

**OBSERVER-BASED PD CONTROLLER
FOR BALANCING ROBOT WITH UNCERTAIN MODEL**
BỘ ĐIỀU KHIỂN PD DỰA TRÊN BỘ QUAN SÁT
CHO HỆ ROBOT CÂN BẰNG ĐỐI VỚI MÔ HÌNH KHÔNG CHẮC CHẮN

Nguyen Van Dong Hai¹, Nguyen Minh Tam², Hoang Ngoc Van³,
Nguyen Thi Yen Tuyet⁴, Mircea Ivanescu⁵

^{1,2,3,4} Ho Chi Minh city University of Technology and Education (HCMUTE), Vietnam

⁵ University of Craiova (UCV), Romania

Received 13/02/2016, Peer reviewed 25/6/2016, Accepted for publication 10/8/2016

ABSTRACT

This research focuses on the observer-based controller of a balancing robot. Firstly, the dynamic model of the robot is inferred. The uncertainty of the model parameters is then introduced as constraints in state space. Practically, it is very difficult to measure state variables as velocity and acceleration. Moreover, system parameters are not always fixed in operating period. In order to avoid this difficulty, a nonlinear observer associated with the uncertain model system is used to estimate the state variables. A PD control algorithm is proposed in order to achieve the control performances. The exponential stabilities are ensured by Lyapunov techniques. Numerical simulations show the control performance successfully.

Keywords: PD controller; Matlab/Simulink; LQR; observer; Lyapunov component; Balancing Robot.

TÓM TẮT

Nghiên cứu chú trọng đến bộ điều khiển dựa trên bộ quan sát đối với robot cân bằng. Trước tiên, một mô hình về robot cân bằng (ở đây là hệ con lắc ngược một bậc) được giới thiệu. Sau đó, sự không chắc chắn của các thông số mô hình được chấp nhận là các tham số bất định cho trước trong vùng trạng thái hoạt động. Thực tế, ta rất khó biết được các biến số hệ thống như vận tốc và gia tốc. Để tránh những khó khăn đó, một bộ quan sát được đề cập để xác định các biến số như vận tốc, gia tốc một cách chính xác. Một phương pháp điều khiển PD được thiết kế để phù hợp cho sự không chắc chắn của mô hình. Tính chất ổn định của bộ điều khiển được đảm bảo bởi phương pháp Lyapunov. Các kết quả mô phỏng cho thấy kết quả điều khiển ổn định, không phụ thuộc sự bất định từ thông số mô hình.

Từ khóa: Bộ điều khiển PD; Matlab/Simulink; LQR; bộ quan sát; thành phần Lyapunov; Robot cân bằng.

1. INTRODUCTION

The study and development of feedback controllers for Balancing Robot (BR) represent a very complex problem and a great number of researchers have tried to offer solutions. There are many papers that treat the control for BR

and a chronological list can be found as: Sliding-Mode Control, back stepping control, fuzzy control (FLC), Linear-quadratic regulator (LQR), Proportional-integral-derivative (PID) [1]. A two-wheeled self-balancing robot is a

special type of wheeled mobile robot. Its balance problem is a hot research topic due to its unstable state for controlling. LQR, FLCM PID are implemented to test the control system [2,3]. A hybrid fuzzy PD controller is proposed in [4]. Other BR models as double-linked Inverted Pendulum are also controlled [5,6]. Many controlling algorithms and many kinds of BR have been implemented successfully.

All research works underline the complexity of control problems, the difficulty in implementing feedback controllers and compensators. But these methods of controller-designing base on the exact model of BR. One of the most difficult problems is determined by the discrepancy between the mathematical model and the actual dynamics of the robot. In real model, the system parameters are not exact because of a remarkable error in measurement. These reasons make the uncertainty of real model. A mathematical model of any real system is always just an approximation of the true, physical reality of the system dynamics. These modelling errors may adversely affect the stability and performances of the control system. Moreover, in those papers, these authors assumed that all velocities and positions are measurable. In real situation, velocities that could not be measured will badly affect quality of controllers.

Depending on these problems, this paper treats the control problem of a class of BR. The dynamic model with uncertain parameters is inferred and the constraints of the state variables and nonlinear components are proved. Because the estimation of gravitational terms and inertial components in a real motion of the robot is very difficult, these components are treated as uncertain components that satisfy the inequality constraints. In BR, practically, it is very difficult to measure state variables as velocity, acceleration. For this

reason, an observer is proposed and implemented. The stability of the global system is proved by Lyapunov techniques.

