MECHANICAL BEHAVIOR FOR STATIC ANALYSIS OF STEEL PLATES INDUCED BY PIEZOELECTRIC ACTUATORS BY FEM

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ABSTRACT

The coupling effects between the mechanical and electric properties of piezoelectric materials have drawn significant attention for their potential applications as sensor and actuators. In this investigation, the piezoelectric actuators and sensor are symmetrically surface bonded on plate. Electric voltage was applied to the piezoelectric actuators, resulting in the bending effect on the plate. The bending moment is derived by using the classical laminate theory and piezoelectricity.

The effects of size, thickness, location and voltage of the piezoelectric actuators on the response of the plate was presented through a parametric study. A simple model incorporating the classical laminate theory and plate theory presented to predict the deformed shape of the simply supported stell plate.

Keywords: *piezoelectric actuator, plate theory, coupling effects, laminate theory.*

TÓM TẮT

Hiệu ứng tương tác giữa các tính chất cơ học và điện của vật liệu áp điện đã cho thấy các ứng dụng tiềm năng của chúng: các cảm biến và cơ cấu điều khiển..., Trong nghiên cứu này, các cảm biến và cơ cấu điều khiển bằng vật liệu áp điện được dán đối xứng trên bề mặt của tấm. Cơ cấu điều khiển sẽ được áp một điện thế kết quả sẽ gây ra biến dạng của tấm. Mô men uốn sẽ được tính toán dựa trên lí thuyết tấm cổ điển và lí thyết về vật liệu áp điện.

Hiệu ứng tác động của bộ kích lên biến dạng của tấm như : chiều dày, kích thước, vị trí và điện thế đã được nghiên cứu, một cách mô hình hóa cho bài toán tấm có dán tấm áp điện với điều kiện biên gối tựa đơn đã được đề xuất.

Từ khóa: *điều khiển áp điện, lí thuyết tấm, hiệu ứng tương tác, lí thuyết tấm nhiều lớp.*

1. INTRODUCTION

Piezo material with some advance about quick response, low energy consumption, high linearity had drawn much attention in in the past decade.

The piezoelectric device are of great interest in structure engineering with applications to the shape control, reduce the noisy and stability control of structure [11]. Piezoceramic is most popular using of smart structure and can be surface boned or embedded in structure without significantly changing the structural stiffness system. Bailey and Hubbard [1] developed adaptive structure using PFDV film as actuator to control the vibration of a cantilever beam. Dimitriadis *et al.* [4] used two dimensions piezoelectric material boned on surface with simply support plate. Luo and Tong [7] developed a finite element model to analysis and simulation twisting and bending shape control using the orthotropic piezoelectricity actuator. Huang. G.H.; Sun, C.T [5] using piezo electricity as actuator to control dynamic adaptive with an anisotropic elastic structure.

The present work investigates the influence of piezoelectric patches symmetrically bonded to the opposite plate surfaces on the static behavior of the composite structure. This propose by some author: Crawley [3] and Luis [8], Dimitriadis [4].

However we was analysis and mode response of the injector in a common rail system.

2. STRUCTURE MODELING

In the present formulation, the following assumptions (Reddy) [10] are considered:

- The piezoelectric layers are perfectly bonded together.

- The formulation is restricted to linear elastic material behavior.

- This formulation uses the Kirchhoff assumption (thin plate) in which the transverse normal remains straight after deformation and they also rotate such that they always remain perpendicular to the mid-surface.

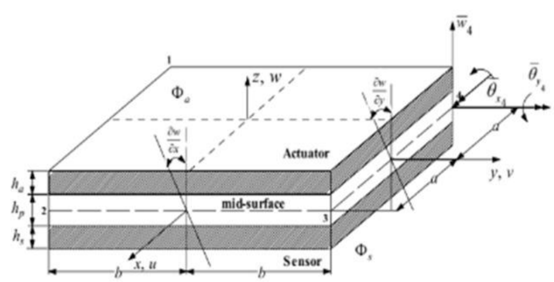

Fig1. *Coordinate system of a laminated finite element with integrated piezoelectric material.*

Therefore, as shown in Fig. 1, the displacements field u, v, and w can be expressed by the Kirchhoff hypotheses as [10]:

$$u = -z\frac{\partial w}{\partial x}, v = -z\frac{\partial w}{\partial y}, w = w(x, y) \quad (1)$$

