ROBUST OBSERVER DESIGN FOR UNCERTAIN POLYNOMIAL SYSTEMS

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ABSTRACT

This research proposes a new method to synthesize a robust observer for the polynomial systems in the presence of the uncertainties. Here, a special mathematical transformation is carried out to convert the uncertain polynomial systems into the unknown input polynomial systems and then the robust observer is designed for estimating the state variables of these systems. Moreover, unlike the existing papers where the bounded conditions of the uncertainties must be given, in this paper, the bounded constraints of the uncertainties are not known in advance and the influences of uncertainties are completely eliminated without using a controller or other techniques. Additionally, a new form of an observer is presented to reduce the complexity of derivation and make the observer synthesis procedure be more relaxed. On the basis of Lyapunov theory and Sum of Square (SOS) technique, the conditions for observer synthesis are derived. Finally, a numerical example is shown to prove the effectiveness of the proposed method.

Keywords: *Observer Design; Uncertain polynomial systems; Sum of Squares (SOS); Unknown Input Systems; Nonlinear systems.*

TÓM TẮT

Trong nghiên cứu này, chúng tôi trình bày một phương pháp mới để thiết kế bộ quan sát bền vững cho hệ thống đa thức với sự xuất hiện của các thành phần bất định. Để làm được điều này, hệ thống đa thức với thành phần bất định sẽ được chuyển thành hệ thống đa thức với đầu vào bất định, sau đó bộ quan sát bền vững sẽ được thiết kế cho hệ thống này. Khác với các nghiên cứu trước đây nơi mà điều kiện bao của các thành phần bất định phải được đưa ra, trong bài báo này, điều kiện bao của thành phần bất định không cần được biết trước, và những ảnh hưởng của thành phần bất định sẽ được loại bỏ hoàn toàn mà không cần sử dụng bộ điều khiển cũng như các kỹ thuậ tkhác. Ngoài ra, trong bài báo này, chúng tôi cũng đưa ra một mô hình mới cho bộ quan sát. Với mô hình mới này, sẽ giúp giảm đi độ phức tạp trong quá trình chứng minh và làm tăng tính linh hoạt của phương pháp so với các phương pháp cũ.Dựa trên lýt huyết Lyapunov và kỹ thuât SOS (tổng của các bình phương), chúng tôi đã xây dựng được các điều kiện cho việc thiết kế bộ quan sát bền vững. Phần cuối của bài báo, một ví dụ minh họa được trình bày để chứng minh tính hiệu quả của phương pháp mới trong bài báo này.

Từkhóa: *Thiết kế bộ quan sát; Hệ thống đa thức với thành phần bất định; Tổng các bình phương (SOS); Hê thống với đầu vào bất định; Hệ phi tuyến.*

1. INTRODUCTION

Solving the problem of nonlinear systems is always a challenge for many researchers. There are many methods developed to resolve the nonlinear systems. Among of them, polynomial systems which represent the nonlinear system as a linear form with polynomial matrices are examples [1]. Fortunately, by the aid of Sum of Squares techniques [2], the polynomial systems become an effective tool for solving the issues of nonlinear systems. Recently, there are many papers studying about polynomial systems. For example, the state feedback controller for the polynomial system was investigated in which a new transformation technique was introduced to make the conditions for controller design more relaxed [3]. In addition, the discrete-time controller synthesis for the discrete time polynomial systems was concerned in [4]. Moreover, a sliding mode controller was investigated in [5] for stochastic polynomial systems where the state variables of this system were unavailable.

In numerous practical systems, some or all state variables are immeasurable due to the reason of technology. However, it is really necessary for controller design or monitoring systems. Therefore, many researchers have seriously paid attention to designing observer. In terms of designing observer for polynomial systems, it has been considered in several papers in recent decades [4], [6-8]. For example, in [4], a filter which was incorporated with an integrator was synthesized for discrete time polynomial systems. Furthermore, the observer synthesis for the polynomial system which bounded disturbance was concerned in [6] or observer was designed for the system with fixed and unknown delay times in [7] and [8], respectively.

