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NONLINEAR ADAPTIVE CONTROLLER OF MECHANICAL TRANSMISSION SYSTEMS WITH BACKLASH

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ABSTRACT

Backlash appears in most of mechanical transmission systems and is one of nonlinearity characteristics leading to the error in position and velocity control. To control a mechanical system with backlash, the system parameters such as: damping, spring stiffness, mass, backlash gap have to be identified exactly. However, it is difficult to measure these parameters in real system because of low measurement accuracy or cost. This paper proposes a new method to compensate backlash by using Model Reference Adaptive Control (MRAC) and a nonlinear compensator without pre-determined backlash parameters. The adaptive controller assures the controllability of the different systems with uncertain parameters. While, the nonlinear compensator is used to overcome the effect of backlash. Based on the results of Gang Tao, the mathematical function of backlash and backlash inverse are presented. After that, the mass-spring-damping model is applied to approximate mathematical transmission systems. Nonlinear adaptive controller is designed and simulated. The simulation results show that errors because of backlash are reduced and the controller tracks the desired input properly.

Keywords: *backlash; adaptive control; mechanical transmission systems; two-mass-spring system; nonlinear compensation.*

TÓM TẮT

Độ rơ xuất hiện trong hầu hết các hệ truyền động cơ khí và là một trong những đặc tính phi tuyến dẫn tới sai số điều khiển vị trí và điều khiển vận tốc. Để điều khiển một hệ cơ khí có rơ, các thông số hệ thống như: giảm chấn, độ cứng, khối lượng, khoảng rơ phải xác định được. Tuy nhiên, các thông số này là khó khăn để xác định trong hệ thống thực vì độ chính xác đo đạc thấp hoặc giá thành cao. Bài báo này đề xuất một phương pháp mới để bù độ rơ bằng việc sử dụng bộ Điều khiển thích nghi theo hàm mẫu và một khâu bù phi tuyến với các độ rơ khác nhau. Bộ điều khiển thích nghi đảm bảo tính điều khiển được với các hệ thống khác nhau có các thông số không chắc chắn. Trong khi đó bộ bù phi tuyến được sử dụng để khắc phục ảnh hưởng của rơ. Đầu tiên, mô hình toán của độ rơ và rơ ngược được trình bày dựa trên các kết quả nghiên cứu của Gang Tao. Sau đó, mô hình khối lượng-lò xo-giảm chấn được áp dụng để xấp xỉ hàm truyền của hệ truyền động. Tiếp theo, bộ điều khiển thích nghi phi tuyến được thiết kế, áp dụng và mô phỏng. Kết quả mô phỏng thể hiện các sai số do độ rơ được giảm xuống trong khi bộ điều khiển vẫn bám được tính hiệu đầu vào mong muốn.

Từ khóa: *độ rơ; điều khiển thích nghi; hệ truyền động cơ khí; mô hình khối lượng lò xo giảm chấn; bù phi tuyến.*

1. INTRODUCTION TO BACKLASH CONTROL

Backlash is the gap between the parts caused imperfection in mechanical systems such as gears or some other mechanical parts. It exists in almost all transmission systems and causes a delay in the system motion. That is the reason why backlash causes error in both position and error control especially the repeatability and accuracy of the transmission systems.

A general method to compensate backlash in common industrial systems is using mechanical structures to reduce the backlash gap when transmission systems reverse. Including xy – positioning tables (Li and Cheng (1996)), robot manipulators (Lewis, Abdallah and Dawson (1993)) are examples [1]. Although mechanical methods show the effectiveness in compensating backlash gap, it remains a high cost for final products because engineers have to overcome the complicated design such as: spring-loaded split gear assemblies and dual motor.

