MODAL ANALYSIS FOR THREE DIMENSIONAL FRAME STRUCTURES ELEMENT USING FEM

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TÓM TẮT
Phân tích động học là việc thiết yếu trong thiết kế, chế tạo và bảo trì của kết cấu. Phương pháp giải tích thì khó đáp ứng cho hầu hết các bài toán trong thực tế và phương pháp phần tử hữu hạn là một công cụ số cho phép giải những bài toán này một cách nhanh hơn. Một đề xuất được đưa ra là tìm những kiểu hình dạng (mode shape) của sự cộng hưởng cho loại kết cấu cố định. Hơn nữa, phương pháp phân tích và dữ liệu trong nghiên cứu này được ứng dụng trong thiết kế động học và đánh giá đặc tính động học của kết cấu. Trong bài báo này, chúng tôi trình bày một phương pháp để tìm đặc tính động học (tần số tự nhiên và kiểu hình dạng) cho một khung không gian bằng phương pháp phần tử hữu hạn sử dụng ngôn ngữ lập trình Matlab
Từ khóa: Tần số tự nhiên, Phương pháp phần tử hữu hạn, Lý thuyết khung dầm, Phân tích động học.

ABSTRACT

The dynamic analysis is a necessity in the design, manufacturing and maintenance of structures. The analytic solutions are not available for most practical cases and the Finite Element Method (FEM) is not reliable due to the joint problem, unclear loading, material properties and numerical error. It was proposed to find the several important modes of resonance peak for these fixed type structures. Furthermore, it is expected that the analysis method and the data in this study can be applied to a dynamic design and dynamic performance evaluation. In this paper, we present a way to find modes shape of 3D Frame by FEM program using Matlab.

Key words: Natural Frequency, Finite Element Method, Frame Theory, Modal analysis.

Introduction

In recent years operational modal testing and analysis has become more popular due to some advantages of the technology compared to traditional modal testing and analysis. However, there still seem to remain some uncertainties, for instance including how the testing should be performed, which techniques should be used, and how reliable the results are. This paper is addresses some of these issues and presents a discussion of some of the advantages of the operational technology.

Adequate testing procedures are discussed. Attention is drawn to the fact that it is of great importance to make sure to be dealing with multiple input loading to improve the quality of the identification process. Typical examples of loading that can contribute to this include moving loads on the structure (the ideal case) or distributed loads with a limited spatial correlation length.

Some of the most well-known identification techniques are presented and the basic concepts of these techniques are discussed. All of the well-known techniques of today can handle multiple input data, and the importance of this aspect is illustrated by a Frequency Domain Decomposition, as well as, a Stochastic Subspace Identification technique. These two techniques represent two very different classes of identification, but they clearly illustrate what is believed to be a common tendency for all techniques: they work much better with multiple-input data.
One of the links that from the beginning was missing in operational identification-the estimation of the mode shape scaling-is addressed. Attention is drawn to the fact that a simple and reliable way of scaling the mode shapes is now available.

Finally the advantages of output only modal testing and analysis are discussed. It is argued that it is a reliable technique that can be used on a large range of structures, and that it can be used for solving a broader range of practical engineering vibration problems.

1. General equation of motion in matrix form[12].

Motion equation of a multidegree of freedom system as following:

$$[m]\ddot{\vec{x}}+[c]\dot{\vec{x}}+[k]\vec{x}=\vec{F}, \quad (1)$$

where

$$\vec{F}=\begin{Bmatrix} F_1 \\ F_2 \\ . \\ . \\ . \\ F_n \end{Bmatrix}, \quad (2)$$

If the system is conservative, there are no nonconservative forces F_i, so the equation of motion becomes:

$$[m]\ddot{\vec{x}}+[k]\vec{x}=\vec{0}, \quad (3)$$

2. Eigenvalue problem[12].

The solution of Equation.(3) corresponds to the undamped free vibration of the system. In this case, if the system is given some energy in the form of initial displacement or initial velocities or both, it vibrates indefinitely because there is no dissipation of energy. We can find the solution of Eq.(3) bu assuming a solution of the form:

$$x_i(t)=X_iT(t), \text{ i=1, 2…n} \quad (4)$$

Where X_i is a constant and T is a function of time t. Equation.(4) shows that the amplitude ratio of two coordinates: $\frac{x_i(t)}{x_j(t)}$

is independent of time. Physically, this means that all coordinates have synchronous motion. The configuration of the system does not change its shape during motion, but its amplitude does.

