APPLICATION OF FREQUENCY CRITERION IN DESIGNING CONTROLLER FOR UNCERTAIN BALANCING ROBOT

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ABSTRACT

In most nonlinear system, it is difficult to identify in a state sapce all system parameters exactly,especially after a period of working operation since system parameters usually vary. So, a system control method is very important in this case. This paper focuses on a method of designing PD controller for a balancing robot-cart. The uncertainty of model parameters was introduced as constraints in state space. PD controller was designed through choosing Lyapunov function which was transformed by Kalman-Yakubovich-Popov lemma, a basic theoretical method based on frequency criterion. After the stability of the control system had been determined by Lyapunov techniques, stability of the controller was proved by mathematics and shown by simulation results.

Keywords: *PD Controller; uncertain model; Kalman-Yakubovich-Popov lemma; balancing robot; Lyapunov method; nonlinear design.*

TÓM TẮT

Trong đa phần các hệ thống phi tuyến, rất khó để ta xác định được toàn bộ các thông số hệ thống chính xác, hoặc sau khi hệ thống hoạt động một thời gian thì thông số hệ thống bị thay đổi trong một khoảng nhất định. Do đó, việc đưa ra một phương pháp để điều khiển hệ thống trong trường hợp như vậy là rất qua trọng. Bài báo chú trọng vào phương pháp thiết kế bộ điều khiển PD cho một hệ robot cân bằng - hệ thống con lắc ngược trên xe. Sự không chắc chắn của các thông số mô hình là các tham số bất định cho trước trong vùng không gian trạng thái. Bộ điều khiển PD được được thiết kế thông qua việc lựa chọn hàm Lyapunov được biến đổi bởi bổ đề Kalman-Yakubovich-Popov, một phương pháp lý thuyết cơ bản của thiết kế phi tuyến theo tiêu chuẩn tần số. Sau khi bộ điều khiển được thiết kế và chứng minh thông qua kỹ thuật Lyapunov, sự ổn định của bộ điều khiển đã được chứng minh bằng toán học và bằng cách các kết quả mô phỏng.

Từ khóa: *Bộ điều khiển PD; Mô hình không chắc chắn; Bổ đề Kalman-Yakubovich-Popov; Robot cân bằng; phương pháp Lyapunov; thiết kế phi tuyến.*

1. INTRODUCTION

Balancing robots are nonlinear models which are very popular in control engineering research. A great number of research results have represented different kinds of control methods for balancing robot (BR). Classical controller was designed successfully [1,2]. Nonlinear controllers and intelligent controllers were also implemented well [3-6]. All these results based on exact models. There existed remarkable errors in measurement between the real system and mathematical model. In most papers, mathematical models were used even though they were just approximate to the true value and physical reality of system dynamics.

In this paper, a PD controller was also designed for uncertain BR with stability assured by Lyapunov technique. The Lyapunov function was selected simply but the nonlinearity of system made the derivative of Lyapunov function more complex. Based on Kalman-Yakubovich-Popov (KYP) lemma, the result of the derivative was equivalent to a more simple form that could be calculated in easier relations. From these relations, control parameters were chosen. Besides being guaranteed by mathematics, the response performances were proved to work well through simulation. With PD controller, the stability of system was still guaranteed by mathematics and controller was simple to be designed.

The paper is organized as follows: Section 2 represents the dynamic model. Method of designing controller is concerned in Section 3. Then, performance response is dedicated in Section 4. Finally, conclusion ends the paper.

2. MODEL OF BR

The dynamic equations of BR have been well studied and there is a great number of papers treating the nonlinear model associated with this robot [5]. The majority of papers have studied BR by using the classical model (Fig 1):

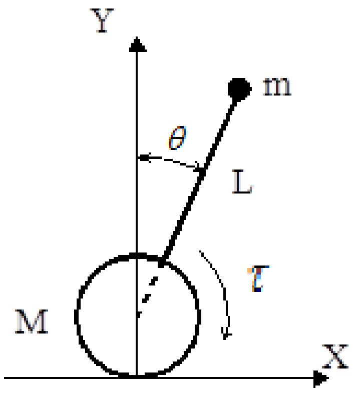

Fig 1. *Classical model of BR*

$$\ddot{x}=\frac{1}{M+m\sin^2\theta}\left[m\sin\theta\left(L\dot{\theta}^2-g\cos\theta\right)+F\right] \tag{1}$$

$$\ddot{\theta}=\frac{1}{L\left(M+m\sin^2\theta\right)}\begin{bmatrix}-mL\dot{\theta}^2\sin\theta\cos\theta+\\+\left(M+m\right)g\sin\theta-F\cos\theta\end{bmatrix} \tag{2}$$