Standard control techniques such as: proportional-derivative (PD) or proportional-integral-derivative (PID) not high effective for backlash compensation. Since, when backlash is present, they can produce limit cycles. Furthermore, closed-loop stability is difficult to prove when backlash is present. Backlash compensation has been studied by many researchers. Rigorous results for motion tracking of such systems are notably sparse. These techniques, which are relying on simulations for verification of effectiveness, are prolific. Besides, backlash inverse approximating method, non-adaptive and adaptive controllers are also used to control the systems. These research results were presented in the survey about controlling with backlash of Mattias Nordian and Per-Olof Gutman [1]. However, most of the research results focus on using backlash inverse method.

In this paper, the authors utilize nonlinear adaptive control for compensating backlash gap in various mechanical systems. The paper has six sections: Section 1 is introduction about backlash impaction and backlash inverse characteristics. Section 2 proposes the model of mechanical transmission system at the view of mass-spring-damper. Section 3 presents nonlinear adaptive control for mechanical transmission system with backlash. In Section 4, the Model Reference Adaptive Control is firstly presented and then is the nonlinear part for compensating backlash gap. Section 5 shows the simulation results of the proposed method. Section 6 gives the conclusion for the new backlash compensation method.

2. BACKLASH AND BACKLASH INVERSE

According to the research of Tao et al. [2], the mathematical of backlash nonlinearity is given by Eq. (1)

$$\dot{\tau} = B(\tau, u, \dot{u})$$

$$= \begin{cases} m\dot{u} if \dot{u} > 0 \; and \tau = mu - md_+ \,, \\ if < 0 \; and \tau = mu - md_+ \,, \\ 0 \qquad otherwise \end{cases} \qquad (1)$$

As it can be seen from the mathematical model, backlash is a first order velocity-driven dynamical system with inputs u and $\dot{u}$ and state of system τ, while m is the slope of control input, and $d+$, $d-$ are the right and left limits respectively. Since the backlash dynamics is given in terms of the derivative of output $\tau(t)$, it contains its own dynamics; therefore, its compensation requires the designs of a dynamic compensator.

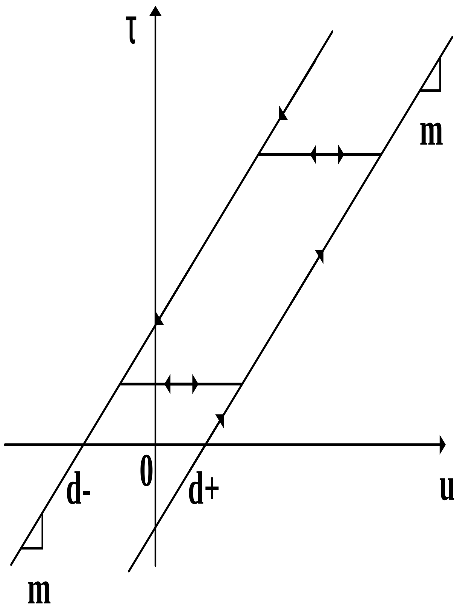

Fig.1 *Backlash*

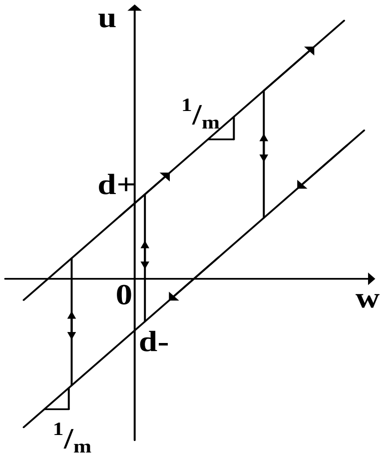

Fig.2 *Backlash inverse*

Whenever the motion $u(t)$ changes its direction, the motion $\tau(t)$ is delayed from the motion $u(t)$. The objective of a backlash compensator is to make this delay as small as possible (i.e., to make the $\tau(t)$ closely follow $u(t)$). In order to cancel the effect of backlash in the system, the backlash pre-compensator generally needs to generate the inverse of the backlash nonlinearity.