In practice, numerous nonlinear systems were affected by the uncertainties. These uncertain terms did make the controller, observer design more complicated. In order to overcome this difficulty, there existed many methods developed in a number of the previous papers [9-13]. For instance, the output feedback controller for the uncertain polynomial system was designed in [9] with iterative SOS approach employed to obtain controller parameters. However, the uncertainties in [9] and [10] must fulfill the bounded constraints and using iterative SOS approach will be time-consuming. Additionally, the robust controller was designed for discrete-time and continuous time polynomial systems in [11] and [12], respectively. In [13], the state feedback controller incorporating with integrator was investigated in [13] where the influence of uncertainties was eliminated by employing the H_∞ technique.

With above analyses and the reviewing literature, it is seen that the observer design for uncertain polynomial systems is not considered. In addition, in previous studies, the upper bounds of uncertainties must be given in advance and controller or H_∞ technique should be used to eliminate the effects of uncertainties, otherwise, it is impossible to get feasible solutions. Recently, an approach based on unknown input method to designing observer has been proposed in [14] that allowed to synthesize the observer and to eliminate the unknown input without knowing bound constraints of them. In this paper, we aim to propose a new approach based on unknown input method to synthesize observer for polynomial systems with uncertainties in which the upper bounds of uncertainties are unknown. Unfortunately, the method in [14] remains the drawback when the parameters of observer form still exist in the constant matrix. In addition, it is very difficult to solve condition (18) of Theorem 1 in [14] because matrix *R(y)* is polynomial matrix while others are constant matrices. We also try to extend the observer form (5) of [14] with polynomial matrix *E(y)* instead of constant matrix *E*, however, this form causes the derivation procedure to be extremely complicated. Due to this reason, we propose a new form of the observer for uncertain polynomial systems that will not only be more relaxed than that the form in [14] but also reduce the complexity during derivation.

The rest of this paper is organized as follows. The system model and problem description are presented in Section 2. The observer design with two main theorems is shown in Section 3. The illustrated example is in Section 4 and a conclusion is presented in Section 5.

Notations: $A>0$ denotes the positive definite matrix A; A^T denotes the transpose of matrix A; A^{-1} denotes the inverse of A; A^{+} denotes the Moore-Penrose pseudo-inverse of A and $A^{+}=(A^TA)^{-1}A^T$. The symbol $\Re^{n\times m}$ denotes the set of $n\times m$ matrices; *I* denotes the identity matrix.

2. SYSTEM MODEL AND PROBLEM DESCRIPTION

2.1 System model

Let us consider the polynomial system with uncertainties as follows:

$$\begin{cases}\dot{x}(t)=(A(\xi(t))+\Delta A(\xi(t)))x(t)\\ \qquad +(B(\xi(t))+\Delta B(\xi(t)))u(t)\\ y(t)=Cx(t)\end{cases}\tag{1}$$

where $x(t)\in\Re^{n}$, $y(t)\in\Re^{p}$ and $u(t)\in\Re^{m}$ are vectors of states, outputs and inputs, respectively. The polynomial matrices $A(\xi(t))\in\Re^{n\times n}$, $B(\xi(t))\in\Re^{n\times m}$ are the system matrices and a constant matrix C is output matrix. $\Delta A(\xi(t))$ and $\Delta B(\xi(t))$ are the uncertainties of matrices $A(\xi(t))$ and $B(\xi(t))$. $\xi(t)$ is the measurable variable that could be the external signal, output *y(t)* or time.

From now on, in order to save the typing space, the brief forms *x*, *y*, *u*, and ξ are used instead of $x(t)$, $y(t)$, $u(t)$, and $\xi(t)$.

Proposition 1[4]: assume $p(x)$ is a SOS, $p(x)$ can be express as $p(x)=\sum_{l=1}^{n}q_l(x)^2$, where $q_l(x)$ is a polynomial in x. Therefore, when $p(x)$ is determined as the SOS, it implies that $p(x)\geq 0$.

2.2 Problem description

Suppose that all or some state variables of the system (1) are unavailable, however, they are necessary for controller design or system supervision. Therefore, this paper addresses to synthesize an observer for (1) to estimate the state variables. However, due to the effects of uncertainties, it is very difficult to design the observer for the system (1). In order to overcome this challenge, several methods have been proposed in the previous papers to eliminate the influences of uncertainties. The upper bounds of the uncertainties in these studies, however, must have been known in advance and the controller was needed to eliminate the impacts of uncertainties. Otherwise, it is infeasible to synthesize observer for the system (1). To overcome this drawback, this paper proses a new approach based on unknown input method to not only eliminate the effects of uncertainties but also guarantee the estimate states approach to real state asymptotically. Before synthesizing observer, the following assumption is needed.