Where in, the following notations are used:

M: mass of Cart (kg)

m: mass of Pendulum (kg)

L: length of equivalent pendulum (m)

g: gravitational acceleration (m/s^2)

$F=\frac{\tau}{r}$: the Force on Cart (N)

x: position of Cart on x-direction (m)

θ: angle of pendulum (rad)

From [5], the position control can be decoupled by the pendulum angle control. Denoting that: $x_1=\theta$, $x_2=\dot{\theta}$, $u=F$, then, (2) becomes:

$$\dot{x}_1=x_2 \tag{3}$$

$$\dot{x}_2=\frac{\begin{bmatrix}-mLx_2^2\sin x_1\cos x_1+\\+\left(M+m\right)g\sin x_1-u\cos x_1\end{bmatrix}}{L\left(M+m\sin^2 x_1\right)} \tag{4}$$

Assumption:

The following technological constraints are assumed:

$$\begin{cases} m \in [m_{\min}, m_{\max}] \\ M \in [M_{\min}, M_{\max}] \\ L \in [L_{\min}, L_{\max}] \\ |x_1| \le \eta_1 \\ |x_2| \le \eta_2 \\ X = x_1 x_2 \in [-\eta_1\eta_2, \eta_1\eta_2] \end{cases} \tag{5}$$

By employing the approximations: $\sin x_1 \approx x_1, \sin^2 x_1 \approx 0$ and $\cos x_1 \approx 1$, (4) becomes:

$$\dot{x}_2 = \gamma_1 x_1 - \gamma_2 x_1 x_2^2 - \gamma_3 u \tag{6}$$

where $\gamma_1 = g\left(\frac{m+M}{ML}\right) > 0;\ \gamma_2 = \frac{m}{M} > 0;\ \gamma_3 = \frac{1}{ML} > 0$ (7)

that satisfy the constraints:

$$\gamma_1 \in \left[\gamma_{1\min} = g\left(\frac{m_{\min} + M_{\min}}{M_{\max} L_{\max}}\right); \gamma_{1\max} = g\left(\frac{m_{\max} + M_{\max}}{M_{\min} L_{\min}}\right)\right]$$

$$\gamma_2 \in \left[\gamma_{2\min} = \frac{m_{\max}}{M_{\min}}; \gamma_{2\max} = \frac{m_{\max}}{M_{\min}}\right];$$

$$\gamma_3 \in \left[\gamma_{3\min} = \frac{1}{M_{\max} L_{\max}}; \gamma_{3\max} = \frac{1}{M_{\min} L_{\min}}\right] \tag{8}$$

Controller Design

From (3) and (6), the dynamic equation of BR are defined as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \gamma_1 & -\gamma_2 x_1 x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\gamma_3 \end{bmatrix} u$$

$$= \begin{bmatrix} 0 & 1 \\ \gamma_1 & -\gamma_2 x_1 x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} u + \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} u \tag{9}$$

Where: $\begin{cases} c_3 = -c_1 \\ c_4 = -\gamma_3 - c_2 \end{cases}$ (10)

The output of the system will be:

$$y = C^T x = [c_5 \quad c_6] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11}$$

Then, (9) becomes:

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -kc_1c_5 & 1 - kc_1c_6 \\ \gamma_1 - kc_2c_5 & -\gamma_2 X - kc_2c_6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} -c_1 \\ -\gamma_3 - c_2 \end{bmatrix}}_{B} u \tag{12}$$

Matrix A is Hurwitz if $c_1 c_5 < 0$ (13)

$$\begin{aligned} &kc_1c_6\{\gamma_{1\min} sign(c_1c_6) + \gamma_{1\max}[1 - sign(c_1c_6)]\} + \\ &+kc_1c_6\{\gamma_{1\min} sign(c_1c_6) + \gamma_{1\max}[1 - sign(c_1c_6)]\} + \\ &-\gamma_{1\max} + kc_2c_5 > k|c_1c_5|\gamma_{2\max} \end{aligned} \tag{14}$$

