MOTION PREDICTION OF LUNG TUMOR USING PREDICTED ERROR-BASED NORMALIZED LEAST MEAN SQUARE ALGORITHM

TIÊN ĐOÁN CHUYỂN ĐỘNG CỦA U PHỔI BẰNG GIẢI THUẬT PE-NLMS

Le Phan Hung[1], Nguyen Truong Thinh[2]
[1]*College of Engineering Technology, Can Tho University*
[2]*Ho Chi Minh City University of Technology and Education*

Received 15/12/2015, Peer reviewed 04/01/2016, Accepted for publication 11/01/2016.

ABSTRACT

In robotic radiotherapy, one of the problems is systematic latencies between the acquisition of the target position and the mechanical response of the system to follow the target position. To compensate the aforementioned latencies while tracing the tumor motion, an accurate algorithm to predict these latencies is therefore required. The prediction algorithm computes the future target position. In this study, we have analyzed the accuracy of three algorithms that predict tumor positions with sufficient lead time to compensate these systematic latencies. The motions have been analyzed for predictability up to 400ms in advance using Least Mean Square (LMS) prediction, Normalized Least Mean Square (NLMS) prediction and the proposed algorithm, named as a Predicted Error-based Normalized Least Mean Square (PE-NLMS) prediction. The performance of three prediction algorithms is evaluated using three real breathing signal data in 30 Hz sampling rate. The results show that the PE-NLMS is outperformed with respect to RMSE by all other algorithms and does not require too much parameter adjustment. The simulation showed that it Jitter needs to be improved for better performance.

Keywords: Prediction algorithm, Lung tumor motion, respiratory compensation, Least Mean Square.

TÓM TẮT

Trong xạ trị robot, một trong những vấn đề chính là thời gian trễ của hệ thống giữa việc xác định vị trí của khối u và đáp ứng của các cơ cấu cơ khí nhằm đạt đến vị trí mong muốn. Để đáp ứng của hệ thống dịch chuyển chính xác, các thuật toán dự đoán chuyển động được sự dụng để bù trừ cho những khoảng thời gian trễ của hệ thống bằng cách tiên đoán trước vị trí của khối u n trong một khoảng thời gian xác định. Trong nghiên cứu này, chúng tôi đã phân tích tính chính xác của ba thuật toán dự đoán vị trí của khối u. Các chuyển động đã được phân tích để dự đoán lên tới 400ms bằng thuật toán dự đoán Least Mean Square (LMS), Normalised Squares Mean Square (NLMs) và dự báo lỗi dựa trên chuẩn hóa Squares Mean nhất (eNLMS). Việc thực hiện các thuật toán dự đoán được đánh giá bằng cách sử dụng tỷ lệ lấy mẫu 30 Hz trong 3 cơ sở dữ liệu hô hấp thực. Kết quả cho thấy thuật toán eNLMS là vượt trội so với tất cả các thuật toán khác và không yêu cầu điều chỉnh thông số quá nhiều.

Từ khóa: *Giải thuật tên đoán, chuyển động khối u phổi, LMS*

1. INTRODUCTION

In recent years, many studies showed that the average amplitude of the tumor motion was greatest in the cranial-caudal direction, the smallest for lateral and anterior-posterior directions, as illustrated in Fig. 1 [1]. Especially, in some cases the motion is known to have amplitude between 5 and 25mm [2], in extreme cases even 50mm [3]. Moreover, the frequency, amplitude and velocity of tumor motion can change over time for the same patient. In addition, the cardiac motion or some muscular motions often induce noise to the signal [4]. The patient movement, respiration, and transient physiological events also change the position of the treatment site during external-beam radiotherapy.

