

A HIGHER-ORDER SHEAR DEFORMATION THEORY FOR BUCKLING AND FREE VIBRATION ANALYSIS OF FUNCTIONALLY GRADED SANDWICH PLATES

LÝ THUYẾT BIẾN DẠNG CẮT BẬC CAO CHO PHÂN TÍCH ỔN ĐỊNH VÀ DAO ĐỘNG TỰ DO TẦM SANDWICH CHỨC NĂNG

Nguyen Trung Kien¹, Vo Phuong Thuc², Thai Huu Tai³, Nguyen Van Hau¹

¹ Ho Chi Minh City University of Technology and Education

² Northumbria University, Newcastle upon Tyne, UK

³ University of New South Wales, Sydney, NSW, Australia

TÓM TẮT

Trong bài báo này, các tác giả đã đề xuất một lý thuyết biến dạng cắt bậc cao cho phân tích ổn định và dao động tự do của tấm sandwich chức năng. Các phương trình chuyển động được rút ra từ nguyên lý Hamilton được thiết lập dựa trên sự phân bố bậc cao của ứng suất cắt ngang thỏa mãn điều kiện biên tự do ứng suất. Lời giải giải tích nhận được đối với các điều kiện biên khác nhau đối với tấm sandwich chức năng cho phép kiểm tra tính hiệu quả của lý thuyết đã phát triển.

Từ khóa: Tấm sandwich chức năng; Dao động; Ổn định

ABSTRACT

In this paper, the authors proposed a higher-order shear deformation theory for buckling and free vibration analysis of functionally graded sandwich plates. Equations of motion derived from Hamilton's principle are established basing on a refined hyperbolic distribution of transverse shear stress which satisfies the traction free boundary conditions. Analytical solutions are obtained with various boundary conditions for functionally graded sandwich plates to verify the validity of the developed theory.

Keywords: Functionally graded sandwich plates; Vibration; Buckling.

I. INTRODUCTION

Increase of applications of functionally graded (FG) sandwich structures in engineering fields led to the development of many plate theories to accurately predict static, buckling and vibration behaviors of FG sandwich plates. By using different shear deformation theories, Zenkour [1]-[2] investigated bending, vibration and buckling behaviors of sandwich plates with FG faces and homogeneous hardcore. Based on four-unknown variable refined shear deformation theories, Meiche et al. [3] studied vibration and buckling analysis of FG sandwich plates. Sobhy [4] studied the vibration and buckling behaviors

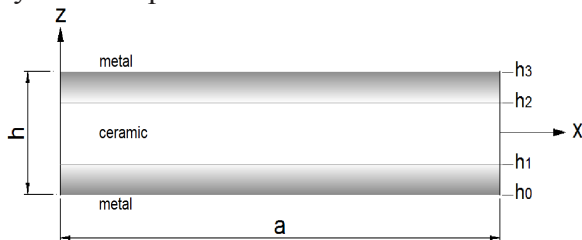
of exponentially FG sandwich plates resting on elastic foundations under various boundary conditions. Neves et al. [5] proposed the static, buckling and free vibration analysis of FG sandwich plates by using a quasi-3D higher-order shear deformation theory. Bessaim et al. [6] focused on both bending and free vibration of FG sandwich plates using a quasi-3D hyperbolic shear deformation theory with five unknowns. Xiang et al. [7] and Li et al. [8] presented a n -order shear deformation theory and a 3D linear theory of elasticity for free vibration of FG sandwich plates, respectively. Nguyen et al. [9] studied vibration and buckling responses of functionally graded

sandwich plates with improved transverse shear stiffness based on the first-order shear deformation theory. Based on the same theory, Thai et al. [10] analyzed the bending, buckling and vibration of FG sandwich plates. Nguyen et al. [11] proposed a new inverse trigonometric shear deformation theory for isotropic and functionally graded sandwich plates.