Theorem 1: The system (12) is stable if the matrix A is Hurwitz, the control law is: $u = -ky$ (15)

and $\exists \gamma, k$ that the following conditions are satisfied:

$$0 < \gamma \le 4 \tag{16}$$

$$0 < k < \frac{1}{2\sqrt{\gamma}} \tag{17}$$

$$0 \le \gamma + 2\operatorname{Re}\left\{\left(\frac{C}{2}\right)^T (j\omega I - A)^{-1} B\right\} \forall X \in [-\eta_1\eta_2 \quad \eta_1\eta_2] \tag{18}$$

Proof: Define a Lyapunov function:

$$V = x^T P x \ge 0 \quad \text{with} \quad P \ge 0 \tag{19}$$

Then, derivative of (28) will be:

$$\begin{aligned} \dot{V} &= \dot{x}^T P x + x^T P \dot{x} \\ &= x^T (A^T P + PA) x + 2x^T PBu = \\ &= x^T (A^T P + PA) x + 2x^T PBu - yu + yu \end{aligned} \tag{20}$$

From KYP lemma [7] yields:

$$\begin{cases} A^T P + PA = -QQ^T \\ PB - \frac{C}{2} = \sqrt{\gamma} Q \end{cases} \quad with: P \ge 0 \tag{21}$$

Substitute (21) and (15) into (20), after several calculations, yields:

$$\begin{aligned} \dot{V} &= -\left[(x^T QQ^T x) - 2x^T \sqrt{\gamma} Qu + \frac{1}{k} u^2\right] \\ &= -(\xi_1 X_1^2 + \xi_2 X_1 X_2 + \xi_3 X_2^2) \end{aligned} \tag{22}$$

Where: $$\begin{cases} \xi_1 = 1 \\ \xi_2 = -2\sqrt{\gamma} \text{ and } \begin{cases} X_1 = x^T Q \\ X_2 = u \end{cases} \\ \xi_3 = \dfrac{1}{k} \end{cases} \tag{23}$$

Substitute Young's Inequalities $\xi_2 X_1 X_2 \geq -|\xi_2| \left(\dfrac{X_1^2}{4} + X_2^2 \right)$ into (22), yields:

$$\dot{V} \leq -\left(\xi_1 - \frac{|\xi_2|}{4} \right) X_1^2 - \left(\xi_3 - |\xi_2| \right) X_1^2 \tag{24}$$

From (16), (17), this inequality can be inferred $\dot{V} \leq 0$ (25)

From (19), (25), the system is stabilized with the assumption (5).

The Lemma 1 in Appendix 2 described the conditions of $\gamma, k, c_1, c_2, c_5, c_6$ that make (18) become true.

3. SIMULATION

System parameters are chosen as:

$M = 0.1(kg)$; $m = 0.01(kg)$; $L = 1(m)$; $M_{\max} = 0.11(kg)$; $M_{\min} = 0.09(kg)$; $m_{\max} = 0.011(kg)$; $m_{\min} = 0.009(kg)$ $L_{\min} = 0.9(m)$; $L_{\min} = 1.1(m)$; $\eta_1 = 1.5(rad)$, $\eta_2 = 10(rad/s)$

A Matlab program has been implemented to select randomly the controller parameters according to (20)-(23) and clause (xiii). One of the solutions is inferred as:

$c_1 = 0.3850$; $c_2 = -1.3970$; $c_5 = -46.8534$; $c_6 = -0.3932$; $k = 2.4584$; $\gamma = 0.0103$

The survey of response performance is implemented in three cases:

Case 1: $m = m_{\min} = 0.009(kg)$;
$M = M_{\min} = 0.09(kg)$; $L = L_{\min} = 0.9(m)$

Case 2: $m = m_{\max} = 0.011(kg)$;
$M = M_{\max} = 0.11(kg)$; $L = L_{\max} = 1.1(m)$

Case 3: $m = m_{\max} = 0.011(kg)$;
$M = M_{\max} = 0.11(kg)$; $L = L_{\min} = 0.9(m)$

The frequency conditions are analyzed in Appendix 1.