Where x and y are the in-plane axes located at the mid-surface of the plate, and z is along the plate thickness direction as seen Fig. 1. In addition, u and v are the displacements in the x and y-axes, respectively, whereas w is the transverse displacement (or also called deflection) along the z-axis.

$$\{\varepsilon\} = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}\}$$
$$= -z\left\{\frac{\partial^2 w}{\partial x^2} \quad \frac{\partial^2 w}{\partial y^2} \quad \frac{\partial^2 w}{\partial x \partial y}\right\}^T \quad (2)$$

For isotropic material, the relation between plane stress (σ) and strain (ε) is given by:

$$\{\sigma\} = [D]\{\varepsilon\} \quad (3)$$

Where:

$$\{\sigma\} = \{\sigma_x \quad \sigma_x \quad \tau_{xy}\}^T, [D] = \frac{E_p}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{bmatrix}$$

s, e, v and E_P are stress, strain field, Poisson ratio and Young's modulus of plate, respectively.

Consider a four-node rectangular plate bending element based on classical plate theory [1,2], where the element is shown in Fig. 1. Each node of the element possesses three degrees of freedom: a displacement w in the z direction; a rotation about x axis (θ_x); and a rotation about y-axis (θ_y) as shown in the Fig. 1. The displacement function w is assumed to be:

$$w(x_i, y_i) = c_1 + c_2x_i + c_3y_i + c_4x_i^2$$
$$+c_5y_ix_i + c_6y_i^2 + c_7x_i^3 + c_8x_i^2y_i \quad (4)$$
$$+c_8x_iy_i^2 + c_{10}y_i^3 + c_{11}x_i^3y_i + c_{12}x_iy_i^3$$

Where

$$\begin{cases} i = 1...4 \\ x1 = -a; y1 = -b; x2 = a; y2 = -b; \\ x3 = a; y3 = b; x4 = -a; y4 = b \end{cases}$$

The transversal displacement field (w) can be expressed by: w={P}^{T} {c}

Where:

{C}={C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8 C_9 C_{10} C_{11} C_{12}}

{P}={1 x y x^2 xy y^2 x^3 x^2y xy^2 y^3 x^3y xy^3}^{T}

Vector $\{d_i\}$ is defined as a nodal displacement field in the rectangular element.

$$\{d_i\} = \{\bar{w}_1 \quad \bar{\theta}_{x1} \quad \bar{\theta}_{y1} ..... \quad \bar{w}_4 \quad \bar{\theta}_{x4} \quad \bar{\theta}_{y4}\}^T \quad (5)$$

Where:

$$\bar{w}_i = w\Big|_{x_i, y_i}, \bar{\theta}_{xi} = \frac{\partial w}{\partial y}\Big|_{x_i, y_i}, \bar{\theta}_{yi} = -\frac{\partial w}{\partial x}\Big|_{x_i, y_i}$$

Displacement field

$$\{d\} = [H][L]^T [X]^{-1} \{d_i\} \tag{6}$$

Where

$$[H] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -z & 0 \\ 0 & 0 & -z \end{bmatrix}$$

$$[L] = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 & 0 & 6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 6xy \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 6y^2 \end{bmatrix}$$

$$[X] = \begin{bmatrix}
1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 & x_1^3 & x_1^2y_1 & x_1y_1^2 & y_1^3 & x_1^3y_1 & x_1y_1^3 \\
1 & 0 & 1 & 0 & x_1 & 2y_1 & 0 & x_1^2 & 2x_1y_1 & 3y_1^2 & x_1^3 & 3x_1y_1^2 \\
0 & -1 & 0 & -2x_1 & -y_1 & 0 & -3x_1^2 & -2x_1y_1 & -y_1^2 & 0 & -3x_1^2y_1 & -y_1^3 \\
1 & x_2 & y_2 & x_2^2 & x_2y_2 & y_2^2 & x_2^3 & x_2^2y_2 & x_2y_2^2 & y_2^3 & x_2^3y_2 & x_2y_2^3 \\
1 & 0 & 1 & 0 & x_2 & 2y_2 & 0 & x_2^2 & 2x_2y_2 & 3y_2^2 & x_2^3 & 3x_2y_2^2 \\
0 & -1 & 0 & -2x_2 & -y_2 & 0 & -3x_2^2 & -2x_2y_2 & -y_2^2 & 0 & -3x_2^2y_2 & -y_2^3 \\
1 & x_3 & y_3 & x_3^2 & x_3y_3 & y_3^2 & x_3^3 & x_3^2y_3 & x_3y_3^2 & y_3^3 & x_3^3y_3 & x_3y_3^3 \\
1 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & x_3^2 & 2x_3y_3 & 3y_3^2 & x_3^3 & 3x_3y_3^2 \\
0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3^2 & -2x_3y_3 & -y_3^2 & 0 & -3x_3^2y_3 & -y_3^3 \\
1 & x_4 & y_4 & x_4^2 & x_4y_4 & y_4^2 & x_4^3 & x_4^2y_4 & x_4y_4^2 & y_4^3 & x_4^3y_4 & x_4y_4^3 \\
1 & 0 & 1 & 0 & x_4 & 2y_4 & 0 & x_4^2 & 2x_4y_4 & 3y_4^2 & x_4^3 & 3x_4y_4^2 \\
0 & -1 & 0 & -2x_4 & -y_4 & 0 & -3x_4^2 & -2x_4y_4 & -y_4^2 & 0 & -3x_4^2y_4 & -y_4^3
\end{bmatrix}$$