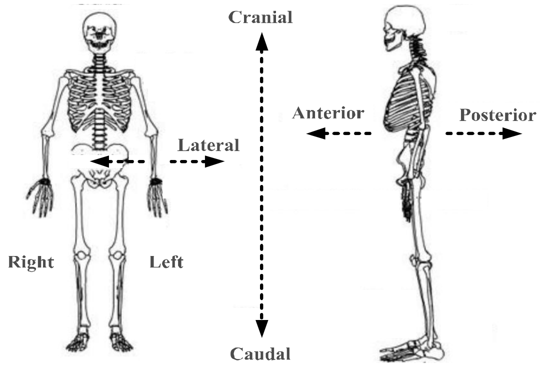

Fig. 1. *Illustrated of motion directions [8].*

Although an accurate treatment planning and a motion following control are used, because of systematic latencies between the acquisition of target position and robot motion, radiotherapy systems cannot respond instantaneously to target's motion [5]. If the acquisition of target position and the response of the radiation beam are not well synchronized, the tumor may be under dosed and a large volume of healthy tissue may be irradiated to be toxic unnecessarily. Therefore, compensating these latencies by using prediction algorithms is proposed as a solution to increase targeting accuracy before or during radiation treatments [6, 7]. The advantage of predicting tumor motion is that radiotherapy can be delivered more accurately to target position, reducing toxicity to the volume of healthy tissue [6].

A number of prediction algorithms were investigated to mitigate the effect of lung tumor motion as well as to improve targeting accuracy by compensating systematic latencies in radiotherapy system [9–11]. Although many prediction algorithms have been proposed for respiratory motion, only a few algorithms such as NLMS are being used in clinical application [11, 12]. For better evaluation of quantitative errors of the system, multiple measuring indices can be used. In this study, first the Root Mean Square Error (RMSE) was used to evaluate the quality of prediction algorithms and to compare with the case when no prediction is used. In the literature, RMSE needs to be less than 2mm to compensate the latency up to 400ms [13]. The other index, i.e. Jitter, measures the average distance (per second) travelled by the system following the commanded predictor output. The both values need to be as small as possible to be implemented in clinical application.

The aim of this study is to propose a new algorithm which uses the Predicted Error-based Normalized Lease Mean Square, the PE-NLMS, for tumor motion prediction. The performance of this algorithm is compared with no prediction case, LMS and NLMS in the case with 400ms prediction time and one-dimensional simulation. This paper is organized as follows: the method and materials are presented in Section II. The results of the prediction methods are discussed in Section III. Finally, in Section IV the conclusion is shown.

2. METHOD AND MATERIAL

2.1. Lung Tumor Motion Data

Lung tumor motion data were obtained from the hospital. All data were collected at a sampling rate of 30 Hz. Due to lung tumor motion and deformation during treatment that are mainly caused by respiration, three different breathing signals named as deep slow, shallow slow and deep fast breathing signals were collected, as shown in Fig. 2. The above-mentioned signals are used as the input data of the prediction algorithms. The average, standard deviation and total sample times of the breathing amplitudes of three signals are listed up in Table 1. The three features of signals have been listed because it can show the rough characteristics of the signals [14].

2.2. Prediction Algorithms

Let us assume that y is the (evenly sampled) real signal's position to be predicted, k is the current sampling index in time, δ is the prediction horizon and $\hat{y}\hat{y}$ is the predicted signal's position [5].