Results of controlling are shown in Fig. 2, 3, 4, respectively and Fig. 5, 6 and 7 are for initial value of state variables that are far and near from balancing position, consequently.

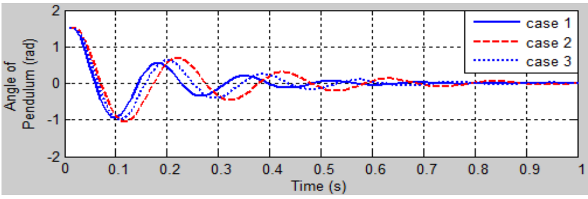

***Fig 2.** Angle Pendulum in three cases*

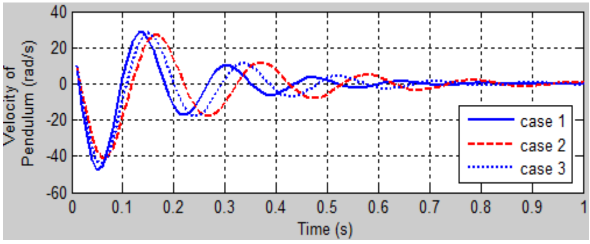

***Fig 3.** Velocity of Pendulum in three cases*

4. CONCLUSION

In this paper, a PD controller was designed for BR models. The robustness of controller was proved by Lyapunov technique. In this case, the complex derivative of Lyapunov function was shortened through KYP lemma. Control parameters were selected through relations through relations of KYP lemma and Lyapunov conditions. The controlling performances were shown that the system was stabilized well according to simulation results.

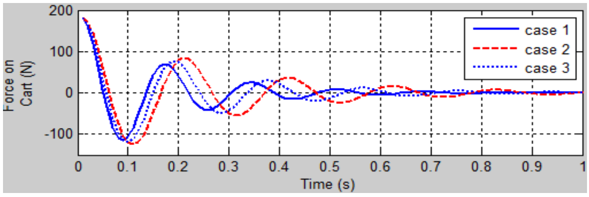

***Fig 4.** Force on Cart (N)*

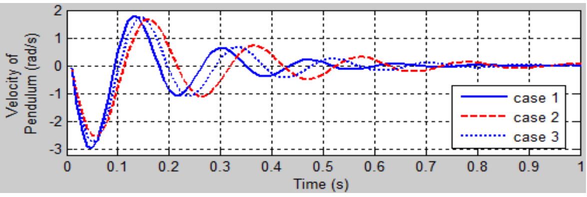

***Fig 6.** Velocity of Pendulum in three cases*

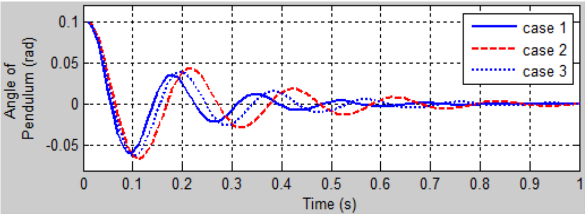

***Fig 5.** Angle Pendulum in three cases*

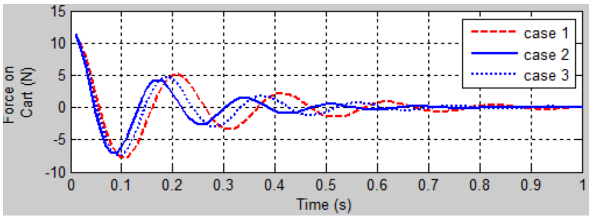

***Fig 7.** Force on Cart (N)*

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APPENDIX 1:

We denote by:

$$\Delta_1 = \gamma\gamma_2^2$$

$$\Delta_2 = 2c_1c_5\gamma_2 + c_2c_6\gamma_2 + c_6\gamma_2\gamma_3 + 2c_2c_6\gamma_2\gamma k$$

$$\Delta_3 = \gamma c_1^2c_2^2k^2 + c1^2c5^2k + 2\gamma c_1c_2c_5c_6k^2 + 2c_1c_2c_5c_6k + \gamma_3c_1c_5c_6k - 2\gamma_1\gamma c_1c_6k + \gamma_1c_1c_6 + \gamma c_2^2c_6^2k^2 + c_2^2c_6^2k - 2\gamma c_2c_5k + c_2c_5 +$$
$$\gamma_3c_2c_6^2k + \gamma_3c_5 + 2\gamma_1\gamma$$