3. PIEZOELECTRIC CONSTITUTIVE EQUATIONS

In this work the following linear constitutive relations for piezoelectric materials are employed:

$$\{\sigma\} = [C^E]\{\varepsilon\} - [e]^T\{E\} \tag{7}$$

$$\{D\} = [e]\{\varepsilon\} + [\xi^s]\{E\} \tag{8}$$

Where

$\{\sigma\}$, $\{D\}$, $\{\varepsilon\}$, $[C^E]$, $[e]$ and $\{\xi^S\}$ are stress field, electric displacement vector dielectric, strain field, elastic constant tensor when the electricity field constant tensor, electricity constant tensor when strain field constant tensor and dielectric coefficient matrix, respectively.

The application of voltage to the element is analogous to the application of heat to a bimetallic strip. The voltage Φa across the bender element forces the bottom layer to expand, while the upper layer contracts, as depicted in Fig. 2.

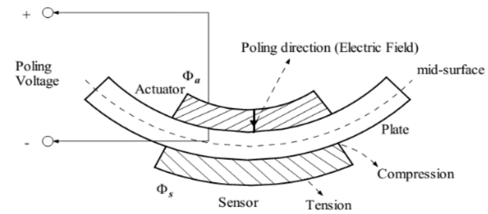

Fig 2. *Curvature of a plate produced by the expansion of one piezoelectric layer and contraction of the other.*

The result of these physical changes is a strong curvature; this implies in a large deflection at the tip when the other end is clamped. Due to the reciprocity effect, deformation of the sensor will produce a charge across the sensor electrode, which is collected through the sensor surface as an electric voltage ϕ_s .

The applied or sensed electric potential through the actuator or sensor element is given by the following equation [8]

$$\phi_z = \left(\frac{z - 0.5h_p}{h}\right)\phi \tag{9}$$

Where

h and ϕ (see Fig. 2) are the thickness and the maximum electric potential at the external surface of the corresponding piezoelectric element (actuator and sensor), and z (z_a and z_s) is defined over the intervals:

$$\frac{h_p}{2} \le z_a \le \frac{h_p}{2} + h_a,\ -\frac{h_p}{2} \ge z_s \ge -\frac{h_p}{2} - h_s$$

Assuming that the electric field (E) is constant through the actuator and sensor elements thickness, the gradient operators are:

$$E = -\frac{d\phi_z}{dz} = -B_z\varphi = -\frac{\phi}{h} \tag{10}$$

In this study, the electric field E, given by:

$$\{E\} = \begin{Bmatrix} 0 \\ 0 \\ E3 \end{Bmatrix}, \quad \text{so that} \quad B_z = \begin{Bmatrix} 0 \\ 0 \\ \frac{1}{h} \end{Bmatrix}$$

where h is thickness of piezoelectricity material.

With the piezoelectric materials. The piezoeletric stress coefficient matrix and dielectric coefficient matrix given by:

$$[e]=\begin{bmatrix}0&0&0\\0&0&0\\e_{31}&e_{31}&0\end{bmatrix}$$

$$[\xi^{s}]=\begin{bmatrix}\xi_{11}&0&0\\0&\xi_{11}&0\\0&0&\xi_{11}\end{bmatrix}$$

4. 'SMART' FINITE ELEMENT FORMULATION

Hamilton's principle is employed here to derive the finite element equations [2].

$$I=\int_{t1}^{t2}Ldt=\int_{t1}^{t2}\left[\delta(T-U+W_e-W_m)+\delta W\right]d \quad (11)$$

Where t_1 and t_2 are two arbitrary instants, T is the kinetic energy, U is the potential energy, We denotes the work done by electrical forces, and W_m is the work done by magnetic forces, which is negligible for piezoelectric materials.

