\right)}{\left(4\lambda + \theta - \beta_1\right)} - \left(\tau_1 + \tau_2\right) \quad (20)$$

$$b = \frac{\left(4\lambda^3 + \frac{12\lambda^2\theta}{5} + \frac{\lambda\theta^2}{5} + \frac{\theta^3}{60} - \frac{3\theta^2\beta_1}{20} + \frac{3\theta\beta_2}{5}\right)}{\left(4\lambda + \theta - \beta_1\right)} - \tau_1\tau_2 - a\left(\tau_1 + \tau_2\right) \quad (21)$$

$$c = \frac{2\theta}{5} \quad (22)$$

$$d = \frac{\theta^2}{20} \quad (23)$$

The lead term $\left(\beta s + 1\right)$ can cause excessive overshoot in the set-point response, which can be eradicated by adding the set-point filter as:

$$f_r(s) = \frac{\gamma\beta s + 1}{\beta s + 1} \quad (24)$$

where $0 \le \gamma \le 1$.

Some important remarks can be described as:

- $\gamma = 0$. For this extreme case, there is no lead term in the set-point filter, which can cause a slow servo response.
- $\gamma = 1$. For this case, there is no set-point filter.
- $0 < \gamma < 1$. That means we adjust γ online to obtain the desired speed of the set-point response.

3. PERFORMANCE AND ROBUSTNESS MEASURE

3.1. Integral absolute error (IAE) criteria

To evaluate the closed-loop performance, the IAE criterion is considered here for both the disturbance rejection and the set-point tracking. The IAE is defined as

$$IAE = \int_0^\infty \left|e(t)\right| dt \quad (25)$$

The IAE value should be as small as possible.

3.2. Overshoot

The overshoot is a good measure for the evaluating the response that exceeds the ultimate value following a step change in the disturbance or the set-point.

3.3. Total Variation (TV)

To evaluate the required control effort, the TV is a good measure of the smoothness of a signal, which is computed the total variation of the manipulated variable by considering the sum of all its moves up and down.

$$\mathrm{TV} = \sum_{i=1}^{\infty} \left| \mathrm{u}_{i+1} - \mathrm{u}_i \right| \quad (26)$$

The TV should be as small as possible.

3.4. Maximum Sensitivity Criterion

To evaluate the robustness of the control system, the peak value of the sensitivity function *Ms*, which has many useful physical interpretations, has been widely used by many researchers. The *Ms* is defined as the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point (-1, 0).

$$Ms = \max_{0\le\omega\le\infty} \left| \frac{1}{\left(1 + \mathrm{G_p}(\mathrm{j}\omega)\mathrm{G_c}(\mathrm{j}\omega)\right)} \right| \quad (27)$$

In order to insure a fair comparison, the model-based controllers should be tuned by adjusting λ so that the *Ms* values are the identical. That means all comparative controllers are designed to have the same robustness level in terms of the maximum sensitivity.

4. SIMULATION STUDY

In this section, an illustrative example is studied to confirm the simplicity and effectiveness of proposed PID tuning rules. Consider the following SOPDT process (Chen and Seborg [4], Shams and Lee [8]):

$$\mathrm{G_p}(s) = \mathrm{G_d}(s) = \frac{2e^{-1s}}{(10s+1)(5s+1)} \quad (28)$$

For this SOPDT process, Shams and Lee [8] previously confirmed the superiority of their method over the method of Chen and Seborg [4]. In this simulation study, the proposed controller was compared with these controllers. In order to guarantee a fair comparison, all controllers were tuned to have the same robustness level by measuring the *Ms* value. For this example, the closed-loop time constant λ_i were adjusted to obtain *Ms* = 1.95 for all of comparative design methods.

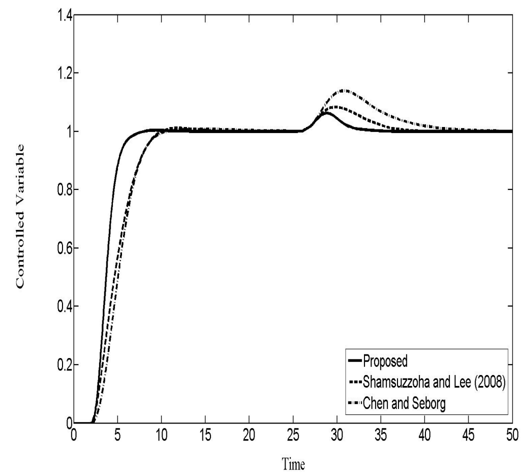

***Figure 2**. Output responses of PID control systems for the SOPDT process model*

The resulting controller parameters together with the performance and robustness indices calculated by the above-mentioned methods are tabulated in Table 1. Unit step changes were introduced in the set-point at t = 0 and in the disturbance at t =25 as shown in Fig. 2, which compares the set-point and load responses afforded by all of above-mentioned methods. It is clear that the set-point filter used for the set-point response has a clear benefit. Therefore, the 2DOF controller using the set-point filter was utilized in all methods except the Rivera et al.'s method [2] to obtain an enhanced set-point response. It should be noted that the method of Rivera et al. [2] has been suggested for the 1DOF controller with the conventional lag filter and we, therefore, used their method without any modification. In the methods of proposed and Shamsuzzoha and Lee [8], in the parameter of set-point filter was selected as γ = 0.45.