$$\Delta_4 = \gamma c_1^2c_5^2\gamma_2^2k^2 - c_1^2c_5^2\gamma_2^2k$$

$$\Delta_5 = c_1c_5\gamma_1\gamma_2 - 2c_1c_2c_5^2\gamma_2k - c_1c_5^2\gamma_2\gamma_3k - 2c_1^2c_5c_6\gamma_1\gamma_2k + 2c_1c_2c_5^2\gamma_2\gamma k^2 - 2c_1c_5\gamma_1\gamma_2\gamma k + 2c_1^2c_5c_6\gamma_1\gamma_2\gamma k^2$$

$$\Delta_6 = \gamma c_1^2c_6^2\gamma 1^2k^2 - c_1^2c_6^2\gamma_1^2k + 2\gamma c_1c_2c_5c_6\gamma_1k^2 - 2c_1c_2c_5c_6\gamma_1k - \gamma_3c_1c_5c_6\gamma_1k - 2\gamma c_1c_6\gamma_1^2k + c_1c_6\gamma_1^2 + \gamma c_2^2c_5^2k^2 - c_2^2c_5^2k - \gamma_3c_2c_5^2k +$$
$$-2\gamma c_2c_5\gamma_1k + c_2c_5\gamma_1 + \gamma_3c_5\gamma_1 + \gamma\gamma_1^2$$

$$\Delta^*_{1\min/\max} = \gamma_{\min/\max}\gamma^2_{2\min/\max}$$

$$\Delta^*_{2\min/\max} = 2c_1c_5(\gamma_{2\min/\max}sign(c_1c_5X) + \gamma_{2\max/\min}sign(c_1c_5X)) + \{\gamma_{2\min/\max}sign(c_2c_6X) +$$
$$+\gamma_{2\max/\min}c_2c_6[1 - sign(c_2c_6X)]\} + \{\gamma_{2\min/\max}\gamma_{3\min/\max}sign(c_6X) + \gamma_{2\max/\min}\gamma_{3\max/\min}[1 - sign(c_6X)]\}c_6 +$$
$$+2\gamma kc_2c_6\{\gamma_{2\min/\max}sign(c_2c_6X) + \gamma_{2\max/\min}[1 - sign(c_2c_6X)]\}$$

$$\Delta^*_3 = c_1^2c_5^2k^2\gamma + kc_1^2c_5^2 + 2\gamma c_1c_2c_5c_6k^2 + 2c_1c_2c_5c_6k + c_1c_5c_6k\{\gamma_{3\min}sign(c_1c_5c_6) + \gamma_{3\max}[1 - sign(c_1c_5c_6)]\} +$$
$$-2k\gamma c_1c_6\{\gamma_{1\max}sign(c_1c_6) + \gamma_{1\min}[1 - sign(c_1c_6)]\} + c_1c_6\{\gamma_{1\min}sign(c_1c_6) + \gamma_{1\max}[1 - sign(c_1c_6)]\} + \gamma c_2^2c_6^2k^2 +$$
$$+c_2^2c_6^2k - 2\gamma kc_2c_5 + c_2c_5 + c_2c_6^2k\{\gamma_{3\min}sign(c_2) + \gamma_{3\max}[1 - sign(c_2)]\} + \{\gamma_{3\min}sign(c5) + [1 - sign(c_5)]\gamma_{3\max}\}c_5 +$$
$$+2\gamma\gamma_{1\min}$$

$$\Delta^*_{4\min/\max} = \gamma c_1^2c_5^2k^2\gamma^2_{2\min/\max} - c_1^2c_5^2k\gamma^2_{2\max/\min}$$