The configuration of the system, given by the vector:

$$\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_n \end{Bmatrix}$$

is known as the mode shape of the system. Substituting Eq.(4) into Eq.(3),we obtain

$$[m]\vec{X}\ddot{T}(t) + [k]\vec{X}T(t) = \vec{0}, \qquad (5)$$

Equation (5) can be written in scalar form as n separate equation:

$$\left(\sum_{j=1}^{n} m_{ij} X_j\right)\ddot{T}(t) + \left(\sum_{j=1}^{n} k_{ij} X_j\right)T(t) = 0, \text{ i=1, 2...n} \qquad (6)$$

from which we obtain the relations

$$-\frac{\ddot{T}(t)}{T(t)} = \frac{\sum_{j=1}^{n} k_{ij} X_j}{\sum_{j=1}^{n} m_{ij} X_j}, \qquad (7)$$

Since the left-hand side of equation (7) is independent of the index I, and the right-hand side is independent of *t*, both sides must be equal to constant. By assuming this constant* as ω^2 we can write (7) as

$$\ddot{T}(t) + \omega^2 T(t) = 0, \qquad (8)$$

$$\sum_{j=1}^{n} \left(k_{ij} - \omega^2 m_{ij}\right) X_j = 0, \qquad \text{i=1, 2, …n}$$

Or

$$\left[[k] - \omega^2 [m]\right]\vec{X} = \vec{0}, \qquad (9)$$

The solution of equation (8) can be expressed as

$$T(t) = C_1 \cos(\omega t + \phi), \qquad (10)$$

where C_1 and ϕ are constants, known as the amplitude and shape angle, respectively. Equation (10) shows that all coordinates can perform a harmonic motion with the same frequency ω and the same phase angle ϕ. However, the frequency ω cannot take any arbitrary value; it has to satisfy equation (9).

Since equation (9) represent a set of n linear homogenous equation in the unknowns X_i (i=1, 2, …n), the trivial solution is $X_i = X_2 = \ldots = X_n = 0$. For nontrivial solution of equation (9), the determinant Δ of the coefficient matrix must be zero. That is,

$$\Delta = \left|k_{ij} - \omega^2 m_{ij}\right| = \left|[k] - \omega^2 \left[m_{ij}\right]\right| = 0, \qquad (11)$$

Equation (9) represents what is known as the eigenvalue or characteristic value problem, equation (11) is called characteristic equation, ω^2 is known as the eigenvalue or the characteristic value, and ω is called the natural frequency of the system.

The expansion of equation (11) leads to an n^{th} order polynomial equation in ω^2. The solution (roots) of this polynomial or characteristic equation gives n values of ω^2. It can be shown that all the n roots are real and positive when the matrices [k] and [m] are symmetric and positive definite $[m]\ddot{\vec{x}}+[k]\vec{x}=\vec{F}$, as in the present case. If $\omega_1^2, \omega_2^2, ..., \omega_n^2$ denote the n roots in ascending order of magnitude, their positive square roots give the n natural frequencies of the system $\omega_1^2 \le \omega_2^2 \le ... \le \omega_n^2$. The lowest value (ω) is called the fundamental or first natural frequency. In general, all the natural frequencies ω_i are distinct, although in some cases two natural frequencies might possess the same value.

3. Solution of the eigenvalue problem [12].

Several methods are available to solve an eigenvalue problem. We shall consider an elementary method in this section.