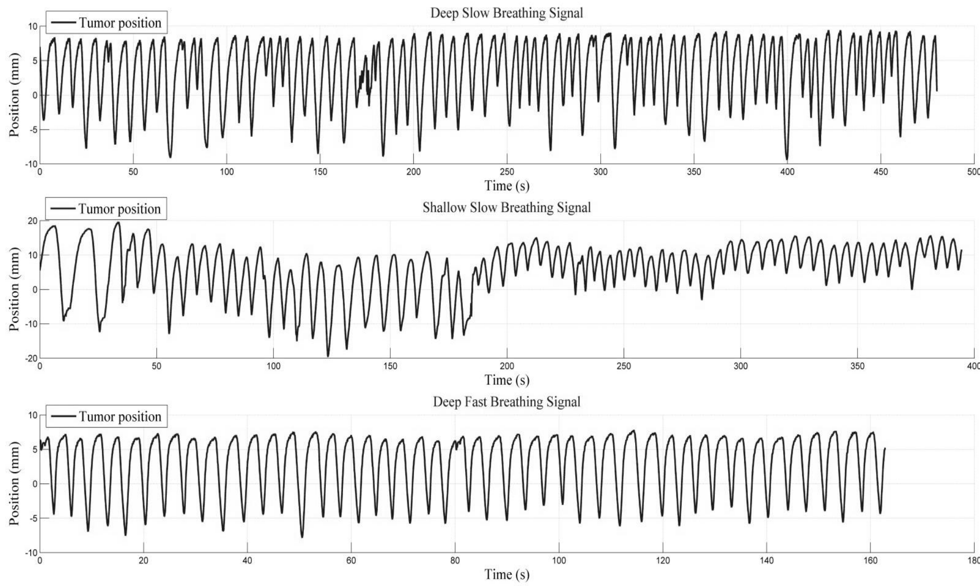

Fig. 2. *Lung tumor motion used for evaluation: deep slow, shallow slow and deep fast breathing signals.*

Table 1. *Average, Standard Deviation and total sample of breathing amplitude for three breathing signals*

Signal	Signal characteristics		
	Mean motion amplitude (mm)	**Standard deviation (mm)**	**Acquisition time (s)**
Deep Slow	4.98	4.50	480.5
Shallow Slow	8.44	7.28	395.0
Deep Fast	4.63	4.10	163.0

2.1.1. The LMS Algorithm [15]

A least mean square (LMS) algorithm is a kind of adaptive filter to find weight vector to minimize the measured signal. A vector-based version of LMS allows utilizing the depth of the past history *M* [5]. The weight vector w_1 w_1 to $w_{M+\delta} w_{M+\delta}$ And the initial predicted position is initialized as follows [16]:

$$w_1 = w_2 = \cdots = w_{M+\delta} = \boldsymbol{e}_M$$
$$= [0,\ldots,0,1]^T \in \mathbb{R}^M \quad (1)$$

$$\hat{y}_1^{LMS} = \cdots = \hat{y}_M^{LMS} = y_1. \quad (2)$$

Next, for each $k \geq M + \delta k \geq M + \delta$ and $k \geq M$ $k \geq M$, two types of signal history variables, whose sizes are both M by 1, are built:

$$u_{k-\delta} = [u_{k-\delta,1}, \ldots, u_{k-\delta,M}]^T \quad (3)$$
$$= [y_{k-M-\delta+1}, \ldots, y_{k-\delta}]^T, \quad k \geq M + \delta.$$

$$u_k = [u_{k,1}, \ldots, u_{k,M}]^T \quad (4)$$
$$= [y_{k-M+1}, \ldots, y_k]^T, \quad k \geq M.$$

Using these variables, the predicted position output can be computed:

$$\hat{y}_{k+\delta}^{LMS} = w_k^T u_k, \quad k \geq M \quad (5)$$

Now, the error between the original signal and the predicted signal is calculated as follows:

$$e_k = y_k - \hat{y}_k^{LMS} e_k = y_k - \hat{y}_k^{LMS}. \quad (6)$$

Finally, the weight vector is updated for $k \geq M + \delta k \geq M + \delta$ as:

$$w_{k+1} = w_k + \mu e_k u_{k-\delta}\,, \qquad k \geq M + \delta \quad (7)$$

where μ is the step size, y_k is the measured signal position at step k, w_k the corresponding weight vector, $u_{k-\delta}$ is the signal history that is used at step $k+\delta$ to update w_k, and u_k is the signal history that is used at step k to compute $\hat{y}_{k+\delta}$, the predicted signal at step $k+\delta$. For stationary signal, if the step size parameter μ is selected properly, the LMS algorithm will perfectly adapt to the system. If we select a wrong step size parameter μ, the system will be broken-down and or the prediction performance will be degraded. The convergence rate of LMS algorithm is known difficult to set a proper value. This problem is significantly reduced by using a normalized LMS algorithm (NLMS).