$$\Delta^*_{5\min/\max} = c_1c_5\{\gamma_{1\min/\max}\gamma_{2\min/\max}sign(c_1c_5) + \gamma_{1\max/\min}\gamma_{2\max/\min}[1 - sign(c_1c_5)]\} - 2kc_1c_2c_5^2\{\gamma_{2max/\min}sign(c_1c_2) +$$
$$+\gamma_{2\min/\max}[1 - sign(c_1c_2)]\} - c_1c_5^2k\{\gamma_{2\max/\min}\gamma_{3\max/\min}sign(c_1) + \gamma_{2\min/\max}\gamma_{3\min/\max}(1 - sign(c_1)]\} +$$
$$-2c_1^2c_5c_6k(\gamma_{1max/\min}\gamma_{2max/\min}sign(c_5c_6) + \gamma_{1\min/\max}\gamma_{2\min/\max}(1 - sign(c_5c_6))) + 2c_1c_2c_5^2k^2\gamma\{\gamma_{2\min/\max}sign(c_1c_2) +$$
$$+\gamma_{2\max/\min}[1 - sign(c_1c_2)]\} - 2c_1c_5k\gamma\{\gamma_{1\max/\min}\gamma_{2\max/\min}sign(c_1c_5) + \gamma_{1\min/\max}\gamma_{2\min/\max}[1 - sign(c_1c_5)]\} +$$
$$+2c_1^2c_5\gamma k^2\{\gamma_{1\min/\max}\gamma_{2\min/\max}sign(c_5c_6) + \gamma_{1\max/\min}\gamma_{2\max/\min}[1 - sign(c_5c_6)]\}$$

$$\Delta^*_{6\min/\max} = \gamma c_1^2c_6^2k^2\gamma^2_{1\min/\max} - c_1^2c_6^2k\gamma^2_{1\max/\min} + 2\gamma c_1c_2c_5c_6k^2\{\gamma_{1\max/\min}[1 - sign(c_1c_2c_5c_6)] + \gamma_{1\min/\max}sign(c_1c_2c_5c_6)\} +$$
$$-2c_1c_2c_5c_6k\{\gamma_{1\max/\min}sign(c_1c_2c_5c_6) + \gamma_{1\min/\max}[1 - sign(c_1c_2c_5c_6)]\} - c_1c_5c_6k\{\gamma_{1\max/\min}\gamma_{3\max/\min}sign(c_1c_5c_6) +$$
$$+\gamma_{1\min/\max}\gamma_{3\min/\max}[1 - sign(c_1c_5c_6)]\} - 2\gamma c_1c_6k\{\gamma^2_{1\max/\min}sign(c_1c_6) + \gamma^2_{1\min/\max}[1 - sign(c_1c_6)]\} + c_1c_6(\gamma^2_{1\min/\max}sign(c_1c_6) +$$
$$+\gamma^2_{1\max/\min}[1 - sign(c_1c_6)]) + \gamma c_2^2c_5^2k^2 - c_2^2c_5^2k - kc_2c_5^2\{\gamma_{3\max/\min}sign(c_2) + \gamma_{3\min/\max}[1 - sign(c_2)]\} +$$
$$-2c_2c_5k\gamma\{\gamma_{1\max/\min}sign(c_2c_5) + \gamma_{1\min/\max}[1 - sign(c_2c_5)]\} + c_2c_5\{\gamma_{1\min/\max}sign(c_2c_5) + \gamma_{1\max/\min}[1 - sign(c_2c_5)]\} +$$
$$+c_5\{\gamma_{1\min/\max}\gamma_{3\min/\max}sign(c_5) + \gamma_{1\max/\min}\gamma_{3\max/\min}[1 - sign(c_5)]\} + \gamma\gamma^2_{1\min/\max}$$

APPENDIX 2:

Lemma 1. The frequency condition (26) will be satisfied if the following conditions are satisfied:

CONDITION A: (i) or (ii) or (iii) or (iv)

CONDITION B: (v) or (vi) or (vii) or (viii) or (ix) or (x) or (xi) or (xii)

then $\forall X \in [-\eta_1\eta_2 \quad \eta_1\eta_2]$:

$$f_1(X) = \Delta_1 X^2 + \Delta_2 X + \Delta_3 0$$
$$f_2(X) = \Delta_4 X^2 + \Delta_5 X + \Delta_6 0$$

and

(i): $$\left(\left(\Delta^*_{1\min}(\eta_1\eta_2)^2 + \Delta^*_{2\min}\eta_1\eta_2 + \Delta^*_{3\min}\right) \geq 0\right) \cap \left(\left(\frac{\Delta^*_{2\max}}{2\Delta^*_{1\max}}\right) \leq -\eta_1\eta_2\right)$$