The total kinetic energy T and the potential energy U of the composite structure are described by the following relations:

$$U=\frac{1}{2}\int\{\varepsilon\}^T\{\sigma\}dV \quad (12)$$

$$T=\frac{1}{2}\int\rho\{\dot{d}\}^T\{\dot{d}\}dV \quad (13)$$

Where $\{\dot{d}\}$ is the differentiation of $\{d\}$ with respect to t, $\{d\}$ is functional of nodal displacement field and dV defined by:

$$dV=dV_a+dV_p+dV_s \quad (14)$$

where the subscript *a, p, s* refer to the actuator, plate, and sensor elements, respectively and dV_a, dV_p and dV_s, given by:

$$dV_p=\int_{-h_p/2}^{h_p/2}\int_{-b}^{b}\int_{-a}^{a}dxdydz \quad (15)$$

$$dV_a=\int_{h_p/2}^{h_p/2+h_a}\int_{-b}^{b}\int_{-a}^{a}dxdydz \quad (16)$$

$$dV_s=\int_{-h_p/2-h_s}^{-h_p/2}\int_{-b}^{b}\int_{-a}^{a}dxdydz \quad (17)$$

The work done by electrical forces:

$$W_e=\frac{1}{2}\int_V\{E\}^T\{D\}dV \quad (18)$$

The work done by mechanical forces:

$$W=\int_V\{d\}^T\{f_b\}dV+\int_A\{d\}^T\{f_A\}dA+\int_A\phi.\sigma_q dA \quad (19)$$

Where: D is the electric displacement vector, f_b is the body force, f_A is the surface force, and σ_q is the surface electrical stress.

Substituting Eq. (7), Eq. (8) into Eq. (11), Eq. (18):

$$U=\frac{1}{2}\int_V\{\varepsilon\}^T[C^E]\{\varepsilon\}dV-\frac{1}{2}\int_V\{\varepsilon\}^T[e]^T\{E\}dV \quad (20)$$

$$W_e=\frac{1}{2}\int_V\{E\}^T[e]\{\varepsilon\}dV+\frac{1}{2}\int_V\{E\}^T[\xi^S]\{E\}dV \quad (21)$$

Substituting Eq. (20), Eq. (21) into Eq. (11):

$$\int_{t1}^{t2}\left\{\begin{matrix}\{\delta q_k\}^T\left[[M_{qq}^e]\{\ddot{q}_k\}+[K_{qq}^e]\{q_k\}+[K_{q\phi}^e]\{\phi\}-\{f\}\right]+\\ \{\delta\phi\}\left[[K_{\phi q}^e]\{q_k\}+[K_{\phi\phi}^e]\{\phi\}+\{Q_a\}\right]\end{matrix}\right\}dt=0$$

Where:

$$[M_{qq}^e]=\rho\int[X]^{-T}[L_M][H]^T[H][L_M]^T[X]^{-1}dV \quad (22.1)$$

$$[K_{qq}^e]=[X]^{-T}\int_V z^2[L_K]^T[D][L_K][X]^{-1}dV \quad (22.2)$$

$$[K_{q\phi}^e]=[K_{\phi q}^e]^T=-[X]^{-T}\int_V z[L_K]^T[e]^T B_z dV \quad (22.3)$$

$$[K_{\phi\phi}^e]=-\int_V B_z[\xi^S]B_z dV \quad (22.4)$$

$$\{\overline{f}\} = \int_V \{f_b\} dV + \int_A \{f_A\} dA \quad (22.5)$$

$$\{Q_a\} = \int_A \sigma_q dA \quad (22.6)$$

Allowing arbitrary variations of $\{q_k\}$ and $\{\Phi\}$, two equilibrium equations written in generalized coordinates are now obtained for the k-th element:

$$\left[M_{qq}^e\right]\{\ddot{d}_k\} + \left[K_{qq}^e\right]\{d_k\} + \left[K_{q\phi}^e\right]\{\phi\} - \{\overline{f}\} = 0 \quad (23)$$

$$\left[K_{\phi q}^e\right]\{d_k\} + \left[K_{\phi\phi}^e\right]\{\phi\} + \{Q\} = 0 \quad (24)$$

5. NUMERICAL RESULTS

The first validation case is the comparison between the displacement field obtained from the numerical techniques are presented.