Backlash inverse method (Fig.2) shows effectiveness to reduce the open gap every reversing time [3]. The backlash parameters need to be determined before applying to an actual system. This work can be done by using backlash identifying method. However, there is always an error existing when identifying the backlash parameters because of random and measured noises. On the other hand, the mechanical system has to run without any tasks to get data for identifying backlash parameters. These tasks require complicated procedures and high cost to control exactly a system with backlash.

3. MECHANICAL TRANSMISSION MODEL

Mechanical transmission can be recognized in a linear model of a mass-spring-damper system. Such system contains a mass $m(kg)$, a spring with spring constant $k\ (N/m)$ that serves to restore the mass to a neural position and a damping element which opposes the motion of the movement with a force proportional to the velocity of the system, the constant of proportionality being the damping constant $c(Ns/m)$. An ideal mass-spring-damper system can be described with the following formulas [4]:

$$F_s = -kx \tag{2}$$

$$F_d = -cv = -c\dot{x} = -c\frac{dx}{dt} \tag{3}$$

This system of equation is derived by the Newton’s law of motion which is

$$F_d = -cv = -c\dot{x} = -c\frac{dx}{dt} \tag{4}$$

$$\sum F = ma = m\ddot{x} = m\frac{d^2x}{dt^2} \tag{5}$$

Where a is the acceleration $[m/s^2]$ of the mass and $x[m]$ is the displacement of the mass relative to a fixed point of reference. The above equations combine to form of the equation motion, a second-order differential equation for displacement Y as a function of time $t[sec]$:

$$m\ddot{y} + c\dot{y} + ky = F \tag{6}$$

Next, to simplify the equation, we define the following parameters $B = c/m$, $K = k/m$ and $f = F/m$, and we obtain the second order system

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{K}{m}y = \frac{F}{m} \tag{7}$$

The mass-spring-damper transfer function is written as

$$G(s) = \frac{Y(s)}{F(s)} = \frac{\frac{1}{m}}{s^2 + Bs + K} \tag{8}$$

To control the mechanical system above, all parameters of the transfer function need to be determined. The author's previous work [5] is used as identifying method to estimate the system parameter. This method has a large computation burden and still has estimation error. For that reason, a nonlinear adaptive method is proposed in the paper.

4. NONLINEAR ADAPTIVE CONTROLLER

4.1 Adaptive control

This paper uses Model Reference Adaptive Control (MRAC) which has been developed by the Instrumentation laboratory at Massachusetts Institute of Technology (MIT). MARC consists of a reference model which produces the desired output and the difference between the plant output and the reference output is then used to adjust the control parameters and the control input directly. Compared with the conventional control design, adaptive control is more involved. MRAC design usually involves the following steps: (i) choose a control law containing variable parameters and (ii) design an adaptation law for adjusting those parameters.

Adaptive control is chosen by the following MIT rule [6], the evaluation function is

$$J(\theta) = \frac{e^2}{2} \tag{9}$$

Whereas, e is the error between the actual output and model reference output, θ is parameter adjustment. The parameter θ must be adjusted to minimize the evaluation function to 0. So, parameter θ can have different sign with gradient of J. Equation of MIT rule is

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J}{\partial \theta} = -\gamma e\frac{\partial e}{\partial \theta} \tag{10}$$

$\gamma -$ adaptive coefficient;

$\frac{\partial e}{\partial \theta} -$ sensitive function of system.