2.1.2. The NLMS Algorithm

To improve the convergence properties of the LMS algorithm (i.e. to make it independent from scaling and to increase the rate of convergence), normalized LMS algorithm are used [5].

For this algorithm, the weight vector w_1 to $w_{M+\delta}$, the initial predicted position $\hat{y}_1^{LMS}$ to $\hat{y}_M^{LMS}$, the history signal $u_{k-\delta}$ and u_k are initialized same as LMS, as shown in (1) – (4).

The updated rule is modified as follows:

$$w_{p,k+1} = w_{p,k} + \mu e_k f_{p,k}. \quad (8)$$

where $p \in \mathbb{N} \cup \{\infty\}$, which corresponds the number p of the p-norm function used for the error correction term.

Here, the error correction term $f_{p,k}$ is defined:

$$\left(f_{p,k}\right)_i = \frac{|u_{k,i}|^{p-1} sgn(u_{k,i})}{\|u_k\|_p^p}. \quad (9)$$

$$\left(f_{\infty,k}\right)_i = \frac{\delta_{i,h}}{u_{k,h}}, \qquad h = \text{maxind}_{j=1,\dots,M}|u_{k,j}| \quad (10)$$

where $\delta_{i,j}$ is the Kronecker delta.

In our study, we only considered the special case of $p = 2p = 2$ (i.e. Normalization with respects to the Euclidean norm).

Hence the algorithm reduces to the following [5]:

$$\hat{y}_{k+\delta}^{NLMS_2} = w_{2,k}^T u_k. \quad (11)$$

Then, the error between no prediction and prediction algorithm is computed by:

$$e_k = y_k - \hat{y}_k^{NLMS_2}. \quad (12)$$

And finally, the update weight vector is built as:

$$w_{2,k+1} = w_{2,k} + \mu e_k f_{2,k-\delta} \quad (13)$$
$$= w_{2,k} + \mu e_k \frac{u_{k-\delta}}{\alpha + \|u_{k-\delta}\|_2^2}.$$

To avoid division by zero, a small parameter α (typically $\alpha = 0.0001$) is introduced in the denominator of the error term [5].

2.1.3. The PE-NLMS Algorithm

In this section, we introduce a novel prediction algorithm is called as PE-NLMS. The newly proposed method PE-NLMS is based on a combination of the signal position and the error that are predicted by NLMS to get more accurate. The basic concept of PE-NLMS is illustrated in Fig.3.

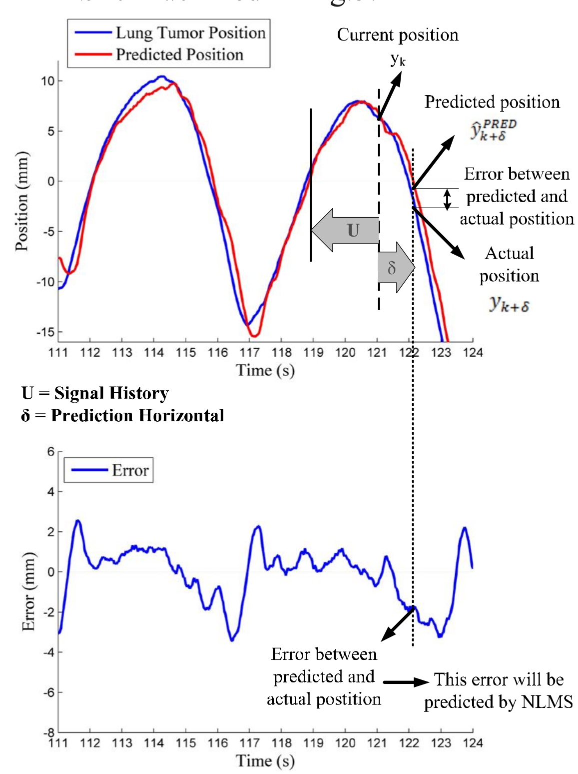

***Fig. 3**. Basic concepts of PE-NLMS.*

In this algorithm, for $M+\delta \le k < 2M+2\delta$ the predicted position is built like NLMS:

$$\hat{y}_{k+\delta}^{PE-NLMS} = \hat{y}_{k+\delta}^{NLMS_2}, \qquad M+\delta \le k < 2M+2\delta. \quad (14)$$