Assumption 1: Assume that $\Delta A(\xi) = D(\xi).\Delta a(\xi)$, $\Delta B(\xi) = D(\xi).\Delta b(\xi)$, $D(\xi)$ is the polynomial matrix, $\Delta a(\xi)$, and $\Delta b(\xi)$ are time-varying uncertainties. $D(\xi)$ and C are full normal column rank and full row rank; rank ($CD(\xi)$)=rank($D(\xi)$).

3. OBSERVER DESIGN

On the basis of the Assumption 1, the uncertain polynomial system (1) is transformed into unknown input polynomial system (2)

$$\begin{cases} \dot{x} = A(\xi)x + B(\xi)u + D(\xi)\gamma(t) \\ y = Cx \end{cases} \tag{2}$$

where $\gamma(t) = \Delta a(\xi)x(t) + \Delta b(\xi)u(t)$.

The system (1) has been turned into unknown input polynomial system (2), therefore, the observer will be synthesized for (2) instead of (1).

For primary thinking, we try to use the conventional observer form in [14], however, this form has brought two drawbacks as we mention in Section 1. Owing to this reason, this paper proposes a new observer form as follows:

$$\begin{cases} \dot{\hat{x}} = N(\xi)\hat{x} + G(\xi)u + L(\xi)y + F(\xi)\dot{y} \\ \hat{y} = C\hat{x} \end{cases} \tag{3}$$

where $\hat{x}$ and $\hat{y}$ are the estimation of $x(t)$ and $y(t)$.

It is seen that all parameters of (3) are the polynomial matrices. It is less conservative with respect to the observer form in [14], because there exist a constant matrix parameter in observer in [14].

The estimation error is defined as follows

$$e(t) = x - \hat{x} \tag{4}$$

Taking the derivative of (4) yields

$$\dot{e}(t) = \dot{x} - \dot{\hat{x}} \tag{5}$$

Theorem 1: Considering system (2), the estimation error (4) with observer (3) is approach zero asymptotically if there exist matrices $N(\xi)$, $G(\xi)$, $L(\xi)$, $F(\xi)$ and a symmetric matrix P satisfying the following conditions:

$$A(\xi) - F(\xi)CA(\xi) - L(\xi)C - N(\xi) = 0 \tag{6}$$

$$B(\xi) - F(\xi)CB(\xi) - G(\xi) = 0 \tag{7}$$

$$D(\xi) - F(\xi)CD(\xi) = 0 \tag{8}$$

$$v^T(P - \varepsilon_1)v \text{ is SOS} \tag{9}$$

$$-v^T(N^T(\xi)P + PN(\xi) + \varepsilon_2(\xi))v \text{ is SOS} \tag{10}$$

where v_1 and v_2 are vectors with appropriate dimensions that do not depend on ς , $\varepsilon_1 > 0$, and $\varepsilon_2(\varsigma) > 0$.

Proof:

Substituting (2) and (3) into (5), it obtains

$$\begin{aligned} \dot{e}(t) &= [A(\xi)x + B(\xi)u + D(\xi)\gamma(t)] \\ &-[N(\xi)\hat{x} + G(\xi)u + L(\xi)y + F(\xi)\dot{y}] \end{aligned} \tag{11}$$

From (2), we have

$$\dot{y} = C\dot{x} = CA(\xi)x + CB(\xi)u + CD(\xi)\gamma(t) \tag{12}$$

Substituting (12) into (11), we obtain

$$\begin{aligned} \dot{e}(t) &= \big[A(\xi)x - F(\xi)CA(\xi)x - L(\xi)Cx \\ &-N(\xi)\hat{x} + B(\xi)u - F(\xi)CB(\xi)u - G(\xi)u \\ &+D(\xi)\gamma(t) - F(\xi)CD(\xi)\gamma(t)\big] \\ &= \big[(A(\xi) - F(\xi)CA(\xi) - L(\xi)C - N(\xi))x \\ &+N(\xi)(x - \hat{x}) + (B(\xi) - F(\xi)CB(\xi) - G(\xi))u \\ &+(D(\xi) - F(\xi)CD(\xi))\gamma(t)\big] \end{aligned} \tag{13}$$

