SEP ANALYSIS OF ESTIMATED CHANNEL IN DECODE-AND-FORWARD COOPERATIVE NETWORKS

PHÂN TÍCH XÁC SUẤT LỖI KÝ TỰ CHO MẠNG HỢP TÁC GIẢI MÃ VÀ CHUYỂN TIẾP DÙNG ƯỚC LƯỢNG KÊNH

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TÓM TẮT

Khi xét đến giới hạn trong thiết kế kích thước của thiết bị cầm tay, mạng chuyển tiếp được xem là hệ thống MIMO ảo đầy hứa hẹn để cải thiện chất lượng mạng vô tuyến. Trong bài báo này, chúng tôi phân tích chất lượng mạng vô tuyến chuyển tiếp hai chặng dùng giao thức Điều chế và Chuyển tiếp (DF). Cụ thể hơn, chúng tôi đưa ra công thức tính gần đúng cho xác suất lỗi ký tự (SEP). Bằng cách sử dụng thuật toán ước lượng kênh bình phương cực tiểu trong tính toán giá trị thông tin trạng thái kênh (CSI), chúng tôi tính công thức SEP xấp xỉ dựa trên giá trị kênh đã ước lượng và trạng thái giải mã ở node chuyển tiếp. Công thức xấp xỉ SEP dùng để đánh giá việc phân bổ công suất trong mạng chuyển tiếp hai chặng. Các kết quả mô phỏng về SEP chứng tỏ ưu điểm của hệ thống.

***Từ khóa**: Điều chế -và- Chuyển tiếp (DF), xác suất lỗi ký tự (SEP), mạng chuyển tiếp hai chặng*

ABSTRACT

When the sizes of end-user equipments are limited, the cooperative relay networks are recommended as promising virtual MIMO systems to improve the performance of wireless communication systems in literature. In this paper, we focus on performance analysis of two-hop relay wireless networks using Decode-and-Forward (DF) scheme. Specifically, we derive the closed-form expression of symbol error probability (SEP). Thanks to utilizing least square-based channel estimation algorithm in calculation of instantaneous channel state information (CSI), we obtain approximate SEP based on the estimated channel states and the decoding state at the relay nodes. The SEP approximation formula is used to assess the power allocation in dual-hop networks. Simulation results for a two-hop system with optimal SEP show significant improvement.

***Keywords**: Decode and Forward (DF); symbol error probability (SEP); dual-hop networks.*

I. INTRODUCTION

The cooperative communication has emerged as a solution to increase the spectral efficiency and coverage of cellular networks. The basic idea of cooperative communications is that all mobile users or nodes in wireless networks help each other to send out not only by the user but also by other users cooperatively. A source node, along with relay nodes or neighborhood nodes that overhear the source node are willing to share their sources. Hence, these nodes create a virtual antenna array and mimic a multi-input-multi-output (MIMO) antenna system. Since its introduction, cooperative communication has attracted much attention, and there are a rich literature [1-4]. When one node helps other to forward data, it is so-called the relay one, and in this process amplify-and-forward (AF) or decode-and-forward (DF) cooperative protocol can be applied in multi-hop networks [5, 6]. The DF relaying node detects and retransmits the received signal. In addition, the DF relay scheme can be further classified into two types: i) fixed DF relay scheme (FDF): the relay always detects and forwards the signal it has received from source and ii) adaptive DF relay scheme (ADF): the relay decides to forward the received signal only when the evaluated signal quality satisfies the predefined metric, otherwise, it keeps silent.

A common underlying assumption in recent literature is the availability of perfect channel state information at the relay or destination receivers [7, 8]. However, in real systems, CSI needs to be precisely estimated and can then be used in the channel equalizers. The performance of multi-hop systems in the presence of imperfect CSI has been investigated [9-11]. In [11], Wu and Patzold have derived and approximate expression for SEP of AF-based multi-hop network over time-varying fading channels thanks to the moment generation function (MGF) approach. Despite the multiplying numbers of literature and recent research on DF-based cooperative scheme, but most of them assumed that having exact CSI. To our best knowledge, very few papers considered channel estimation error [12]. In this paper, we consider DF relay and derive a closed-form expression for power allocation subject to minimum SEP over flat fading channel. We also assume the deployment of the pilot-symbol-assisted channel estimation algorithm [13]. The analysis here is carried for two-state phase-shift keying (BPSK), but it can be extended to general M-ary quadrature amplitude modulation.

The remainder of this paper is organized as follows. In section II, we present the system model under consideration. In section III, we present the details on the approximate average SEP derivation. In next section, we provide numerical results to confirm the analytical derivations and provide insight into system performance and conclusions are drawn in section V.