For $k \ge 2M+2\delta k \ge 2M+2\delta$ the predicted position is computed:

$$\hat{y}_{k+\delta}^{PE-NLMS} = \hat{y}_{k+\delta}^{NLMS_2} + \hat{e}_{k+\delta}^{NLMS_2}, \qquad k \ge 2M+2\delta \quad (15)$$

where $\hat{e}_{k+\delta}^{NLMS_2}$ is the predicted error using NLMS and is used as to calculated $e_k^{PE-NLMS}$ $e_k^{PE-NLMS}$ as shown in (16).

Here, the error between no prediction and prediction algorithm of PE-NLMS is:

$$e_k^{PE-NLMS} = y_k - \hat{y}_k^{PE-NLMS}. \quad (16)$$

Finally, the update weight vector is same as NLMS.

2.3. Evaluation Measures

2.3.1. Root Mean Square Error (RMSE):

The first measure is the RMSE that computes the difference between the tumor position and the predicted position. The RMSE indicates an average error over the squared differences between the signal itself and the value returned by the predictor as in (17).

$$RMSE(y, y^{PRED}) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left\|y_k - y_k^{PRED}\right\|^2} \quad (17)$$

Where N is the total number of time steps [16, 17].

To evaluate prediction algorithms more accurately, only RMSE is not sufficient, thus the other measure is introduced.

2.3.2. Jitter

The Jitter of the predicted position which the robotics system will follow should be as small as possible. The Jitter measures accumulated distances between two consecutive (discrete) points along the signal [16, 17]:

$$Jitter(y^{PRED}) = \frac{1}{N}\frac{1}{\Delta t}\sum_{k=0}^{N}\left\|y_k^{PRED} - y_{k+1}^{PRED}\right\|^2 \quad (18)$$

Where, the factor *1/Δt* is used to normalize the measure with respect to the spacing between the samples. Thus the Jitter value J measures the average distance (per second) travelled by the robot to follow the predicted output [16]. Therefore, we can determine whether the average distance the system would have to move in each cycle is acceptable by using the jitter and the motion tracking device's clock rate [5].

3. RESULTS AND DISCUSSION

The algorithms were evaluated for the prediction horizon $\delta = 12$ corresponding to latency 400ms. For evaluation of accuracy, the algorithms were stabilized with the first 2,000 sampling points corresponding to 66.67 second [5]. After the prediction algorithms were stabilized, the prediction performance was evaluated on the remaining data [5]. In addition, the depth of the past history, M, was set to 90 that corresponds to 3 second.

In family of LMS algorithm, the usual approach of selecting μ is based on try-and-error framework. Therefore, to determine the optimal output of the prediction algorithms, optimization of the μ parameters should be performed. Two different case approaches were taken as follows:

3.1 Case 1

In this case, each algorithm was tested with its own optimal parameter μ that produces its best result. The values of μ were listed in Table 2.

First, in deep slow breathing signal, the RMSE of PE-NLMS was best with 42.11%, which was better than no prediction and twice better than LMS and NLMS. For Jitter, no prediction was the best among four and slightly better than LMS, approximately by 1.46%. The PE-NLMS performed worst with 47.21% less than no prediction, shown in Table 3.

Regarding the shallow slow breathing signal, the PE-NLMS was better than NLMS, LMS and no prediction by 17.35%, 18.45% and 47.37%, respectively. Unfortunately, when we looked at the Jitter, the algorithm which performed worst was also PE-NLMS with 65.18% less than no prediction, as shown in Table 4.

At last, regarding the deep fast breathing, the best result was achieved by the PE-NLMS algorithm, which was better than no prediction by 52.22%. Similarly, the PE-NLMS was also worst in case of Jitter with 61.06% less than no prediction as shown in Table 5. The result of case 1 was shown in Fig. 4.