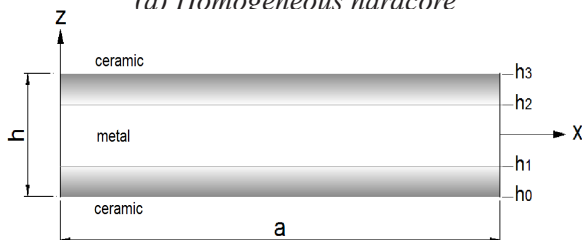
This paper aims to propose a refined four-unknown inverse hyperbolic shear deformation theory for vibration and buckling analysis of FG sandwich plates with various boundary conditions. The theory contains four unknowns, accounts for a hyperbolic distribution of transverse shear stress and satisfies the traction free boundary conditions. Equations of motion are derived from Hamilton's principle. Analytical solutions are obtained for various boundary conditions. Numerical results for FG sandwich plates with homogeneous hardcore and softcore are compared to those of earlier works.

II. PROBLEM FORMULATION

Consider a three-layer sandwich plate as in Fig. 1. The face layers are made of a ceramic-metal material and the core layer is constituted by an isotropic material.



(a) Homogeneous hardcore



(b) Homogeneous softcore

Figure 1. Geometry of functionally graded sandwich plates.

The effective material properties of FG sandwich plates according to the power-law form can be expressed by:

$$P^{(j)}(z) = (P_b - P_t)V_b^{(j)} + P_t \quad (1)$$

where P_t and P_b are the Young's moduli (E), Poisson's ratio (ν), mass densities (ρ) of materials located at the top and bottom surfaces, and at the core, respectively. The

volume fraction function $V_b^{(j)}$ defined by the power-law form as follows:

$$\begin{cases} V_b^{(1)} = \left(\frac{z-h_0}{h_1-h_0}\right)^p & \text{for } z \in [h_0, h_1] \\ V_b^{(2)} = 1 & \text{for } z \in [h_1, h_2] \\ V_b^{(3)} = \left(\frac{z-h_3}{h_2-h_3}\right)^p & \text{for } z \in [h_2, h_3] \end{cases} \quad (2)$$

where p is a power-law index, which is positive. Distribution of material with V_b through the plate thickness for the thickness ratio of layers (1-1-1) is displayed in Fig. 2.

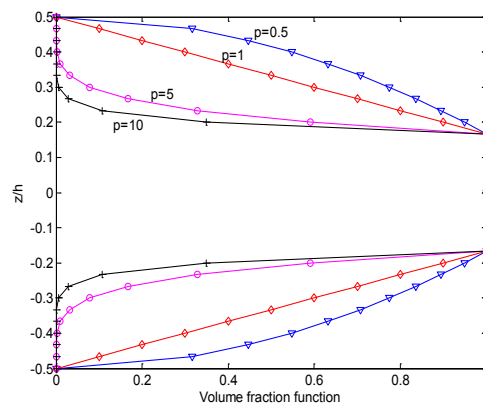


Figure 2. Volume fraction function V_b with respect to the thickness ratio of layers (1-1-1).

1. Higher-order shear deformation theory

The displacement field of the present study is expressed by:

$$\begin{aligned} u_1(x, y, z) &= u(x, y) - zw_{b,x} - f(z)w_{s,x} \\ u_2(x, y, z) &= v(x, y) - zw_{b,y} - f(z)w_{s,y} \\ u_3(x, y, z) &= w_b(x, y) + w_s(x, y) \end{aligned} \quad (3)$$

where the comma indicates partial differentiation with respect to the coordinate subscript that follows, the shape function $f(z)$ at location z is given by:

$$f(z) = z \left(1 + \frac{16rz^2}{3h^2(r^2 + 4)} \right) - h \arctan\left(\frac{rz}{h}\right) \quad (4)$$

and u, v, w_b and w_s are the four unknown displacements of the mid-plane of the plate. The in-plane and out-of-plane strains associated with the displacement field in Eq. (3) are obtained:

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \epsilon_{xx}^{(1)} \\ \epsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + f \begin{Bmatrix} \epsilon_{xx}^{(2)} \\ \epsilon_{yy}^{(2)} \\ \gamma_{xy}^{(2)} \end{Bmatrix} \quad (5a)$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \mathbf{g} \begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} \quad (5b)$$

where $\mathbf{g}(z) = 1 - f_{,z}$; $\epsilon^{(0)}$ are membrane strains; $\epsilon^{(1)}$, $\epsilon^{(2)}$ are curvatures and $\mathbf{a}^{(0)}$ are transverse shear strains. These strains are related to the displacements as follows:

$$\begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix}, \begin{Bmatrix} \epsilon_{xx}^{(1)} \\ \epsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} -w_{b,xx} \\ -w_{b,yy} \\ -2w_{b,xy} \end{Bmatrix} \quad (6a)$$

$$\begin{Bmatrix} \epsilon_{xx}^{(2)} \\ \epsilon_{yy}^{(2)} \\ \gamma_{xy}^{(2)} \end{Bmatrix} = \begin{Bmatrix} -w_{s,xx} \\ -w_{s,yy} \\ -2w_{s,xy} \end{Bmatrix} \quad (6a)$$

$$\begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} = \begin{Bmatrix} w_{s,x} \\ w_{s,y} \end{Bmatrix} \quad (6b)$$

The linear constitutive relations of the j -th layer of FG sandwich plates are written as:

$$\begin{Bmatrix} \sigma_{xx}^{(j)} \\ \sigma_{yy}^{(j)} \\ \sigma_{xy}^{(j)} \end{Bmatrix} = \begin{bmatrix} C_{11}^{(j)} & C_{12}^{(j)} & 0 \\ C_{12}^{(j)} & C_{22}^{(j)} & 0 \\ 0 & 0 & C_{66}^{(j)} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}^{(j)} \\ \epsilon_{yy}^{(j)} \\ \gamma_{xy}^{(j)} \end{Bmatrix} \quad (7a)$$

$$\begin{Bmatrix} \sigma_{xz}^{(j)} \\ \sigma_{yz}^{(j)} \end{Bmatrix} = \begin{bmatrix} C_{55}^{(j)} & 0 \\ 0 & C_{44}^{(j)} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (7b)$$

where

$$C_{11}^{(j)}(z) = C_{22}^{(j)}(z) = \frac{E^{(j)}(z)}{1 - \nu^{(j)}(z)^2}$$

$$C_{12}^{(j)}(z) = \nu^{(j)}(z) C_{11}^{(j)}(z)$$

$$C_{44}^{(j)}(z) = C_{55}^{(j)}(z) = C_{66}^{(j)}(z) = \frac{E^{(j)}(z)}{2[1 + \nu^{(j)}(z)]} \quad (8)$$

The application of Hamilton's principle enables to derive the equilibrium equations of the plate as:

$$N_{xx,x} + N_{xy,y} = I_0 \ddot{u} - I_1 \ddot{w}_{b,x} - J_1 \ddot{w}_{s,x} \quad (9a)$$

$$N_{xy,x} + N_{yy,y} = I_0 \ddot{v} - I_1 \ddot{w}_{b,y} - J_1 \ddot{w}_{s,y} \quad (9b)$$

$$M_{xx,xx}^{b} + 2M_{xy,xy}^{b} + M_{yy,yy}^{b} + \bar{N} + q = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 (\ddot{u}_{,x} + \ddot{v}_{,y}) - I_2 (\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - J_2 (\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \quad (9c)$$

$$M_{xx,xx}^{s} + 2M_{xy,xy}^{s} + M_{yy,yy}^{s} + Q_{x,x} + Q_{y,y} + \bar{N} = I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 (\ddot{u}_{,x} + \ddot{v}_{,y}) - J_2 (\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - K_2 (\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \quad (9d)$$

where the dot-superscript convention indicates the differentiation with respect to the time variable t ; N , M , and Q are the stress resultants defined by:

$$(N_{xx}, N_{yy}, N_{xy}) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (\sigma_{xx}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{xy}^{(j)}) dz \quad (10a)$$

$$(M_{xx}^b, M_{yy}^b, M_{xy}^b) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} z (\sigma_{xx}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{xy}^{(j)}) dz \quad (10b)$$

$$(M_{xx}^s, M_{yy}^s, M_{xy}^s) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} f(\sigma_{xx}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{xy}^{(j)}) dz \quad (10c)$$