Transfer function of mechanical transmission systems has a model as shown in Eq. (8). However, in an actual system, it is hard to identify damper and stiffness coefficients. Therefore, adaptive controller is applied to adapt with various systems. The gain–damper–spring process is a second order element, and the model reference function is

$$G_m = \frac{Y_m(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n} \tag{11}$$

Control law is

$$u(t) = k_1(t)r(t) - k_2(t)y(t) - k_3\dot{y}(t) \tag{12}$$

Doing Laplace transformation for (12) we can obtain

$$U(s) = k_1R(s) - k_2Y(s) - k_3sY(s) \qquad (13)$$

The transfer function of mechanical transmission can be rewritten as:

$$G(s) = \frac{\frac{1}{m}}{s^2 + Bs + K} = \frac{\alpha_1}{s^2 + \alpha_2 s + \alpha_3} \qquad (14)$$

$$Y(s) = G(s)U(s) = \frac{\alpha_1\big(k_1R(s) - k_2Y(s) - k_3sY(s)\big)}{s^2 + \alpha_2 s + \alpha_3}$$

$$Y(s) = \frac{\alpha_1 k_1 R(s)}{s^2 + (\alpha_2 + \alpha_1 k_3)s + (\alpha_1 k_2 + \alpha_3)}$$

$$y(t) = \frac{\alpha_1 k_1 r(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}$$

As the consequence, the error is

$$e = \left(\frac{\alpha_1 k_1 r(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)} - \frac{\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n}\right) r(t)$$

The sensitivity derivatives are obtained by taking the partial derivatives of the error and considering the controller parameters

$$\frac{\partial e(t)}{\partial k_1} = \frac{\alpha_1 r(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}$$

$$\frac{\partial e(t)}{\partial k_2} = \frac{-\alpha_1^2 k_1 r(t)}{\big(p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)\big)^2} = \frac{\alpha_1 y(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}$$

$$\frac{\partial e(t)}{\partial k_3} = \frac{-\alpha_1^2 p k_1 r(t)}{\big(p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)\big)^2} = \frac{\alpha_1 \dot{y}(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}$$

Due to the fact that the process parameters are unknown, none of the three equations above can be used. Furthermore, the transfer function of the system is close to the transfer function of the reference model in adaptive control, it means that Eq. (10) will approximate to Eq. (8), so:

$$p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3) = p^2 + 2\xi\omega_n p + \omega_n$$

The adjustment for the controller parameters is:

With $\gamma = \alpha_1$is adaptive gain parameters.

$$\frac{dk_1(t)}{dt} = -\gamma\left(\frac{\alpha_1 r(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}\right)e(t)$$

$$\frac{dk_2(t)}{dt} = \gamma\left(\frac{\alpha_1 y(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}\right)e(t)$$

$$\frac{dk_3(t)}{dt} = \gamma\left(\frac{\dot{y}(t)}{p^2 + (\alpha_2 + \alpha_1 k_3)p + (\alpha_1 k_2 + \alpha_3)}\right)e(t)$$

4.2 Nonlinear backlash compensator

The backlash pre-compensator generates the inverse signal of the backlash nonlinearity. The backlash inverse function is shown in Fig.2. Thus if the signals w enters the backlash inverse and generates signal $u(t)$, which is subsequently sent into the backlash to produce $\tau(t)$, the throughput in the ideal case from $w(t)$ and $t(t)$ will be unity. It means that when a mechanical system reverses its direction, the controller needs to create a larger control signals to compensate the open gap.

The paper used a nonlinear compensator to alternate backlash inverse. The compensator used error between of the actual output signal and the model output signal to compensate backlash gap. When a mechanical system inverses its direction, the error will be large. The nonlinear compensator amplifies the error and multiplies to k_1 in control signal of adaptive output. The exponential of the error is used, the characteristic of the exponential is small and large gain respectively with the small and large error values. The nonlinear compensator is

$$u_{nc} = \epsilon^{k|e(t)|} \tag{15}$$

With ϵ is natural logarithm coefficient, k is constant parameter.