Table 2. *setting parameter μ for each signal*

Algorithms	Signal		
	Deep Slow	**Shallow Slow**	**Deep Fast**
LMS	0.000002	0.000001	0.000005
NLMS	0.005	0.006	0.015
PE-NLMS	0.005	0.005	0.009

Table 3. *performance of prediction deep slow breathing for optimal parameter μ*

Algorithms	Evaluation measures			
	RMSE		**Jitter**	
	(mm)	Gain(%)	(mm/s)	Gain(%)
No pred	1.81	reference	4.16	reference
LMS	1.42	21.59	4.22	-1.46
NLMS	1.42	21.63	4.40	-5.72
PE-NLMS	1.05	42.11	6.12	-47.21

Table 4. *performance of prediction shallow slow breathing for optimal parameter μ*

Algorithms	Evaluation measures			
	RMSE		**Jitter**	
	(mm)	Gain (%)	(mm/s)	Gain (%)
No pred	2.21	reference	5.07	reference
LMS	1.57	28.92	5.18	-2.18
NLMS	1.55	30.02	5.32	-5.02
PE-NLMS	1.17	47.37	8.37	-65.18

Table 5. *performance of prediction deep fast breathing for optimal parameter μ*

Algorithms	Evaluation measures			
	RMSE		**Jitter**	
	(mm)	**Gain(%)**	**(mm/s)**	**Gain(%)**
No pred	3.17	reference	7.30	reference
LMS	1.70	46.45	6.50	10.92
NLMS	1.70	46.28	6.56	10.07
PE-NLMS	1.51	52.22	11.75	-61.06

3.2. Case 2

In case 1, we can find the best value μ manually. Therefore, it is not clear how to select the value of step size μ. Note that, selecting off-optimum values for μ will not only degrade the prediction performance, but may even lead to complete breakdown of the prediction. In case 2, this disadvantage will be solved by automatically update μ and selected a fixed μ.

First, for LMS algorithm, the limitation of $\mu\mu$ was investigated in [18].

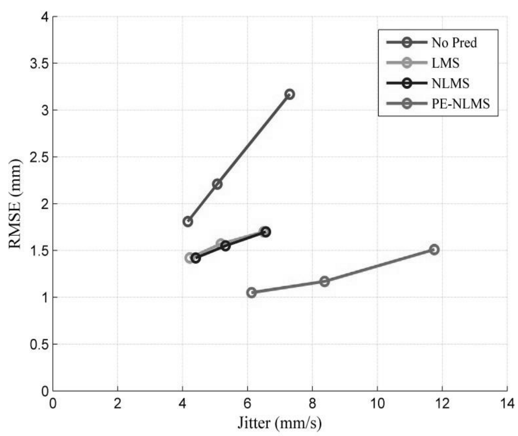

***Fig. 4**. Results of case 1 for three prediction algorithms, in each line, from left to right corresponding to deep slow breathing, shallow slow breathing and deep fast breathing signal.*

$$0 < \mu < 1/Tr(\boldsymbol{R}) \quad (19)$$

where $Tr(\boldsymbol{R})$ denotes trace of $\boldsymbol{R}$ and $\boldsymbol{R}$ is autocorrelation matrix:

$$\boldsymbol{R} = E[u_{k-\delta}u_{k-\delta}^T] \quad (20)$$

In our case, after calculating (6), μ will be computed and updated as follows:

$$\mu = 1/Tr(\boldsymbol{R}). \quad (21)$$

The second approach, since the convergence of NLMS is known that easier to select μ than LMS. In our experiments, for the NLMS and PE-NLMS algorithms, μ was set a constant value to 0.01 for all signals.

Now, we analyze the RMSE and Jitter, first in deep slow breathing signal. The RMSE of PE-NLMS, NLMS and LMS were 41.49%, 21.08% and 15.8% better than no prediction, respectively. However, in Jitter, no prediction was the best with significantly better than PE-NLMS, approximate 41.91%, as shown in Table 6.