In the upcoming validations a rectangular plate and piezoelectric material with the following characteristics are used (see Table.1):

Table 1. *Material and geometrical properties of plate and piezoelectric.*

Properties	Piezoelectricity material		Steel plate
	sensor	actuator	
E(Young's modulus) (N.m-2)	2	69	207
ρ (Density) (kg.m^{-3})	1780	7700	7870
v (Poisson)	0.3	0.3	0.29
h Thickness) (m)	0.205x10^{-3}	0.254x10^{-3}	1x10^{-3}
ξ^s (Dielectric constant) (F.m^{-1})	1.06x10^{-10}	1.6x10^{-8}	-
e (Piezoelectric stress) (C.m^{-2})	0.046	-12.5	-
C (Capacitance) (F)	5.2x10^{-9}	6.3x10^{-7}	-
Geometry ($Lx\times Ly$) (m)	0.1x0.1	0.1x0.1	0.6x0.4

The influence of the piezoelectric patches on the static behavior of the structure is investigated.

The configuration, illustrated in Fig. 3, shows the corresponding layout of the piezoelectric actuators and sensors symmetrically bonded to the simply support plate.

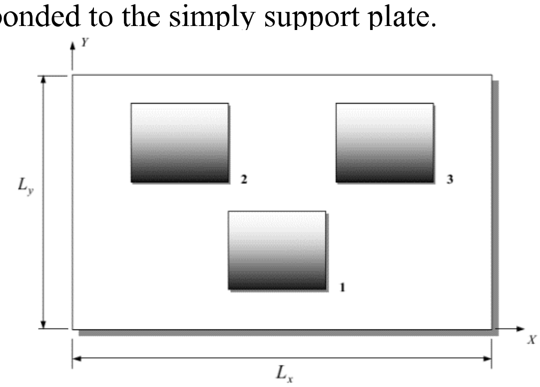

Fig 3*. Piezoelectric actuators and sensors test configuration.*

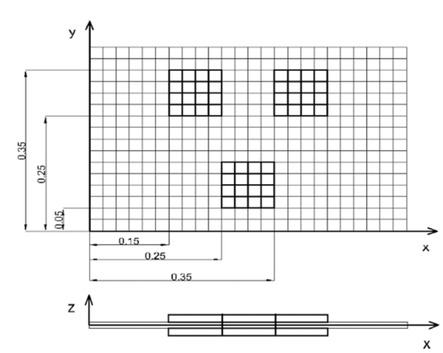

Fig 4. *Mesh gird (24x16) and position of actuators and sensors*

Table 2. *Piezoelectric element positions on the plate structure.*

	Actuators and sensors		
Position (*m*)	**1**	**2**	**3**
*x*1	0.25	0.15	0.35
*y*1	0.05	0.25	0.25

Figure.5 shows the total plate displacement amplitude calculated, when a static input voltage (Φ_a) is applied to actuators (1, 2 and 3) for the following magnitude: $\{\Phi_a\}=\{1,1,1\}$

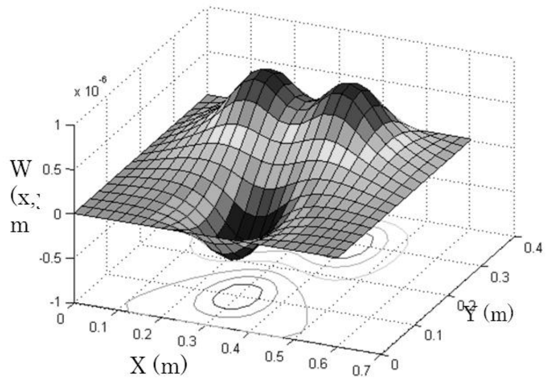

Fig.5 *Displacement of plate*

Table 3*. Electric potentials generated by the piezoelectric sensors***.**

	Electric Potential (*Volts*)		
sensors	**Present**	**Ref [13]**	**Err(V)**
1	+0.0164	+0.0162	0.002
2	-0.0164	-0.0162	0.002
3	-0.0164	-0.0162	0.002

The second validation case is the simulation about the static behavior of plate with center boned actuator and sensor.