$$(Q_x, Q_y) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} g(\sigma_{xz}^{(j)}, \sigma_{yz}^{(j)}) dz \quad (10d)$$

$$\text{and } \bar{N} = -N_0 [w_{b,xx} + w_{s,xx} + \eta(w_{b,yy} + w_{s,yy})]$$

(η is non-dimensional load parameter, N_0 is in-plane compressive load). The inertia terms I_i, J_i, K_i are expressed by:

$$(I_0, I_1, I_2) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (1, z, z^2) \rho(z) dz \quad (11a)$$

$$(J_1, J_2, K_2) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (f, zf, f^2) \rho(z) dz \quad (11b)$$

where $\rho(z)$ is the mass density. Substituting Eq. (7a) into Eqs. (10a)-(10c), the stress resultants are obtained in terms of strains as following compact form:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M}^b \\ \mathbf{M}^s \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{B}^s \\ \boldsymbol{\varepsilon} \mathbf{B} & \mathbf{D} & \mathbf{D}^s \\ \mathbf{B}^s & \mathbf{D}^s & \mathbf{H}^s \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^{(0)} \\ (1) \\ \boldsymbol{\varepsilon}^{(2)} \end{Bmatrix} \quad (12)$$

where $\mathbf{A}, \mathbf{B}, \mathbf{D}, \mathbf{B}^s, \mathbf{D}^s, \mathbf{H}^s$ are the stiffnesses of the FG sandwich plate given by:

$$(\mathbf{A}, \mathbf{B}, \mathbf{D}) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (1, z, z^2) \mathbf{C}^{(j)}(z) dz \quad (13a)$$

$$(\mathbf{B}^s, \mathbf{D}^s, \mathbf{H}^s) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (f, zf, f^2) \mathbf{C}^{(j)}(z) dz \quad (13b)$$

where $\mathbf{C}^{(j)}(z)$ is reduced stiffness matrix of the j -th layer. Similarly, using Eqs. (7b) and (10d), the transverse shear forces can be calculated from the constitutive equations as:

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} \quad (14)$$

or in a compact form as:

$$\mathbf{Q} = \mathbf{A}^s \boldsymbol{\gamma}^{(0)} \quad (15)$$

where the shear stiffnesses \mathbf{A}^s of the FG sandwich plate are defined by:

$$A_{44}^s = A_{55}^s = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} g^2 C_{44}^{(j)}(z) dz \quad (16)$$

2 Analytical solutions

In order to obtain analytical solutions, the solution field is approximated as of form:

$$u(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{mn} X'(x) Y(y) e^{i\omega t} \quad (17a)$$

$$v(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{mn} X(x) Y'(y) e^{i\omega t} \quad (17b)$$

$$w_b(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{bmn} X(x) Y(y) e^{i\omega t} \quad (17c)$$

$$w_s(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{smn} X(x) Y(y) e^{i\omega t} \quad (17d)$$

where ω is the frequency of free vibration of the plate, $i^2 = -1$ the imaginary unit. The shape functions $X(x)$ and $Y(y)$ are given for various boundary conditions (SS: simply supported, CC: clamped-clamped plates). For SSSS plates: $X(x) = \sin \alpha_m x$, $Y(y) = \sin \beta_n y$ with $\alpha_m = m\pi/a$ and $\beta_n = n\pi/b$ while for CCCC plates, the shape functions are:

$$X(x) = \sin \alpha_m x - \sinh \alpha_m x - \varphi_m (\cos \alpha_m x - \cosh \alpha_m x)$$

$$\varphi_m = \frac{\sin \alpha_m a - \sinh \alpha_m a}{\cos \alpha_m a - \cosh \alpha_m a}, \alpha_m = \frac{(m+0.5)\pi}{a}$$

and,

$$Y(y) = \sin \beta_n y - \sinh \beta_n y - \delta_n (\cos \beta_n y - \cosh \beta_n y)$$

$$\delta_n = \frac{\sin \beta_n b - \sinh \beta_n b}{\cos \beta_n b - \cosh \beta_n b}, \beta_n = \frac{(n+0.5)\pi}{b}$$

Substituting Eqs. (17a)-(17d) into Eqs. (9a)-(9d) by taking into account Eqs. (12) and (14) leads to:

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{U} = \mathbf{0} \quad (18)$$

where the components of the stiffness matrix

K_{ij} and mass matrix M_{ij} are given by:

$$\begin{aligned}
 k_{11} &= -A_{12}e_{12} - A_{66}e_{11}, k_{12} = -(A_{12} + A_{66})e_{11} \\
 k_{13} &= B_{11}e_{12} + (B_{12} + 2B_{66})e_{11}, k_{14} = B_{11}^s e_{12} + (B_{12}^s + 2B_{66}^s)e_{11} \\
 k_{21} &= -(A_{12} + A_{66})e_9, k_{24} = B_{22}^s e_8 + (B_{12}^s + 2B_{66}^s)e_9 \\
 k_{22} &= -A_{22}e_8 - A_{66}e_9, k_{23} = B_{22}e_8 + (B_{12} + 2B_{66})e_9 \\
 k_{31} &= -B_{11}e_6 - (B_{12} + 2B_{66})e_5, k_{32} = -B_{22}e_3 - (B_{12} + 2B_{66})e_5 \\
 k_{33} &= D_{11}e_6 + 2(D_{12} + 2D_{66})e_5 + D_{22}e_3 + N_0(e_4 + \eta e_2) \\
 k_{34} &= D_{11}^s e_6 + 2(D_{12}^s + 2D_{66}^s)e_5 + D_{22}^s e_3 + N_0(e_4 + \eta e_2) \\
 k_{41} &= -B_{11}^s e_6 - (B_{12}^s + 2B_{66}^s)e_5, k_{42} = -B_{22}^s e_3 - (B_{12}^s + 2B_{66}^s)e_5 \\
 k_{43} &= D_{11}^s e_6 + 2(D_{12}^s + 2D_{66}^s)e_5 + D_{22}^s e_3 + N_0(e_4 + \eta e_2) \\
 k_{44} &= H_{11}^s e_6 + 2(H_{12}^s + 2H_{66}^s)e_5 + H_{22}^s e_3 + N_0(e_4 + \eta e_2) \\
 &\quad - A_{55}^s e_4 - A_{44}^s e_2 \\
 m_{11} &= I_0 e_{10}, m_{13} = -I_1 e_{10}, m_{14} = -J_1 e_{10}, m_{22} = I_0 e_7 \\
 m_{23} &= -I_1 e_7, m_{24} = J_1 e_7, m_{31} = I_1 e_4, m_{32} = I_1 e_2 \\
 m_{33} &= I_0 e_1 - I_2(e_4 + e_2), m_{34} = I_0 e_1 - J_2(e_4 + e_2) \\
 m_{41} &= J_1 e_4, m_{42} = J_1 e_2, m_{43} = I_0 e_1 - J_2(e_4 + e_2) \\
 m_{44} &= I_0 e_1 - K_2(e_4 + e_2) \\
 (e_1, e_2, e_3) &= \int_0^a \int_0^b (X_m Y_n, X_m Y_n'', X_m Y_n''') X_m Y_n dx dy \\
 (e_4, e_5, e_6) &= \int_0^a \int_0^b (X_m'' Y_n, X_m'' Y_n'', X_m''' Y_n) X_m Y_n dx dy \\
 (e_7, e_8, e_9) &= \int_0^a \int_0^b (X_m' Y_n', X_m' Y_n''', X_m'' Y_n') X_m Y_n dx dy \\
 (e_{10}, e_{11}, e_{12}) &= \int_0^a \int_0^b (X_m' Y_n, X_m' Y_n'', X_m''' Y_n) X_m Y_n dx dy \quad (19)
 \end{aligned}$$

Eq. (18) is a general form for buckling and free vibration analysis of FG sandwich plates in which the stability problem can be carried out by neglecting the mass matrix while the free vibration problem is achieved by omitting the in-plane load.