Similarly, for the shallow slow breathing signal, although it has the best result in RMSE and it is 46.46% better than no prediction. But considering Jitter, PE-NLMS performed worst with 57.84%, 51.11% and 48.63% less than NLMS, LMS and no prediction, respectively, as shown in Table 7.

In case of deep fast breathing signal, all algorithms performed well. RMSE of NLMS was slightly better than that of LMS by 0.59%. Regarding RMSE, the best algorithm is PE-NLMS, since its RMSE is much smaller than no prediction by 52.19%. Only in this case, the LMS and NLMS gave a better performance than no prediction for Jitter with 9.50% and 11.41%, respectively. Once again, the Jitter of PE-NLMS worst with 58.51% less than no prediction, as shown in Table 8. The results of all algorithms for case 2 were shown in Fig. 5.

***Table 6.** performance of prediction deep slow breathing for case 2*

Algorithms	Evaluation measures			
	RMSE		Jitter	
	(mm)	Gain(%)	(mm/s)	Gain(%)
No pred	1.81	reference	4.16	reference
LMS	1.53	15.80	4.60	-10.83
NLMS	1.43	21.08	4.52	-8.66
PE-NLMS	1.06	41.49	5.90	-41.91

Table 7. *performance of prediction shallow slow breathing for case 2*

Algorithms	Evaluation measures			
	RMSE		Jitter	
	(mm)	Gain(%)	(mm/s)	Gain(%)
No pred	2.21	reference	5.07	reference
LMS	1.75	20.77	5.53	-9.21
NLMS	1.57	29.12	5.41	-6.73
PE-NLMS	1.19	46.46	8.00	-57.84

Table 8. *performance of prediction deep fast breathing for case 2*

Algorithms	Evaluation measures			
	RMSE		Jitter	
	(mm)	Gain(%)	(mm/s)	Gain(%)
No pred	3.17	reference	7.30	reference
LMS	1.72	45.60	6.60	9.50
NLMS	1.70	46.19	6.46	11.41
PE-NLMS	1.51	52.19	11.57	-58.51

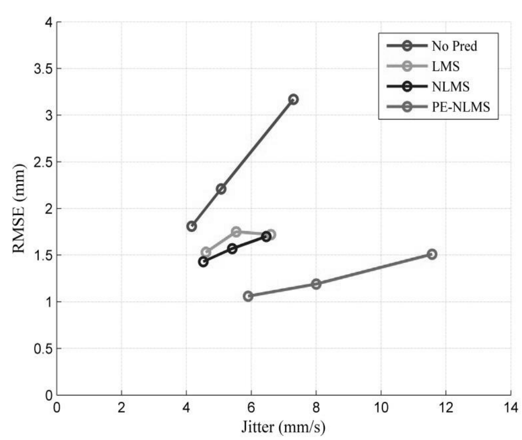

Fig. 5. *Results of case 2 for three prediction algorithms, in each line, from left to right corresponding to deep slow breathing, shallow slow breathing and deep fast breathing signal.*

4. CONCLUSION

In this study, we proposed a new prediction algorithm PE-NLMS that is modified the normalized lease mean square (NLMS) algorithm. In addition, we evaluated three prediction algorithms, including a basic LMS algorithm in the case where the latency up to 400ms exists. The results showed clearly that the proposed algorithm, PE-NLMS, outperforms the other two algorithms. Since the PE-NLMS produces the RMSE less than 2mm when the latency up to 400ms exists, the PE-NLMS is expected to be used in clinical application. In the future, we not only need to improve the PE-NLMS to decrease the Jitter but also optimize the algorithm to get better accuracy.

REFERENCES

[1] Y. Seppenwoolde, H. Shirato, K. Kitamura, S. Shimizu, M. van Herk, J.V. Lebesque and K. Miyasaka, *"Precise and real-time measurement of 3D tumor motion in lung due to breathing and heartbeat, measured during radiotherapy,"* International Journal of Radiation Oncology Biology Physics, vol. 53, no. 4, pp. 822-834, 2002.