If the conditions (6)-(8) hold, (13) is equivalent to

$$\dot{e}(t) = N(\xi)e(t) \tag{14}$$

Select the Lyapunov function:

$$V(t) = e^T(t)Pe(t) \tag{15}$$

Taking derivation of (15) yields

$$\dot{V}(t) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \tag{16}$$

Substitute (14) into (16), one obtains

$$\begin{aligned}\dot{V}(t) &= e^T N^T(\xi)Pe + e^T PN(\xi)e \\ &= e^T\{N^T(\xi)P + PN(\xi)\}e\end{aligned} \tag{17}$$

From condition (10), it implies that $\dot{V}(t) < 0$ then the estimation error (4) is guaranteed to converge to zero asymptotically. Proof is completed.

Because (10) is Polynomial Bilinear Matrix Inequality (PBMI) which is difficult to solve by SOS tool in Matlab, therefore, it needs to transform into Polynomial Linear Matrix Inequality (PLMI). The Theorem 2 is necessary for transforming PBMI (10) into PLMI.

Theorem 2: Considering system (2), the estimation error (5) with observer (3) is approach zero asymptotically if there exist matrices $N(\xi)$, $G(\xi)$, $L(\xi)$, $F(\xi)$, $X(\xi)$, $Q(\xi)$ and a symmetric matrix P satisfying the following conditions:

$$\begin{aligned}&(A(\xi) - (U(\xi)CA(\xi)))^T P - (V(\xi)CA(\xi))^T X^T(\xi) \\ &- C^T Q^T(\xi) + P(A(\xi) - (U(\xi)CA(\xi))) \\ &- X(\xi)(V(\xi)CA(\xi)) - Q(\xi)C < 0\end{aligned} \tag{18}$$

where:

$$(CD(\xi))^+ = ((CD(\xi))^T(CD(\xi))^{-1}(CD(\xi))^T$$

$$U(\xi) = D(\xi)(CD(\xi))^+ \tag{19}$$

$$V(\xi) = (I - (CD(\xi))(CD(\xi))^+) \tag{20}$$

$$X(\xi) = PY(\xi) \tag{21}$$

$$\mathrm{Q}(\xi) = PL(\xi) \tag{22}$$

And the parameters of observer (3) are computed as follows

$$F(\xi) = U(\xi) + Y(\xi)V(\xi) \tag{23}$$

$$G(\xi) = B(\xi) - F(\xi)CB(\xi) \tag{24}$$

$$L(\xi) = P^{-1}Q(\xi) \tag{25}$$

$$N(\xi) = A(\xi) - F(\xi)CA(\xi) - L(\xi)C \tag{26}$$

Proof:

From (8), we have:

$$F(\xi)CD(\xi) = D(\xi) \tag{27}$$

General solution of (27) is

$$\begin{aligned}F(\xi) &= D(\xi)(CD(\xi))^+ \\ &+ Y(\xi)(I - (CD(\xi))(CD(\xi))^+)\end{aligned} \tag{28}$$

where $Y(\xi)$ is arbitrary chosen and the Moore-Penrose pseudo-inverse of $CD(\xi))$

$$(CD(\xi))^+ = ((CD(\xi))^T(CD(\xi)))^{-1}(CD(\xi))^T.$$

Denote

$$U(\xi) = D(\xi)(CD(\xi))^+ \tag{29}$$

$$V(\xi) = (I - (CD(\xi))(CD(\xi))^+) \tag{30}$$

Substituting (29) and (30) into (28) yields

$$F(\xi) = U(\xi) + Y(\xi)V(\xi) \tag{31}$$

From (31) and (6), we have

$$\begin{aligned}N(\xi) &= A(\xi) - (U(\xi)CA(\xi)) \\ &- (Y(\xi)V(\xi)CA(\xi)) - L(\xi)C\end{aligned} \tag{32}$$