The paper is organized as follows. In Section 2, the dynamic model is presented. Section 3 concerns about the control. Section 4 is dedicated to numerical simulations. Finally, a conclusion section ends the paper.

2. DYNAMIC MODEL

The dynamic equations of BR were well studied and a great number of papers treated the nonlinear model associated with this robot [7]. Most of the papers were based on the classical model presented in Fig 1.

According to [7], the general nonlinear equations of BR (Fig. 1) are:

$$\begin{cases} \ddot{x} = \frac{1}{M + m \sin^2 \theta} [m \sin \theta (L \dot{\theta}^2 - g \cos \theta) + F] \\ \ddot{\theta} = \frac{1}{L(M + m \sin^2 \theta)} [-mL \dot{\theta}^2 \sin \theta \cos \theta + (M + m)g \sin \theta - F \cos \theta] \end{cases} \quad (1)$$

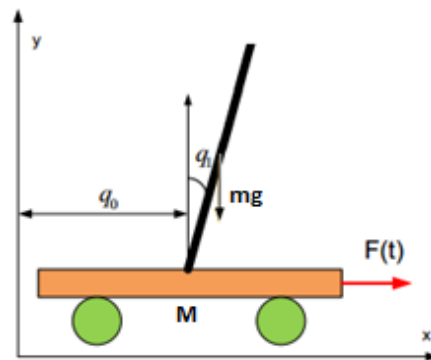


Fig.1 Classical model of BR

Table 1. System parameters

Name	Description	Unit
M	Mass of Cart	kg
m	Mass of Pendulum	kg
L	length of Pendulum	m
g	Gravitational Acceleration	m/s ²
F	Force on Cart	N
x	Position of Cart on x-direction	m
θ	angle of pendulum	rad

Also from [7], the position control can be decoupled by the pendulum angle control:

$$\ddot{\theta} = \frac{1}{L(M+m\sin^2\theta)} \begin{bmatrix} -mL\dot{\theta}^2 \sin\theta \cos\theta + \\ +(M+m)g \sin\theta - F \cos\theta \end{bmatrix} \quad (2)$$

We denote by: $x_1 = \theta$, $x_2 = \dot{\theta}$, $u = F$. Then, (2) becomes:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{\begin{bmatrix} -mLx_2^2 \sin x_1 \cos x_1 + \\ +(M+m)g \sin x_1 - u \cos x_1 \end{bmatrix}}{L(M+m\sin^2 x_1)} \end{cases} \quad (3)$$

Assumption: It is assumed here that the following technological constraints:

$$\begin{aligned} m &\in [m_{\min}, m_{\max}]; \quad M \in [M_{\min}, M_{\max}]; \\ L &\in [L_{\min}, L_{\max}]; \quad |x_1| \leq \eta_1; \quad |x_2| \leq \eta_2 \end{aligned} \quad (4)$$

Approximating $\sin x_1 \approx x_1$; $\sin^2 x_1 \approx 0$; $\cos x_1 \approx 1$, (3) becomes:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \gamma_1 x_1 - \gamma_2 x_1 x_2^2 - \gamma_3 u \end{cases} \quad (5)$$

$$\text{where: } \gamma_1 = g \left(\frac{m+M}{ML} \right) > 0; \quad \gamma_2 = \frac{m}{M} > 0; \quad \gamma_3 = \frac{1}{ML} > 0 \quad (6)$$

The assumptions from (4) determine the following constraints of the model parameters:

$$\gamma_1 \in \left[\gamma_{1\min} = g \left(\frac{m_{\min} + M_{\min}}{M_{\max} L_{\max}} \right); \gamma_{1\max} = g \left(\frac{m_{\max} + M_{\max}}{M_{\min} L_{\min}} \right) \right]$$

$$\gamma_2 \in \left[\gamma_{2\min} = \frac{m_{\max}}{M_{\min}}; \gamma_{2\max} = \frac{m_{\min}}{M_{\max}} \right]$$

$$\gamma_3 \in \left[\gamma_{3\min} = \frac{1}{M_{\max} L_{\max}}; \gamma_{3\max} = \frac{1}{M_{\min} L_{\min}} \right] \quad (7)$$

3. CONTROL SYSTEM

The control problem consists of finding the control law $u(t)$, such that (x_1, \dot{x}_1)

converge to zero. The block of observer-based control system is presented in Fig 2.