(ii): $$\left(\Delta^*_{2\max} \leq 0\right) \cap \left(\left(\frac{\Delta^*_{2\min}}{2\Delta^*_{1\min}}\right) \geq -\eta_1\eta_2\right) \cap \left(\Delta^*_{3\min} \geq 0\right)$$

(iii): $\left(\Delta^*_{2\min} \ge 0\right)\cap\left(\left(\frac{\Delta^*_{2\max}}{2\Delta^*_{1\min}}\right)\le \eta_1\eta_2\right)\cap\left(\Delta^*_{3\min}\ge 0\right)$

{iv): $\left(\left(\frac{\Delta^*_{2\min}}{2\Delta^*_{1\max}}\right)\ge\eta_1\eta_2\right)\cap\left(\left(\left(\Delta^*_{1\min}\right)^2\left(\eta_1\eta_2\right)^2-\Delta^*_{2\max}\eta_1\eta_2+\Delta^*_{3\min}\right)\ge 0\right)\cap\left(\Delta^*_{2\min}\ge 0\right)$

(v): $\left(\Delta^*_{5\min}\ge 0\right)\cap\left(\Delta^*_{4\max}\le 0\right)\cap\left(\left(\Delta^*_{4\min}\left(\eta_1\eta_2\right)^2+\Delta^*_{5\min}\eta_1\eta_2+\Delta^*_{6\min}\right)\ge 0\right)\cap\left(\left(-\frac{\Delta^*_{5\min}}{2\Delta^*_{4\min}}\right)\ge\left(\eta_1\eta_2\right)\right)$

(vi): $\left(\Delta^*_{5\min}\le 0\right)\cap\left(\Delta^*_{4\max}\ge 0\right)\cap\left(\left(\Delta^*_{4\min}\left(\eta_1\eta_2\right)^2+\Delta^*_{5\min}\eta_1\eta_2+\Delta^*_{6\min}\right)\ge 0\right)\cap\left(\left(-\frac{\Delta^*_{5\max}}{2\Delta^*_{4\max}}\right)\ge\eta_1\eta_2\right)$

(vii): $\left(\Delta^*_{5\min}\ge 0\right)\cap\left(\Delta^*_{4\min}\ge 0\right)\cap\left(\left(\frac{\Delta^*_{5\max}}{2\Delta^*_{4\min}}\right)\le\left(\eta_1\eta_2\right)\right)$

(viii): $\left(\Delta^*_{5\min}\ge 0\right)\cap\left(\Delta^*_{4\max}\le 0\right)\cap\left(\left(-\frac{\Delta^*_{5\max}}{2\Delta^*_{4\max}}\right)\le\eta_1\eta_2\right)$

(ix): $\left(\Delta^*_{5\max}\le 0\right)\cap\left(\Delta^*_{4\min}\ge 0\right)\cap\left(\left(-\frac{\Delta^*_{5\min}}{2\Delta^*_{4\min}}\right)\le\eta_1\eta_2\right)$

(x): $\left(\Delta^*_{5\max}\le 0\right)\cap\left(\Delta^*_{4\max}\le 0\right)\cap\left(\left(\frac{\Delta^*_{5\min}}{2\Delta^*_{4\max}}\right)\le\eta_1\eta_2\right)$

(xi): $\left(\Delta^*_{5\min}\ge 0\right)\cap\left(\Delta^*_{4\min}\ge 0\right)\cap\left(\left(\Delta^*_{4\min}\left(\eta_1\eta_2\right)^2-\Delta^*_{5\max}\eta_1\eta_2+\Delta^*_{6\min}\right)\ge 0\right)\cap\left(\left(\frac{\Delta^*_{5\max}}{2\Delta^*_{4\min}}\right)\ge\eta_1\eta_2\right)$

(xii): $\left(\Delta^*_{5\max}\le 0\right)\cap\left(\Delta^*_{4\max}\le 0\right)\cap((\Delta^*_{4\min}(\eta_1\eta_2)^2-\Delta^*_{5\max}\eta_1\eta_2+\Delta^*_{6\min})\ge 0)\cap\left(\left(\frac{\Delta^*_{5\min}}{2\Delta^*_{4\max}}\right)\ge\eta_1\eta_2\right)$

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