From (10) of Theorem 1, it implies that

$$N^T(\xi)P + PN(\xi) < 0 \tag{33}$$

Substituting (33) into (34) obtains

$$\begin{aligned}&(A(\xi) - (U(\xi)CA(\xi)))^T P \\ &- (V(\xi)CA(\xi))^T Y^T(\xi)P - C^T L^T(\xi)P \\ &+ P(A(\xi) - (U(\xi)CA(\xi))) \\ &- PY(\xi)(V(\xi)CA(\xi)) - PL(\xi)C < 0\end{aligned} \tag{34}$$

Let:

$$\begin{aligned}X(\xi) &= PY(\xi) \\ \mathrm{Q}(\xi) &= PL(\xi)\end{aligned} \tag{35}$$

Substitute (31) into (30), we obtain

$$\begin{aligned}&(A(\xi)-(U(\xi)CA(\xi)))^T P-(V(\xi)CA(\xi))^T X^T(\xi)\\&-C^T Q^T(\xi)+P(A(\xi)-(U(\xi)CA(\xi)))\\&-X(\xi)(V(\xi)CA(\xi))-Q(\xi)C<0\end{aligned} \qquad (36)$$

It is seen that (36) is PLMIs and is the same with (18) of Theorem 2, it means that PBMI (10) is transformed PLMI which is solved by SOS tool easily. Proof of Theorem 2 is completed.

4. ILUSSTRATIVE EXAMPLE

Consider the nonlinear system as follows

$$\begin{cases}\dot{x}_1(t)=-0.2x_2(t)+0.5x_3{}^3(t)+0.5x_3{}^2u(t)\\ \dot{x}_2(t)=(0.3-x_3(t))x_1(t)-0.1x_2(t)+0.3u(t)\\ \dot{x}_3(t)=(0.3-x_3(t))x_2(t)-0.6x_3(t)+0.4u(t)\\ y(t)=x_3(t)\end{cases} \qquad (37)$$

Nonlinear system (37) can be rewritten as the form (1) with $\xi=y$ and we assume that this system is affected by uncertainties.

$$\begin{cases}\dot{x}=(A(y)+\Delta A(y))x+(B(y)+\Delta B(y))u(t)\\ y=Cx\end{cases} \qquad (38)$$

where

$$A(y)=\begin{bmatrix}0 & -0.2 & 0.5y^2\\ 0.3-y & -0.1 & 0\\ 0 & 0.3-y & -0.6\end{bmatrix}$$

$$B(y)=\begin{bmatrix}0.5y^2\\ 0.3\\ 0.4\end{bmatrix},\quad C=\begin{bmatrix}0 & 0 & 1\end{bmatrix},$$

The uncertainties:

$$\Delta A(y)=\begin{bmatrix}-0.01y & -0.01 & -0.02y-0.1\\ -0.02y^2-0.2y & 0.01y+0.1 & 0.02y^2+0.1y-1\\ -0.2y & 0.01 & 0.02y-0.1\end{bmatrix}$$

$$\Delta B(y)=\begin{bmatrix}0.1y\\ 0.1y^2+y\\ 0.1y\end{bmatrix}.$$

Based on Assumption 1, we may transform (38) into the form of system (2) with

$$D(y)=\begin{bmatrix}0.1\\ 0.1y+1\\ 0.1\end{bmatrix}.$$

The conditions of Theorem 2 are solved by SOS tool of Matlab and the parameters of observer (3) are obtained as follows

$$L(y)=\begin{bmatrix}-2588.508y^2+36949.196y-99509.890\\ -36056.363y^2+438484.346y-895216.651\\ 1.698y^2-1.729y+9.300\end{bmatrix}$$

$$N(y)=\begin{bmatrix}0 & y-0.5 & \begin{matrix}2589.0y^2-36949.1y\\+99510.4\end{matrix}\\ 0.3-y & y^2+9.7y-2.9 & \begin{matrix}36056.3y^2-438483.7y\\+895222.6\end{matrix}\\ 0 & 0 & \begin{matrix}-1.6y^2+1.7y\\-9.3\end{matrix}\end{bmatrix}$$

$$F(y)=\begin{bmatrix}1\\ y+10\\ 1\end{bmatrix},\quad G(y)=\begin{bmatrix}0.5y^2-0.4\\ -0.4y-3.7\\ 0\end{bmatrix}.$$

Simulink of Matlab has been used to simulate and obtain the following results.