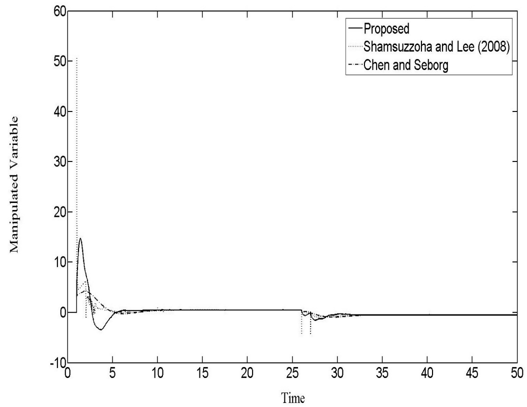

***Figure 3**. Manipulated variables of PID control systems for the SOPDT process model*

It is indicated from Fig. 2 that the proposed controller shows a fast and well-balance response in compared with the other methods. The values of the performance indices in Table 1 also confirm the advanced performance of the proposed method for the disturbance rejection as well as the set-point tracking.

Table 1. PID controller parameters and performance indices

	Proposed	**Shams & Lee**	**Chen & Seborg**
K_C	17.57	10.60	6.839
τ_I	3.869	5.215	7.300
τ_D	1.314	1.636	2.036
λ	0.745	1.115	2.278
a	0.064	0.032	-
b	0.052	0.0	-
c	0.40	0.50	-
d	0.05	0.0	-
γ	0.30	0.30	0.50
Ms	1.950	1.950	1.950
IAE_{sp}	2.842	3.980	4.314
OVT_{sp}	0.004	0.011	0.008
TV_{sp}	37.74	158.7	10.49
IAE_d	0.221	0.495	1.065
OVT_d	0.064	0.083	0.139
TV_d	3.174	20.35	1.873
Noted: *sp* denotes set-point tracking; *d* denotes disturbance rejection			

To demonstrate the robust performance of the proposed method, the simulation study is also done by inserting a perturbation uncertainty of + 10% in the process gain, time constant, and time delay in the worst direction simultaneously, whereas the controller settings are those provided for the nominal process. The simulation results for the plant-model mismatch are listed in Table 2. It is obvious that the proposed method consistently affords the advanced robust performance both for the disturbance and set-point changes.

Table 2. Robustness analysis for the SOPDT process model (+ 10%)

	Proposed	**Shams & Lee**	**Chen & Seborg**
IAE_{sp}	3.146	4.040	4.526
OVT_{sp}	0.089	0.025	0.050
TV_{sp}	45.922	171.715	11.977
IAE_d	0.235	0.499	1.068
OVT_d	0.069	0.089	0.146
TV_d	4.051	20.915	2.160
Noted: *sp* denotes set-point tracking; *d* denotes disturbance rejection			

Table 3. Robustness analysis for the SOPDT process model (- 10%)

	Proposed	**Shams & Lee (2008)**	**Chen & Seborg**
IAE_{sp}	2.907	4.001	4.344
OVT_{sp}	0.012	0.009	0.008
TV_{sp}	49.779	168.381	10.023
IAE_d	0.223	0.497	1.070
OVT_d	0.058	0.078	0.132
TV_d	4.023	20.955	1.777
Noted: *sp* denotes set-point tracking; *d* denotes disturbance rejection			

Various kinds of SOPDT processes are also studied and the simulation results shows that the proposed IMC-PID controllers can enhanced the stability and performance of control systems for both nominal and robust cases.

4. CONCLUSIONS

The analytical design methods for the PID controller cascaded with the lead/lag filter were proposed for a variety of the FOPDT, the DIP, and the unstable FOPDT process models. On the basis of the renowned IMC theory, the proposed controller can provide an excellent improvement of performance for the disturbance rejection. In order to enhance the set-point response, the proposed method also utilized a set-point filter like the 2DOF controller introduced by a number of above-mentioned authors. The simulation results indicate that the proposed method consistently affords more advanced performance with a fast and well-balanced closed-loop time response for both the disturbance rejection and set-point tracking in compared with the other methods, since the various controllers are all tuned to have the same degree of robustness in terms of the peak value of the sensitivity function. The robustness study was also conducted by inserting a perturbation uncertainty of ±10% in all three process parameters simultaneously. The results showed that the proposed control systems held robust stability well in both the nominal and the plant-model mismatch cases.

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