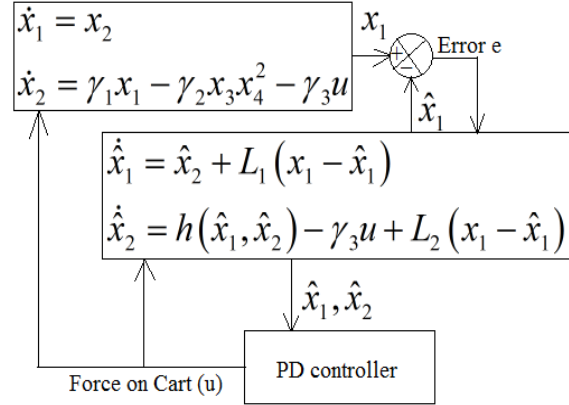


Fig.2 Control system

3.1 Nonlinear Observer

In order to estimate the state parameters, the following observer is proposed

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 = h(\hat{x}_1, \hat{x}_2) - \gamma_3 u + L_2(x_1 - \hat{x}_1) \end{cases} \quad (8)$$

Where \hat{x}_1 , \hat{x}_2 represents the estimated states of x_1 , x_2 and L_1 , L_2 are the observer gains. We also define function:

$$h(x_1, x_2) = \gamma_1 x_1 - \gamma_2 x_1 x_2^2 \quad (9)$$

and we denote by $e = x_1 - \hat{x}_1$ the associated error. In terms of this error, (9) becomes:

$$h(x_1, x_2) \approx h(\hat{x}_1, \hat{x}_2) + \left(e \frac{\partial}{\partial x_1} + \dot{e} \frac{\partial}{\partial x_2} \right) h(x_1, x_2) \quad (10)$$

$$h(x_1, x_2) - h(\hat{x}_1, \hat{x}_2) = e(\gamma_1 - \gamma_2 x_2^2) + \dot{e}(-2\gamma_2 x_1 x_2) \quad (11)$$

The observer gains will be calculated in order to ensure the observer convergence:

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (12)$$

After simple calculations, the above equations become:

$$\begin{cases} \dot{x}_1 - \dot{\hat{x}}_1 = x_2 - \hat{x}_2 - L_1 e \\ \dot{x}_2 - \dot{\hat{x}}_2 = h(x_1, x_2) - h(\hat{x}_1, \hat{x}_2) - L_2 e \end{cases} \quad (13)$$

$$\begin{cases} \dot{e} = (x_2 - \hat{x}_2) - L_1 e \rightarrow \dot{x}_2 - \dot{\hat{x}}_2 = \ddot{e} + L_1 \dot{e} \\ \dot{x}_2 - \dot{\hat{x}}_2 = [h(x_1, x_2) - h(\hat{x}_1, \hat{x}_2)] - L_2 e \end{cases} \quad (14)$$

$$\ddot{e} + (L_1 + 2\gamma_2 x_1 x_2) \dot{e} + (L_2 + \gamma_2 x_4^2 - \gamma_1) e = 0 \quad (15)$$

The error convergence can be easily analyzed by Routh method (Table 1) below. In the table, c_{11} , c_{12} , c_{21} , c_{22} are directly recognized from equation (15), then, a_{13} and c_{31} are recognized:

Table 1. Routh table

	$c_{11} = 1$	$c_{12} = L_2 + \gamma_2 x_4^2 - \gamma_1$
	$c_{21} = L_1 + 2\gamma_2 x_1 x_2$	$c_{22} = 0$
$a_3 = \frac{1}{L_1 + \gamma_2 x_1 x_2}$	$c_{31} = L_2 + \gamma_2 x_4^2 - \gamma_1$	

From Table 1 and the constraints (4) we obtain the observer error convergence conditions as:

$$L_1 > 2\gamma_{2\max} \eta_1 \eta_2; \quad L_2 > \gamma_{1\max} \quad (16)$$

3.2 PD Controller

Theorem 1: For the system described by (5), if the control law is given by

$$u(t) = \frac{k_1}{\gamma_3} x_1(t) + \frac{k_2}{\gamma_3} x_2(t) \quad (17)$$

where, k_1 and k_2 are positive controller gains that satisfy the conditions:

$$k_1 > \gamma_{1\max} \quad (18)$$

$$\alpha + \gamma_{1\max} \beta < k_1 + k_2 \delta \quad (19)$$

$$(\gamma_{\min} - k_1) \delta + \frac{1}{4} (k_1 + k_2 \delta - \alpha - \gamma_{1\max} \beta) < 0 \quad (20)$$

$$\delta - k_2 \beta + \gamma_2 \beta + \gamma_2 \beta \eta_1 \eta_2 + k_1 + k_2 \delta - \alpha - \gamma_{1\max} \beta < 0 \quad (21)$$

and α, β, γ are positive constants that satisfy the relations $\alpha > \frac{\delta}{4} > 0; \beta > 2\delta$ (22)

Then, the system is stable.