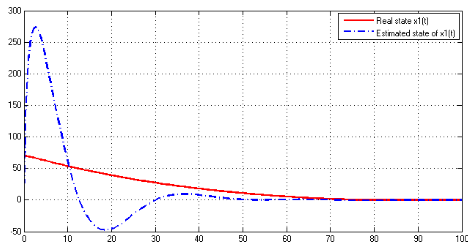

Fig.1 *Real state* $x_1(t)$ *and estimated* $\hat{x}_1(t)$

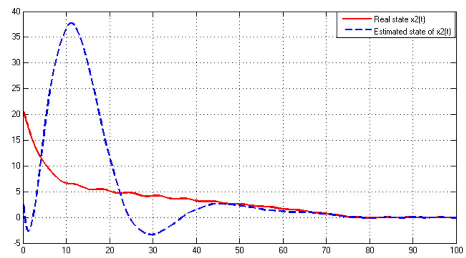

Fig.2 *Real state* $x_2(t)$ *and estimated* $\hat{x}_2(t)$

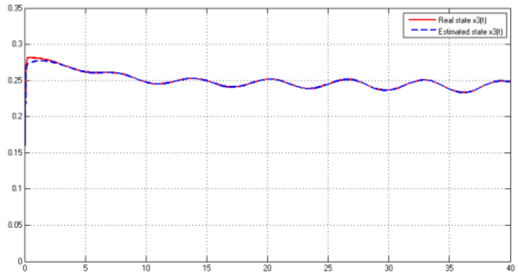

Fig.3 *Real state* $x_3(t)$ *and estimated* $\hat{x}_3(t)$

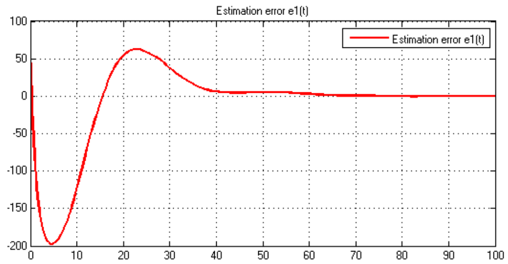

Fig.4 *Estimation error* $e_1(t)$

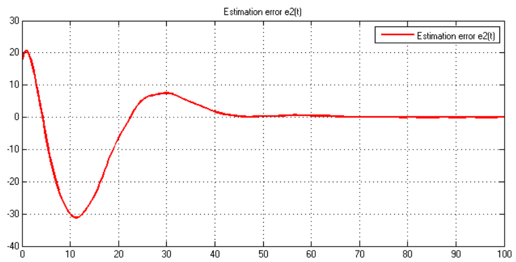

Fig.5 *Estimation error* $e_2(t)$

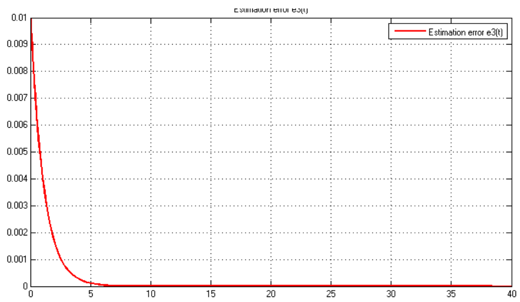

Fig.6 *Estimation error* $e_3(t)$

The simulation results are demonstrated in Fig 1 to Fig 6. The real states x_1, x_2, x_3 and their estimations are expressed in Fig. 1, Fig. 2 and Fig.3, respectively. Additionally, the estimation errors e_1, e_2, and e_3 are presented in Fig 4-6. From these figures, it is clearly seen that the estimated states approach to real states asymptotically. It means that the proposed method successfully synthesized observer for the uncertain polynomial system (1).

5. CONCLUSION

In this paper, the approach based on unknown input to design observer for the uncertain polynomial system is presented. The bounded constrains of uncertainties are unknown in advance and the effects of uncertainties are eliminated without using the controller. By aids of Lyanopunov and SOS techniques, two Theorems for observer design are derived. The simulation results of the example haveproved the effectiveness of the proposed method.

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