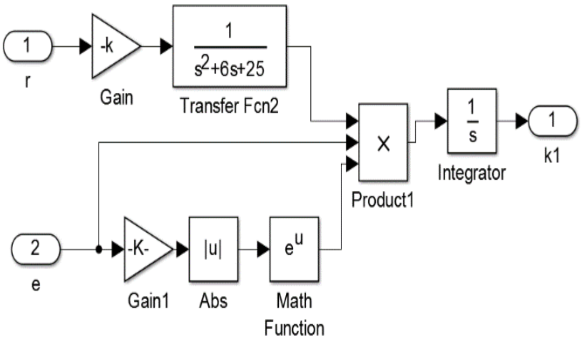

Fig.3 *Nonlinear backlash compensator*

The backlash non-linear compensator combines with the adaptive control based on MRAC to fully create controller for mechanical transmission system with backlash. Fig. 4 shows the blocks of whole proposed controller on MATLAB/Simulink.

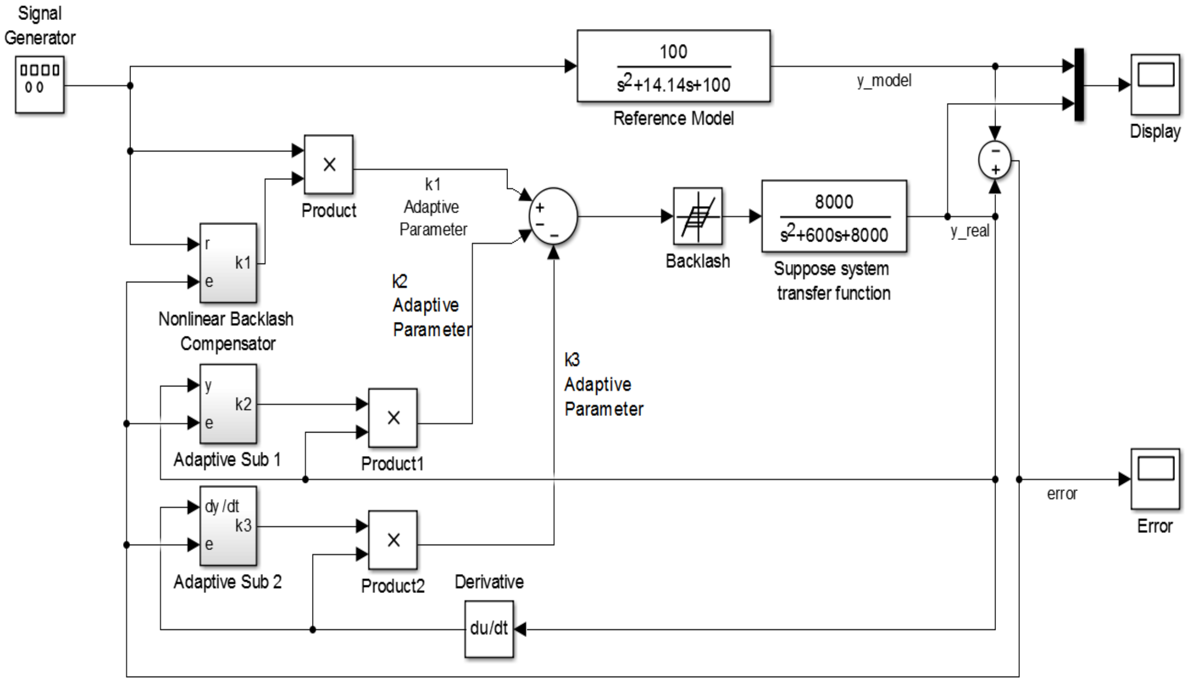

Fig.4 *Nonlinear adaptive controller*

5. SIMULATION

The closed loop control system has been simulated in MATLAB/Simulink for the nominal values of the uncertain parameters as Fig. 5. Using adaption gain equal to $\gamma = 3.5$ and the desired input is a sinusoidal signal. The constant of backlash nonlinear compensator is $k = 100$. The model reference is chosen

$$G_m = \frac{100}{s^2 + 14.14s + 100}$$

And the choose the transfer function of mechanical transmission is

$$G_m = \frac{800}{s^2 + 600s + 800}$$

The transmission system parameters can be changed widely.