III. NUMERICAL RESULTS

In this section, a number of numerical examples are analyzed to verify the accuracy of present study and investigate the natural frequencies and critical buckling loads of FG sandwich plates. Two cases of FG sandwich plates are considered: hardcore with homogeneous core made of Al_2O_3 (E_b, ν_b, ρ_b) and FG faces with top and bottom surfaces made of Al (E_t, ν_t, ρ_t), softcore with homogeneous core made of Al (E_b, ν_b, ρ_b) and FG faces with top and bottom surfaces made of Al_2O_3 (E_t, ν_t, ρ_t). The properties of Al_2O_3 are: $E=380$ Gpa, $\nu = 0.3$, $\rho = 3800$ kg/m³ and those of Al are $E=70$ Gpa, $\nu = 0.3$, $\rho = 2707$ kg/m³. For convenience, the following non-dimensional parameters are used:

$$\bar{N}_{cr} = \frac{N_{cr} a^2}{100 E_0 h^3}, \bar{\omega} = \omega \frac{a^2}{h} \sqrt{\rho_0 / E_0} \quad (20)$$

where $E_0 = 1$ GPa, $\rho_0 = 1$ kg/m³.

Table 1. Nondimensional fundamental frequency $\bar{\omega}$ of Al/Al₂O₃ sandwich square plates with homogeneous hardcore (b/h=10).

BC	p	Theory	1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
SSSS	0	Present	1.82445	1.82445	1.82445	1.82445	1.82445
		Nguyen et al. (HSDT) [11]	1.82489	1.82489	1.82489	1.82489	1.82489
		Bessaim et al. (Quasi-3D) [6]	1.82682	1.82682	1.82682	1.82682	1.82682
		Li et al. (3D) [8]	1.82682	1.82682	1.82682	1.82682	1.82682
	1	Present	1.24323	1.30013	1.35334	1.39566	1.43933
		Nguyen et al. (HSDT) [11]	1.24332	1.30024	1.35345	1.39579	1.43948
		Bessaim et al. (Quasi-3D) [6]	1.24495	1.30195	1.35527	1.39987	1.44143
		Li et al. (3D) [8]	1.24470	1.30181	1.35523	1.39763	1.44137
	5	Present	0.94606	0.98189	1.04469	1.10899	1.17397
		Nguyen et al. (HSDT) [11]	0.94611	0.98193	1.04473	1.10905	1.17403

		Bessaim et al. (Quasi-3D) [6]	0.94716	0.98311	1.04613	1.11723	1.17579
		Li et al. (3D) [8]	0.94476	0.98103	1.04532	1.10983	1.17567
	10	Present	0.92846	0.94302	0.99555	1.06110	1.12315
		Nguyen et al. (HSDT) [11]	0.92854	0.94305	0.99558	1.06114	1.12320
		Bessaim et al. (Quasi-3D) [6]	0.92952	0.94410	0.99684	1.07015	1.12486
		Li et al. (3D) [8]	0.92727	0.94078	0.99523	1.06104	1.12466
CCCC	0	Present	3.17398	3.17398	3.17398	3.17398	3.17398
		Thai et al. (FSDT) [10]	3.29360	3.29360	3.29360	3.29360	3.29360
		Li et al. (3D) [8]	3.13800	3.13800	3.13800	3.13800	3.13800
	1	Present	2.19951	2.30008	2.39165	2.46282	2.53734
		Thai et al. (FSDT) [10]	2.28140	2.38640	2.48180	2.55560	2.63300
		Li et al. (3D) [8]	2.19020	2.29110	2.38190	2.45110	2.53980
	5	Present	1.67955	1.75105	1.86227	1.97289	2.08623
		Thai et al. (FSDT) [10]	1.72930	1.80640	1.92690	2.04150	2.16290
		Li et al. (3D) [8]	1.66190	1.73930	1.85790	1.96720	2.15720
	10	Present	1.64133	1.68225	1.77671	1.88979	1.99884
		Thai et al. (FSDT) [10]	1.68580	1.72680	1.83290	1.94970	2.07030
		Li et al. (3D) [8]	1.62120	1.66330	1.76860	1.88080	1.99860

Table 2. Nondimensional fundamental frequency $\bar{\omega}$ of Al/Al_2O_3 sandwich square plates with homogeneous softcore ($b/h=10$).