Proof: The following Liapunov function will be considered:

$$V(t) = \frac{1}{2} (\alpha x_1^2 + \beta x_2^2 + 2\delta x_1 x_2) \quad (23)$$

After small calculations, (23) becomes:

$$V(t) \geq \frac{1}{2} \left[\left(\alpha - \frac{\delta}{2} \right) x_1^2 + (\beta - 2\delta) x_2^2 \right] \quad (24)$$

and from the conditions (22), it is positive definite: $V > 0$ (25)

The derivative of (24) will be:

$$\dot{V} = \alpha x_1 \dot{x}_1 + \beta x_2 \dot{x}_2 + \delta \dot{x}_1 x_2 + \delta x_1 \dot{x}_2 \quad (26)$$

Substitute $\dot{x}_2 = \gamma_1 x_1 - \gamma_2 x_1 x_2^2 - \gamma_3 u$ in (5) into (26). After simple calculations, results become:

$$\begin{aligned} \dot{V} = & -[\delta(k_1 - \gamma_1)] x_1^2 - (\beta k_2 - \delta) x_2^2 + \\ & + (\alpha + \gamma_1 \beta - k_1 \beta - k_2 \delta) x_1 x_2 - (\gamma_2 \beta) x_1 x_2^3 - (\gamma_2 \delta) x_1^2 x_2^2 \\ = & -[\delta(k_1 - \gamma_1)] x_1^2 - (\beta k_2 - \delta + \gamma_2 \beta x_1 x_2) x_2^2 + \\ & + (\alpha + \gamma_1 \beta - k_1 \beta - k_2 \delta) x_1 x_2 - (\gamma_2 \delta) x_1^2 x_2^2 \end{aligned} \quad (27)$$

From Young's inequality: $|x_1 x_2| \leq \frac{x_1^2}{4} + x_2^2$, the relation (27) becomes:

$$(\alpha + \gamma_1 \beta - k_1 \beta - k_2 \delta) x_1 x_2 \leq |\alpha + \gamma_1 \beta - k_1 \beta - k_2 \delta| \left(\frac{x_1^2}{4} + x_2^2 \right) \quad (28)$$

If we substitute (28) into (27), we obtain:

$$\begin{aligned} \dot{V} \leq & -[\delta(k_1 - \gamma_1)] x_1^2 - (\beta k_2 - \delta + \gamma_2 \beta x_1 x_2) x_2^2 + \\ & + (\alpha + \gamma_1 \beta - k_1 \beta - k_2 \delta) x_1 x_2 - (\gamma_2 \delta) x_1^2 x_2^2 \\ \leq & -[\delta(k_1 - \gamma_1)] x_1^2 - (\beta k_2 - \delta + \gamma_2 \beta x_1 x_2) x_2^2 + \\ & + |\alpha + \gamma_1 \beta - k_1 \beta - k_2 \delta| \left(\frac{x_1^2}{4} + x_2^2 \right) - (\gamma_2 \delta) x_1^2 x_2^2 \\ \Rightarrow \dot{V} \leq & \left[(\gamma_1 - k_1) \delta + \frac{1}{4} |\alpha + \gamma_1 \beta - k_1 - k_2 \beta| \right] x_1^2 + \\ & + [\delta - k_2 \beta - \gamma_2 \beta x_1 x_2 + |\alpha + \gamma_1 \beta - k_1 - k_2 \delta|] x_2^2 - (\gamma_2 \delta) x_1^2 x_2^2 \end{aligned} \quad (29)$$

From (19) and (22), we have:

$$|\alpha + \gamma_1 \beta - k_1 - k_2 \delta| = k_1 + k_2 \delta - \alpha - \gamma_1 \beta \quad (30)$$