Without loss generality, we assume the sinusoidal signal has $5mm$ - amplitude and 0.0375Hz - frequency. The backlash gap is from $0.1mm$ to $0.2mm$ for both right and left limitation.

Fig. 6 shows the tracking error using MRAC without the backlash nonlinear compensator. The maximum error is $0.1mm$. The error is reduced when using both MRAC and the backlash nonlinear compensator.

The error between the reference model and the actual signals increases when inversing the direction of the system. However, it will rapidly converge to zero and be stable. The maximum inversing error is $0.03mm$, less than the backlash gap $0.1mm$. In steady state, the error keeps less than $0.01mm$.

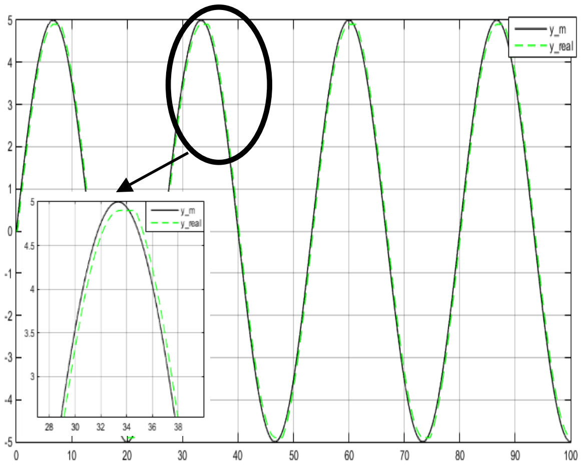

Fig.5 *MRAC without backlash nonlinear compensator*

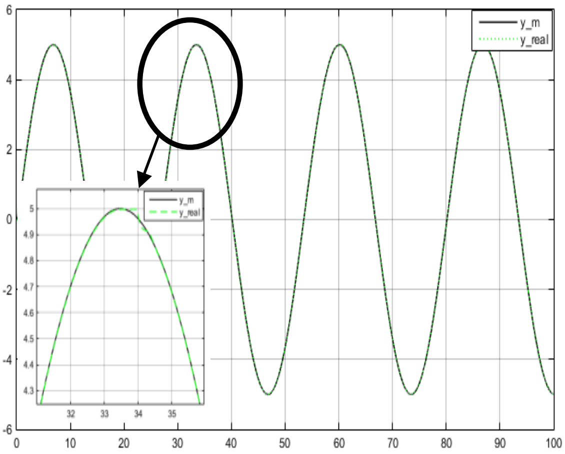

Fig.6 *MRAC with backlash nonlinear compensator*

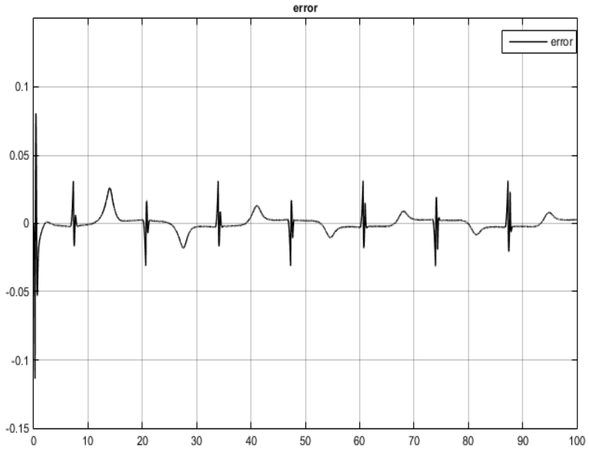

Fig.7 *MRAC with backlash nonlinear compensator error*

6. CONCLUSION

The paper has proposed a new nonlinear adaptive control method for mechanical transmission system with backlash. The simulation shows that the error between the model output and actual output signals is reduced. The backlash parameters and system's parameter are not necessary to be determined before compensating. This is an advantage of the new method.

ACKNOWLEGMENT

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