Notation: the upper letter stands for matrices, $(.)^T$ stands for the transpose, $E\{.\}$ denotes the expectation operation.

II. SYSTEM MODEL

Consider a dual hop relay network with a single source with single antenna transmit data to a single destination through N relay nodes. The relay nodes are assumed to be half-duplex, that is, the relays with transmit or receive the signal at the same frequency at time instant. In relay network, data transmission between the source and the destination occurs in two phases. In the first phase, the source node transmits message to the destination and the relay nodes. Whereas, the second phase, relay one retransmit the exact decoded signal to the destination. The channels in this system are including of channel from source to relay, direct link between source and destination channels from relay to destination. These channels are assumed to be Rayleigh flat fading and independent from each other. The source and relay nodes utilize BPSK modulation. In this paper, we use DF cooperation scheme for transmit signal from source to destination through processing algorithm in DF-based relay nodes. The received signal at the destination and the i^{th} relay node in the first phase can be written as follow [7]

$$\begin{aligned} y_{s,d} &= \sqrt{p_0} h_{s,d} x + n_{s,d} \\ y_{s,i} &= \sqrt{p_0} h_{s,i} x + n_{s,i}, \quad i = 1, \ldots, N \end{aligned} \tag{1}$$

where x is the source message with unit power, p_0 is the transmit power of the source node, $h_{s,d}$ and $h_{s,i}$ denote the channel coefficients between the source and the destination and between the source and the i^{th} relay node, respectively, and $n_{s,d}$ and $n_{s,i}$ are the complex additive white Gaussian noises (AWGN) in the destination and in the i^{th} relay node, respectively.

The channel gain of each hop is chosen as a complex Gaussian random variable with zero mean and variance $\sigma_i^2/2$ per complex dimension. In general, this channel parameter includes path loss, shadowing, and Rayleigh fading. Assuming that the channel variations are slow compared to the length of a packet. We also assume each i^{th} relay node has the same transmitting power of p_i, and the variance of the additive white Gaussian noise is $N_0/2$ per complex dimension. The average SNR of the channel including the path loss and shadowing at the receiver node can be written as follow [7]

$$\gamma_i = p_i \sigma_i^2 / N_0 \tag{2}$$

After decoding the received signal based on estimated channel from the source nodes, only the relay nodes which have decodes the source message correctly retransmit it to the destination. Ultilizing cyclic redundancy check (CRC) scheme, the relay node can decode the received signal correctly or not. The probability that i^{th} relay node can decode the received signal correctly conditioned on the instantaneous estimated CSI can be described as

$$\alpha_i\left(\hat{h}_{s,i}\right) = 1 - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{\left|\hat{h}_{s,i}\right|^2 p_0}{\sin^2(\theta) N_0}} d\theta \tag{3}$$

where $\hat{h}_{s,i}$ is estimated channel coefficient calculated by least square-based estimation technique. This parameter can be found as

$$\hat{h}_{s,i} = \left(x^H x\right)^{-1} x^H y \tag{4}$$

We denote $\boldsymbol{\varphi} = \left(\phi_1, \phi_2, \ldots, \phi_N\right)^T$ as a vector indicating whether each relay has decoded the source message correctly or not. It can be seen that $\boldsymbol{\varphi}(i) = 1$ if i^{th} relay node decodes the source message correctly while $\boldsymbol{\varphi}(i) = 0$, otherwise. We refer to the vector $\boldsymbol{\varphi}$ as the vector of decoding state at the relay nodes and it consists of only binary numbers. The received signal from the i^{th} relay node which is able to decode the source message correctly, that is, $\boldsymbol{\varphi}(i) = 1$, at the destination can be written as

$$y_{i,d} = \sqrt{p_i} h_{i,d} x + n_{i,d} \tag{5}$$

where p_i is the transmit power of the i^{th} relay node, $h_{i,d}$ is the channel between the i^{th} relay node and the destination, and $n_{i,d}$ is the AWGN with zero mean and variance N_0. The channel $h_{i,d}$ is an independent complex Gaussian random variable with zero mean and variance of $\sigma_i^2/2$. It is worth stressing that the assumption that channel are independent from each other is applicable for dual hop relay network because the distances between different relay nodes are typically large enough.

At destination node, we apply the maximal ratio combining (MRC) to combine received signals from the source and relay nodes. The received SNR at the destination conditioned on the decoding state at the relay nodes can be written as

$$\gamma_D(\phi_k) = \gamma_s + \sum_{i|\phi_k(i)=1} \gamma_i \tag{6}$$

where γ_s and γ_i are the received SNRs at the destination from, respectively, the source and the i^{th} relay node.