The material properties given in table 1. With mesh (12x8) and position of actuator in Fig.7, the displacement of plate with distribute load P=100 N, given in Fig.8, The center deflection with diffent voltage apply to actuator given in Fig.9 and Fig.10

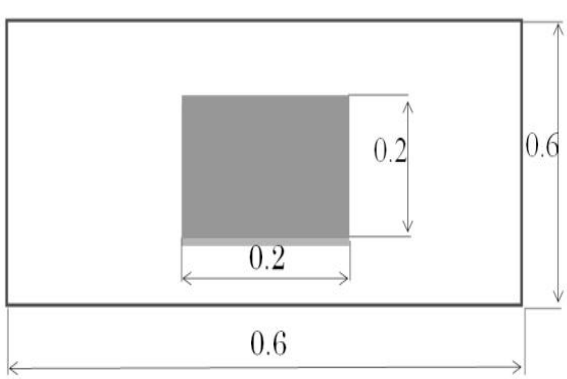

Fig.6. *The geometrical of plate*

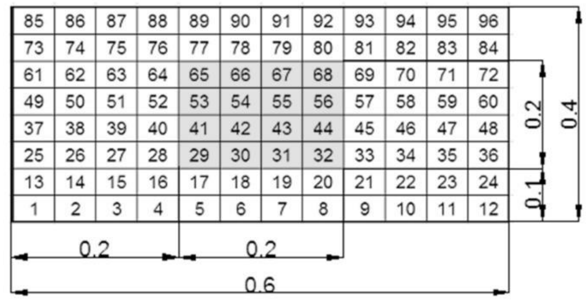

Fig.7. *The mesh of finite element*

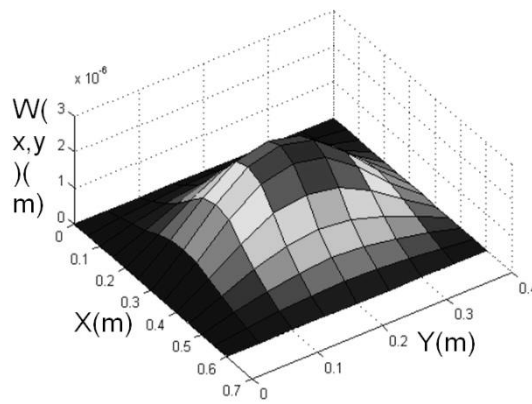

Fig.8. *Displacement of plate*

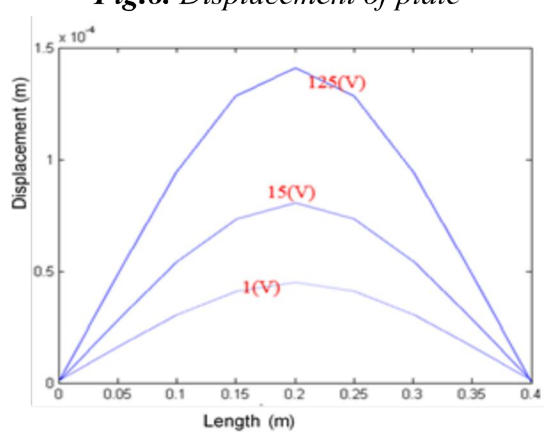

Fig.9. Displacement at $y=L_y/2$

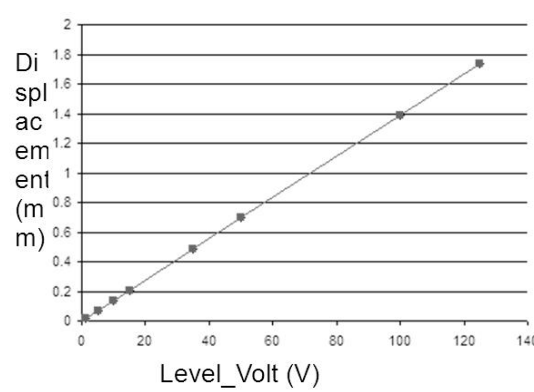

Fig.10*. The center deflection with difference voltage*

6. CONCLUSION

A finite element formulation based on Kirchhoff's plate model has been developed for the analysis of smart composite structures with piezoelectric materials. The modeling technique used can be easily understood. Besides, computational tests were performed demonstrating the efficiency of the methodology used.

Results were presented for a simply supported rectangular thin plate excited and sensed by various rectangular actuators and sensors bonded symmetrically to both sides of the plate. The present solutions are useful for understanding the electromechanical coupling in intelligent structures for using in the design of vibration control devices.

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