We can conclude that:

$$\dot{V} \leq \left[(\gamma_1 - k_1)\delta + \frac{1}{4}(k_1 + k_2\beta - \alpha - \gamma_1\beta) \right] x_1^2 + [\delta - k_2\beta - \gamma_2\beta x_1 x_2 + k_1 + k_2\delta - \alpha - \gamma_1\beta] x_2^2 - (\gamma_2\delta) x_1^2 x_2^2 \quad (31)$$

Also, from the Assumption (4) and (22), we have $|x_1 x_2| \leq \eta_1 \eta_2$ and:

$$-(\gamma_2\beta x_1 x_2) x_2^2 \leq (\gamma_2\beta \eta_1 \eta_2) x_2^2 \quad (32)$$

Substitute (32) into (31), it results:

$$\dot{V} \leq \left[(\gamma_1 - k_1)\delta + \frac{1}{4}(k_1 + k_2\beta - \alpha - \gamma_1\beta) \right] x_1^2 + (\delta - k_2\beta + \gamma_2\beta \eta_1 \eta_2 + k_1 + k_2\delta - \alpha - \gamma_1\beta) x_2^2 - (\gamma_2\delta) x_1^2 x_2^2 \quad (33)$$

From (20), (21), we have:

$$\dot{V} \leq 0 \quad (34)$$

and from (18)- (22), (25), (34), we can conclude that system is stable with:

$$\lim_{t \rightarrow \infty} x_1(t) = 0; \quad \lim_{t \rightarrow \infty} x_2(t) = 0 \quad (35)$$

4. SIMULATION

4.1 Simulation of observer-based PD controller

System parameters were chosen as:

$$M = 0.1 \pm 0.08(kg); \quad m = 0.01 \pm 0.008(kg); \quad L = 1 \pm 0.7(m)$$

A Matlab program was implemented to select randomly the controller parameters in according with (18)-(22) and observer parameters in according with (16). One of the solutions was: $k_1 = 454.4657$; $k_2 = 919.9060$; $L_1 = 60$; $L_2 = 350$ and Lypunov parameters: $\alpha = 39.3479$; $\beta = 19.5419$; $\delta = 8.6780$. With these results and according to (17), PD controller parameters would be $K_p = 45.4657$ and $K_d = 91.9906$.

In the case of $M = 0.1(kg)$; $m = 0.01(kg)$; $L = 1(m)$, the control performance can be appreciated in Fig. 3, 4 and observer error is shown in Fig. 5:

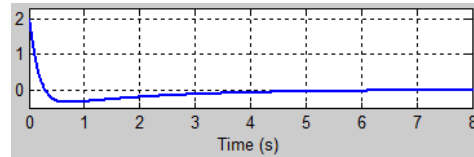


Fig.3 Angle of Pendulum (rad)

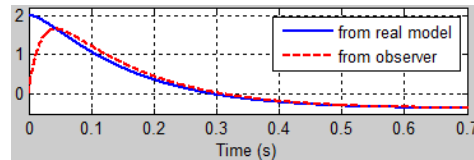


Fig.4 Comparison of pendulum's angles from real system and observer

4.2 Comparison of LQR controller and observer-based PD controller

The controller presented in Part 4.1 will be compared to a LQR control [8]. Based on [9], the linearized model will be: $\dot{x} = Ax + Bu$

where $A = \begin{bmatrix} 0 & 1 \\ \gamma_1 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ -\gamma_3 \end{bmatrix}$; $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

with weighing matrices: $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $R = 1$

Using Matlab, control parameters would be:

$$K = [-2.5503 \quad -1.2288]$$

The control signal is: $u = -Kx$

Table 2 below describes the comparison of response of pendulum angle between PD controller and LQR controller when the value of system parameters is not always fixed. From Fig. 6-21, with uncertain model, response of pendulum angle with PD controller is stabilized and nearly the same in all case. But, with the LQR controller, the response is affected much and not stabilized in case 2, 6, 8.