III. ANALYSIS OF AVERAGE SEP

Assuming that data transmission in cooperative network using BPSK modulation, the calculation of SEP at the destination node conditioned on the estimated channel $EC = \left\{\hat{h}_{s,d}, \hat{h}_{s,i}, \hat{h}_{i,d}\right\}$ and the decoding state at the relay nodes, ϕ_k , can be calculated as [7]

$$P_e\{EC, \phi_k\} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma_D(\phi_k)}{\sin^2\theta}\right) d\theta \tag{7}$$

We have the total probability rule, and hence the expression of conditioned SEP can be rewritten

$$P_e\{EC\} = \sum_{k=0}^{2^N-1} P_r(\phi_k) P_e\{EC,\phi_k\} \qquad (8)$$

where $P_r(\phi_k)$ is the probability of the decoding state ϕ_k that can be described as

$$P_r(\phi_k) = \prod_{i|\phi_k(i)=1}\alpha_i \cdot \prod_{i|\phi_k(i)=0}(1-\alpha_i) \qquad (9)$$

The average SEP can be calculated with the fact that $P_r(\phi_k)$ is statistically independent from $P_e\{EC,\phi_k\}$ while $P_r(\phi_k)$ depends only on $\hat{h}_{s,i}, i = 1,\ldots,N$ and $\hat{h}_{s,d}$ conditioned on channel estimation error $\left|h-\hat{h}\right| \le \delta$, where acceptable threshold. Then the average SEP can be written as

$$P_e = \sum_{k=0}^{2^N-1} E\{P_r(\phi_k)\} E\{P_e(EC,\phi_k)\} \qquad (10)$$

Using results from [7], we have the first expectation can be computes as

$$E\{P_r(\phi_k)\} = E\left\{\prod_{i|\phi_k(i)=1}\alpha_i \cdot \prod_{i|\phi_k(i)=0}(1-\alpha_i)\right\}$$
$$= \prod_{i|\phi_k(i)=1}\beta_i \cdot \prod_{i|\phi_k(i)=0}(1-\beta_i) \qquad (11)$$

where

$$\beta_i = 1 - \frac{1}{\pi}\int_0^{\pi/2} \frac{\sin^2\theta}{\sin^2\theta + c_i p_0}$$

and $c_i = \sigma_{s,i}/N_0, i = 1,\ldots,N$

In addition, we also have the second expectation can be written as

$$E\{P_e(EC,\phi_k)\} = \int_0^{\pi/2} \prod_{i|\phi_k(i)=1\cup i=0} \frac{\sin^2\theta}{\sin^2\theta + b_i p_i} d\theta \qquad (12)$$

where $b_i = \sigma_{s,i}/N_0, i = 1,\ldots,N$

Finally, we can find out the following closed-form expression for the average SEP [7]

$$P_e = \frac{1}{8}\sum_{i=0}^{N}\left(\beta_i\left(\prod_{j|j=0,\ldots,N-\{i=j\}}\left(\frac{\beta_j}{1-\frac{b_j p_j}{b_i p_i}} + (1-\beta_j)\right)\right)\right) \qquad (13)$$

IV. SIMULATION RESULTS

We consider a dual hop relay network consisting of a source-destination pair. In the first phase, source transmits its message to the destination and the relay nodes, while in the second phase, only the relay nodes, which decoded the source message correctly, retransmit it to the destination. All kind of nodes use BPSK modulation technique and noise power is assumed to be equal to 1. Assuming that the channel coefficients are modeled as zero mean complex Gaussian random variables. Positions of the dual hop network with 7 relay nodes calculated from source node are randomly selected according to the uniform distribution and are (0.0125, 0.1378, 0.2963, 0.4298, 0.5721, 0.6901, 0.8251). The total power is equally devided among the source and the delay nodes.

The first experiment (Figure 1) shows the relationship between total power versus and symbol error probability. This simulation result illustrates comparison of approximate SEP corresponding to the closed-form expression (13) of different number of relay nodes using LS-based estimated channel. It is can be seen that the larger relay network lead to better performance than small relay network. As a result, the gap in performance increases further with increase in number of relay nodes

The second experiment (in Figure 2) depicts the SEP of 7 relay node using different modulation technique. Using more level signal modulation will get lower SEP than another.

V. CONCLUSION

In this paper, we briefly studied a new simple closed-form expression for the DF two-hop network. Hence, a new power allocation algorithm has been developed by minimizing the average SEP under constraints on the total power of the relays and the maximum powers of each relay node. Through the simulated results, the power allocation depend on the average probability of correct decoding by relay node and average channel gain-to-noise rate for selected relay node.