BC	p	Theory	1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
SSSS	0	Present	0.92777	0.92777	0.92777	0.92777	0.92777
		Bessaim et al. (Quasi-3D) [6]	0.92897	0.92897	0.92897	0.92897	0.92897
		Li et al. (3D) [8]	0.92897	0.92897	0.92897	0.92897	0.92897
	1	Present	1.72533	1.68324	1.63906	1.57849	1.56060
		Bessaim et al. (Quasi-3D) [6]	1.72814	1.68625	1.64199	1.58430	1.56301
		Li et al. (3D) [8]	1.72227	1.67437	1.63053	1.57037	1.55788
	5	Present	1.84180	1.84103	1.81640	1.75281	1.74762
		Bessaim et al. (Quasi-3D) [6]	1.84465	1.84456	1.82032	1.75972	1.75143
		Li et al. (3D) [8]	1.84198	1.82611	1.78956	1.72726	1.72670
10	Present	1.83848	1.85148	1.83581	1.77492	1.77474	
	Bessaim et al. (Quasi-3D) [6]	1.84113	1.85489	1.83973	1.78163	1.77878	
	Li et al. (3D) [8]	1.84020	1.83987	1.80813	1.74779	1.74811	
CCCC	0	Present	1.61402	1.61402	1.61402	1.61402	1.61402
		Li et al. (3D) [8]	1.59667	1.59667	1.59667	1.59667	1.59667
	1	Present	2.92463	2.82087	2.72875	2.64242	2.58842
		Li et al. (3D) [8]	2.84447	2.71205	2.61914	2.53960	2.50113
	5	Present	3.17064	3.11899	3.03820	2.94505	2.88063
		Li et al. (3D) [8]	3.11435	2.98733	2.85602	2.77151	2.71561
	10	Present	3.18030	3.15221	3.08354	2.99255	2.93066
		Li et al. (3D) [8]	3.13632	3.03681	2.89948	2.81400	2.74524

Table 3. Nondimensional critical buckling loads \bar{N}_{cr} of Al/Al_2O_3 sandwich square plates subjected to biaxial compressive load ($\eta = 1$) with homogeneous hardcore ($b/h=10$).

BC	p	Theory	1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
SSSS	0	Present	6.50251	6.50251	6.50251	6.50251	6.50251
		Nguyen et al. (HSDT) [11]	6.50566	6.50566	6.50566	6.50566	6.50566
		Thai et al. (FSDT) [10]	6.50220	6.50220	6.50220	6.50220	6.50220
	1	Present	2.58369	2.92013	3.23241	3.47473	3.75323
		Nguyen et al. (HSDT) [11]	2.58410	2.92060	3.23299	3.47544	3.75403
		Thai et al. (FSDT) [10]	2.58240	2.91930	3.23200	3.47420	3.75280
	5	Present	1.32932	1.52144	1.78989	2.05612	2.36735
		Nguyen et al. (HSDT) [11]	1.32948	1.52155	1.79002	2.05633	2.36760
		Thai et al. (FSDT) [10]	1.32080	1.51410	1.78550	2.05120	2.36520
	10	Present	1.24384	1.37331	1.59747	1.85386	2.14001
		Nguyen et al. (HSDT) [11]	1.24406	1.37341	1.59758	1.85403	2.14020
		Thai et al. (FSDT) [10]	1.23330	1.36120	1.58970	1.84500	2.13640
CCCC	0	Present	15.9139	15.9139	15.9139	15.9139	15.9139
		Thai et al. (FSDT) [10]	15.9226	15.9226	15.9226	15.9226	15.9226
	1	Present	6.54655	7.39815	8.17136	8.75748	9.43980
		Thai et al. (FSDT) [10]	6.54340	7.39900	8.17530	8.76120	9.44430
	5	Present	3.39243	3.91888	4.60625	5.26967	6.05338
		Thai et al. (FSDT) [10]	3.34000	3.87380	4.58130	5.24170	6.04450
	10	Present	3.14660	3.53967	4.12089	4.76227	5.48856
		Thai et al. (FSDT) [10]	3.08250	3.46290	4.07320	4.70840	5.46960