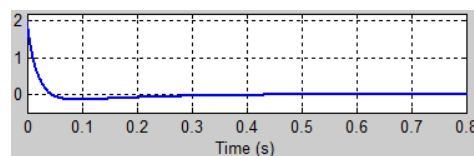


Fig.5 Error of pendulum's angle from observer and real model

Table 2. Pendulum angle comparison between PD controller and LQR controller with uncertain model

PD-based Controller

LQR Controller

Case 1:

$$M = 0.1(kg);$$

$$m = 0.01(kg);$$

$$L = 1(m)$$

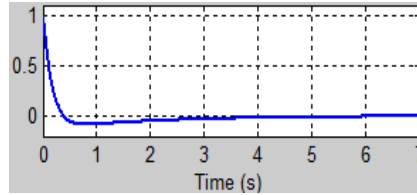


Fig. 6

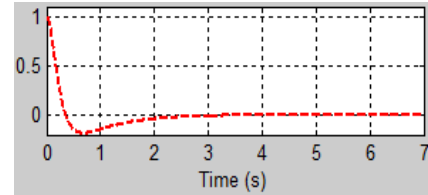


Fig. 7

Case 2:

$$M = M_{\max} = 0.18(kg);$$

$$m = 0.01(kg);$$

$$L = 1(m)$$

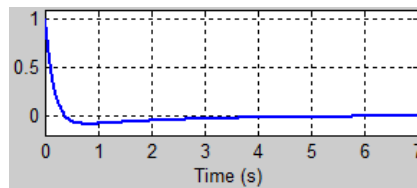


Fig. 8

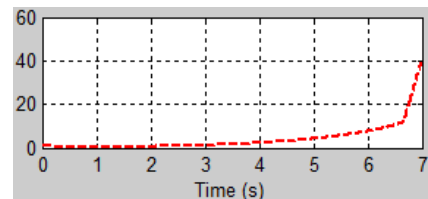


Fig. 9

Case 3:

$$M = 0.1(kg);$$

$$m = m_{\max} = 0.018(kg);$$

$$L = 1(m)$$

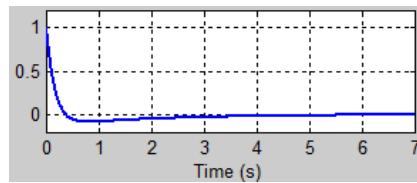


Fig. 10

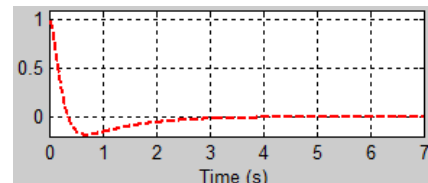


Fig. 11

Case 4:

$$M = 0.1(kg);$$

$$m = 0.01(kg);$$

$$L = L_{\max} = 1.7(m)$$

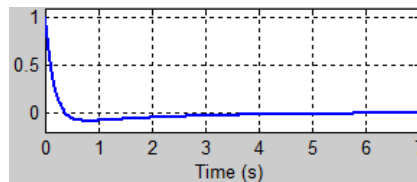


Fig. 12

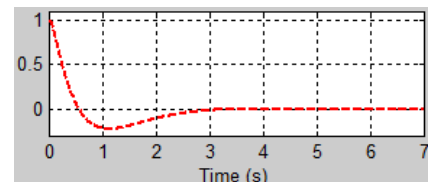


Fig. 13

Case 5:

$$M = M_{\min} = 0.02(kg);$$

$$m = 0.01(kg);$$

$$L = 1(m)$$

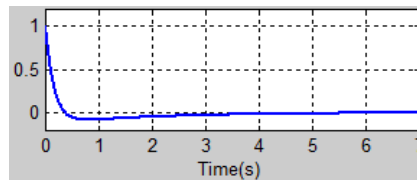


Fig. 14

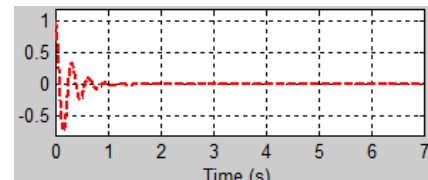


Fig. 15

Case 6:

$$M = 0.1(kg);$$

$$m = m_{\min} = 0.002(kg);$$

$$L = 1(m)$$

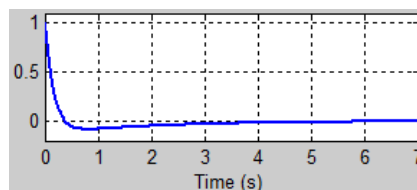


Fig. 16

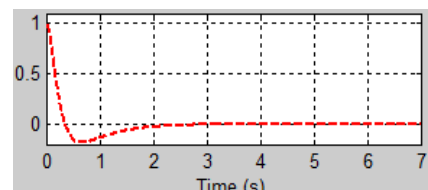


Fig. 17

Case 7:

$$M = 0.1(kg);$$

$$m = 0.01(kg);$$

$$L = L_{\min} = 0.3(m)$$

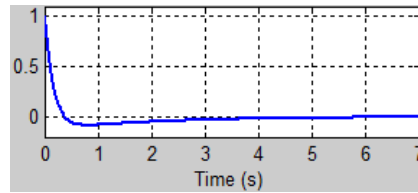


Fig. 18

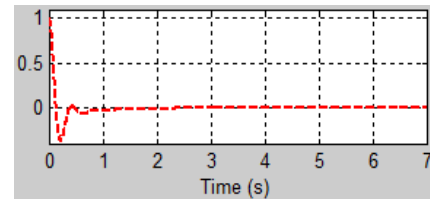


Fig. 19

Case 8:

$$M = M_{\max} = 0.18(kg);$$

$$m = m_{\max} = 0.018(kg);$$

$$L = L_{\min} = 0.3(m)$$

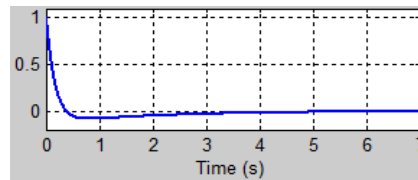


Fig. 20

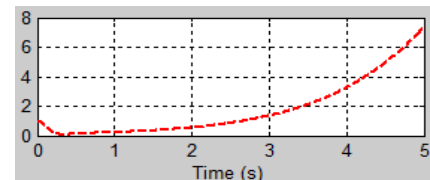


Fig. 21

5. CONCLUSION

A lot of control algorithm were presented and discussed in the literature but all the authors when treated the certain models only considering that all the parameters are measurable ones. To overcome this problem, in this paper, the observer-based PD controller for uncertain models has been proposed. An

observer-based PD controller is designed and implemented for uncertain BR model. The robustness of the controller with uncertain model has been investigated. Also, numerical simulations results were illustrated in Section 4 and an observer-based PD controller was compared to a LQR control. The controller performances were specified.

REFERENCES

- [1] Olfar Boubaker, *The inverted Pedulum: a fundamental Benchmark in Control Theory and Robotics*, pp 1-6, International Conference on Education and e-Learning Innovations (ICEELI), IEEE, 2012.
- [2] Ahmad Nor Kasrudin Nasir, Mohd, Ashraf Ahmad, Riduwan Ghazali, Nasrul Salim Pakheri, *Performance Comparison Between Fuzzy Logic Controller (FLC) and PID Controller for a Highly Nonlinear Two-wheels Balancing Robot*, pp. 176-181, First International Conference on Informatics and Computaional Intelligence (IEEE), 2011.
- [3] Amir A. Bature, Mustafa Muhammad, *A comparision of controllers for balancing two-whelled inverted pendulum robot*, pp. 62-68, Vol. 14, Issue. 3, International Journal of Mechanical and Mechatronics Engineering, 2014.
- [4] Song Xin, Min Gong, Yang Sun, Zhongqiu Zhang, *Control system design for two-wheeled self-balanced robot based on Fuzzy-PD control*, pp. 169-174, Fifth International Conference on Intelligent Control and Information Processing (IEEE), 2014.
- [5] Zhi-hui Li, Yong-li Zhang, Hong-xing Li, *Design of Optimal Cost Fuzzy Controller for Spatial Double Inverted Pendulum System*, pp. 129-138, Vol. 147, Series of Advances in Intelligent and Soft Computing, Book of Fuzzy Engineering and Operations Research, 2012.

- [6] Kwakernaak, Huibert and Sivan, Raphael, *Linear Optimal Control Systems, First Edition*. Wiley-Interscience. ISBN 0-471-51110-2, 1972.
- [7] Fantoni, I. and Lozano, R., *Nonlinear Control for Underactuated Mechanical System*, Springer-Verlag, London, 2002.
- [8] Chenxi Sun, Tao Lu, Kui Yuan, *Balance control of two-wheeled self-balancing robot based on Linear Quadratic Regulator and Neural Network*, Fourth International Conference on Intelligent Control and information Processing (ICICIP), IEEE, 2013.
- [9] M. Young, *The Technical Writer's Handbook*. Mill Valley, CA: University Science, 1989.
- [10] Quanser, *Linear experiment #5: LQR Control, Linear Motion Servo Plants: IP01 or IP02* (Student Handout).

Corresponding author:

Nguyen Minh Tam

Ho Chi Minh City University of Technology and Education

Email: tammn@hcmute.edu.vn