[2] M.J. Murphy. *"Tracking moving organs in real time,"* Seminar in Radiation Oncology, vol. 14, no. 1, pp. 91-100, 2004.

[3] P. Giraud and et al. "*Conformal radiotherapy (CRT) planning for lung cancer: analysis of intrathoracic organ motion during extreme phases of breathing,"* International Journal of Radiation Oncology Biology Physics, vol. 51, no. 4, pp. 1081-1092, 2001.

[4] H. Wu, et al, *"A finite state model for respiratory motion analysis in image guided radiation therapy,"* Physics in Medicine and Biology, vol. 49, no. 23, pp. 5357–5372, 2004.

[5] F. Ernst, *Compensating for Quasi-periodic Motion in Robotic Radiosurgery*, Springer, 2011.

[6] P.J. Keall, G.S. Mageras, J.M. Balter, R.S. Emery, K.M. Forster, S.B. Jiang, J.M. Kapatoes, D.A. Low, M.J. Murphy, B.R. Murray, C.R. Ramsey, M.B. Van Herk, S.S. Vedam, J.W. Wong, E. Yorke, *"The management of respiratory motion in radiation oncology report of AAPM Task Group 76,"* Medical Physics, vol. 33, no. 10, pp. 3874–3900, 2006.

[7] Y.D. Mutaf, C.J. Scicutella, D. Michalski, K. Fallon, E.D. Brandner, G. Bednarz, M.S. Huq, *"A simulation study of irregular respiratory motion and its dosimetric impact on lung tumors,"* Physics in Medicine and Biology, vol. 56, no. 3, pp. 845–859, 2011.

[8] *http://www.imaios.com/en/e-Anatomy/Limbs/Planes-and-motions-diagrams*

[9] Lee, Suk Jin, Motai, Yuichi, Prediction and Classification of Respiratory Motion, Springer, 2013.

[10] P.S. Verma, H. Wu, M.P. Langer, I.J. Das, G. Sandison, "Survey: real-time tumor motion prediction for image guided radiation treatment," Computing in Science & Engineering, vol. 13, no. 5, pp. 24-35, 2011.

[11] E. Floris, D. Robert, S. Alexander and S. Achim, *"Evaluating and comparing algorithms for respiratory motion prediction,"* Physics in Medicine and Biology, vol.58, no.11, pp. 3911-3929, 2013.

[12] S. Sayeh, J. Wang, W.T. Main, W. Kilby, Jr. Maurer, C.R.: Robotic Radiosurgery, Treating Tumors that Move with Respiration, 1st ed., chap. *"Respiratory motion tracking for robotic radiosurgery"*, pp. 15-30. Springer, Berlin, 2007.

[13] S.S. Vedam, P.J. Keall, A. Docef, D.A. Todor, V.R. Kini, R. Mohan, *"Predicting respiratory motion for four-dimensional radiotherapy,"* Medical Physics, vol. 31, no. 8, pp. 2274–2283, 2004.

[14] G. .C. Sharp, S.B. Jiang, S. Shimizu, H. Shirato, *"Prediction of respiratory tumour motion for real-time image-guided radiotherapy,"* Physics in Medicine and Biology, vol. 49, no. 3, pp. 425–440, 2004.

[15] S. Haykin.: *Adaptive Filter Theory, 4th ed., Prentice Hall, Englewood Cliffs*, NJ, 2002.

[16] A. Schweikard, Medical Robotics, course notes, summer 2013.

[17] F. Ernst, R. Bruder, A. Schlaefer and A. Schweikard, *"Performance Measures and Pre-Processing for Respiratory Motion Prediction,"* In: 53rd Annual Meeting of the AAPM, Medical Physics, vol. 38, no.6, pp. 3857, 2011.

[18] S. Haykin, B. Widrow, Least-Mean-Square Adaptive Filter, John Wiley & Sons, 2003.