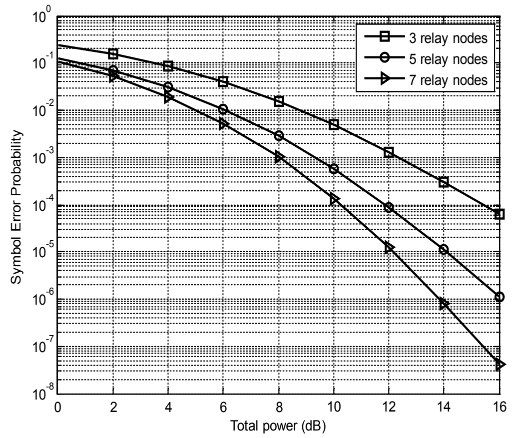

Fig. 1. The average SEP of the different number of relay nodes.

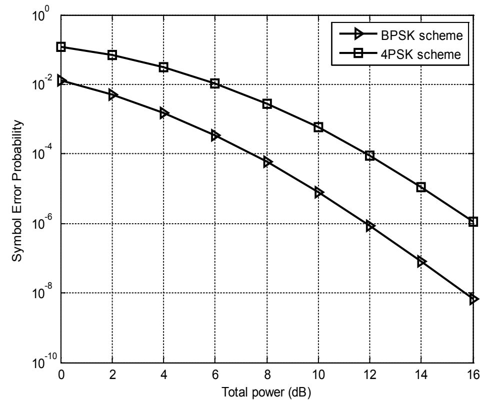

Fig. 2. The performances of dual hop cooperative networks with 7 relay nodes.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory,* vol. 50, pp. 3062–3080, 2004.

[2] A. Sendonaris, E. Erkrip, and B. Aazhang, "User cooperation diversity - Part I: System description," *IEEE Trans. Commun.,* vol. 51, pp. 1927–1938, 2003.

[3] A. Sendonaris, E. Erkrip, and B. Aazhang, "User cooperation diversity - Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.,* vol. 51, pp. 1939–1948, 2003.

[4] B. Barua, M. Abolhasan, and F. Safaei, "On the Symbol Error Probability of Multi-hop Parallel Relay Networks," *IEEE Commun. Letter,* vol. 15, pp. 719-721, July 2011.

[5] V. Asghari, A. Maaref, and S. Aissa, "Symbol Error Probability Analysis for Multi-hop Relaying over Nakagami Fading Channels " in *Proc. of Wireless Communications and Networking Conference (WCNC)* Sydney, Australia 2010, pp. 1 - 6

[6] S. Ikki and M. H. Ahmed, "Performance of decode-and-forward coop-erative diversity networks over Nakagami-m fading channels," in *Proc. of IEEE GLOBECOM'07*, Washington, DC, Nov. 2007, pp. 4328–4333.

[7] A. Khabbazibasmenj and S. A. Vorobyov, "Power Allocation in Decode-and-Forward Cooperative Networks via SEP Minimization," in *Proc. of IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, Puerto Vallarta, Mexico 2009, pp. 328-331.

[8] W. Su, A. K. Sadek, and K. J. R. Liu, "SER performance analysis and optimum power allocation for decode-and-forward cooperation protocol in wireless networks," in *Proc. of IEEE Wireless Communications and Networking Conference*, New Orleans, LA, U.S.A., 2005, pp. 984-989.

[9] C. S. Patel and G. L. Stuber, "Channel estimation for amplify and forward relay based cooperation diversity systems," *IEEE Trans. Wireless Com-mun.,* vol. 6, pp. 2348–2356, 2007.

[10] F. Gao, T. Cui, and A. Nallanathan, "On channel estimation and optimal training design for

amplify and forward relay networks," *IEEE Trans.Wireless Commun.,* vol. 7, pp. 1907–1916, 2008.

[11] Y. Wu and M. Pätzold, "Performance analysis of cooperative communication systems with imperfect channel estimation," in *Proc. IEEE ICC*, Dresden, Germany, Jun. 2009, pp. 1-6.

[12] S. S. Ikki, S. Al-Dharrab, and M. Uysal, "Error Probability of DF Relaying with Pilot-Assisted Channel Estimation over Time-Varying Fading Channels," *IEEE Trans on Vehicular Technology,* vol. 61, pp. 393-397, 2012.

[13] J. K. Cavers, "An analysis of pilot-symbol-assisted modulation for Rayleigh fading channels," *IEEE Trans. Veh. Technol.,* vol. 40, pp. 686–693, 1991.