Tables 1-3 present the comparisons of nondimensional fundamental frequency and critical buckling load under biaxial compressions derived from the present model and those obtained from earlier works (quasi-3D [6], 3D [8], FSDT [10] and HSDT [11]). It can be seen that the results obtained from the present theory is in well agreement with those of previous studies for both SSSS and CCCC FG sandwich plates. The effect of the power-law index and thickness ratio of layers on the fundamental frequency and critical buckling load is also showed in Tables 1-3. It is observed that with an increase of the power-law index, the fundamental frequency and critical buckling load decrease for FG sandwich plates with homogeneous hardcore and inversely for FG sandwich plates with homogeneous softcore. That is due to the fact that for the case of homogeneous hardcore, the increase of the power-law index corresponds

to the decrease of the ceramic portion, that thus makes the plate become softer and inversely for that of homogenous softcore.

IV. CONCLUSION

A refined higher-order shear deformation theory for vibration and buckling analysis of FG sandwich plates is proposed in this paper. It contains only four unknowns, accounts for a hyperbolic distribution of transverse shear stress and satisfies the traction free boundary conditions. Analytical solutions are derived for various boundary conditions and compared with the existing solutions to verify the validity of the developed theory. The obtained numerical results are in well agreement with those of previous studies. The present theory with four unknowns is found to be simple and efficient in predicting buckling and vibration behaviors of FG sandwich plates.

ACKNOWLEDGEMENTS

This research is funded by Vietnam National Foundation for Science and Technology

Development (NAFOSTED) under Grant No. 107.02-2012.07. The authors gratefully acknowledge this support.

REFERENCES

- [1] A. M. Zenkour, A comprehensive analysis of functionally graded sandwich plates: Part 1 -Deflection and stresses, *Int. J. Solids Struct.*, 42, 5224–5242, 2005.
- [2] A. M. Zenkour, A comprehensive analysis of functionally graded sandwich plates: Part 2 - Buckling and free vibration, *Int. J. Solids Struct.*, 42, 5243–5258, 2005.
- [3] N. E. Meiche, A. Tounsi, N. Ziane, I. Mechab, E. A. Adda Bedia, A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate, *Int. J. Mech. Sci.*, 53, 237 – 247, 2011.
- [4] M. Sobhy, Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions, *Compos. Struct.*, 99, 76 – 87, 2013.
- [5] A. M. A. Neves, A. J. M. Ferreira, E. Carrera, M. Cinefra, C. M. C. Roque, R. M. N. Jorge, C. M. M. Soares, Static, free vibration and buckling analysis of isotropic and sandwich functionally graded plates using a quasi-3D higher-order shear deformation theory and a meshless technique, *Compos. Part B: Eng.*, 44, 657–674, 2013.
- [6] A. Bessaim, M. S. Houari, A. Tounsi, S. Mahmoud, E. A. Adda Bedia, A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets, *J. Sandw. Struct. Mater.*, In press, 2013.
- [7] A. Xiang, Y. Jin, Z. Bi, S. Jiang, M. S. Yang, A n-order shear deformation theory for free vibration of functionally graded and composite sandwich plates, *Compos. Struct.*, 93, 2826–2832, 2011.
- [8] Q. Li, V. P. Iu, K. P. Kou, Three-dimensional vibration analysis of functionally graded material sandwich plates, *J. Sound Vib.*, 311, 498–515, 2008.
- [9] T. K. Nguyen, T. P. Vo, H. T. Thai, Vibration and buckling analysis of functionally graded sandwich plates with improved transverse shear stiffness based on the first-order shear deformation theory, *Proc IMechE Part C: J. Mech. Eng. Sci.*, 228, 2110–2131, 2014.
- [10] H. T. Thai, T. K. Nguyen, T. P. Vo, J. Lee, Analysis of functionally graded sandwich plates using a new first-order shear deformation theory, *Eur. J. Mech. A/Solids*, 45, 211 – 225, 2014.
- [11] V. H. Nguyen, T. K. Nguyen, H. T. Thai, T. P. Vo, a new inverse trigonometric shear deformation theory for isotropic and functionally graded sandwich plates, *Compos. Part B: Eng.*, 66, 233-246, 2014.