Equation (9) can also be expressed as

$$\left[\lambda[k]-[m]\right]\overrightarrow{X}=\vec{0}, \quad (12)$$

where

$$\lambda=\frac{1}{\omega^2}, \quad (13)$$

By premultiplying equation (12) by $[k]^{-1}$, we obtain

$$\left[\lambda[I]-[D]\right]\overrightarrow{X}=\vec{0},$$

Or

$$\lambda[I]\overrightarrow{X}=[D]\overrightarrow{X}, \quad (14)$$

Where [I] is the identify matrix and $[D]=[k]^{-1}[m]$, (15)

is called the dynamic matrix. The eigenvalue problem of equation (14) is known as the standard eigenvalue problem. For a nontrivial solution of $\vec{X}$, the characteristic determinant must be zero-that is,

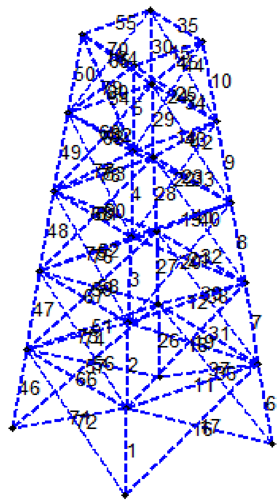
$$\Delta=\left|\lambda[I]-[D]\right|, \quad (16)$$

On expansion, equation (16) gives an nth degree polynomial in λ, known as the characteristic or frequency equation. If the degree of freedom of the system (n) is the large, the solution of this polynomial equation becomes quite tedious. We must use some numerical method, several of which are available to find the roots of a polynomial equation.

4. Numerical examples.

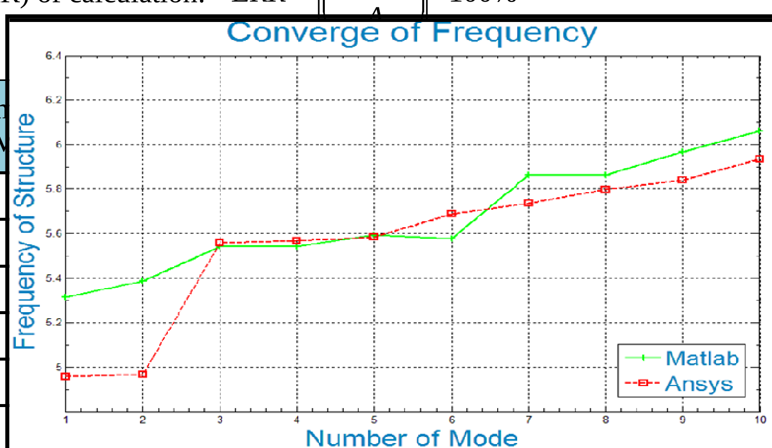
We consider a 4-leg steel jacket is numerically modeled with the fixed boundary condition at the sea bottom.

No.	Item	Value
1	Jacket Type Tower total length(m)	58.665
2	Number of leg [pieces]	4
3	Top dimension [m]	10x10
4	Bottom dimension [m]	21.36x21.36
5	Out diameter [cm]	50
6	E(Young's modulus) [Kg/sq cm]	2100×1000
7	G(Shear modulus) [Kg/sq cm]	840×1000
8	Density [kg/m3]	7850
9	K factor	1



The following result of frequency was calculated by Matlab program comparison with Ansys software.

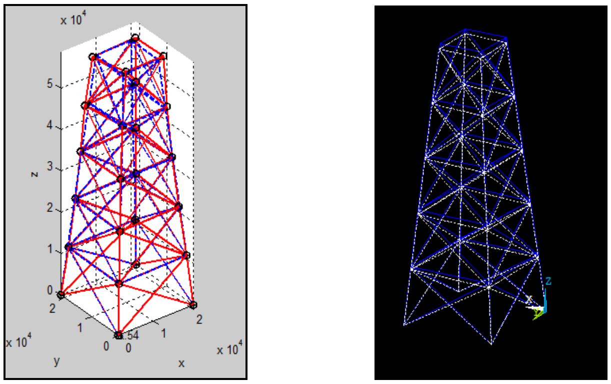
The error (ERR) of calculation: $ERR = \left|\left(\frac{A-B}{A}\right)\right| *100\%$



Nun M			ERR
			6.7241%
			7.7780%
			0.3285%
			0.4819%
			0.1430%
			1.4960%
7	5.8618	5.5987	2.1324%
8	5.8623	5.5987	1.0848%
9	5.9658	5.6393	2.1204%
10	6.0598	5.6393	2.0495%

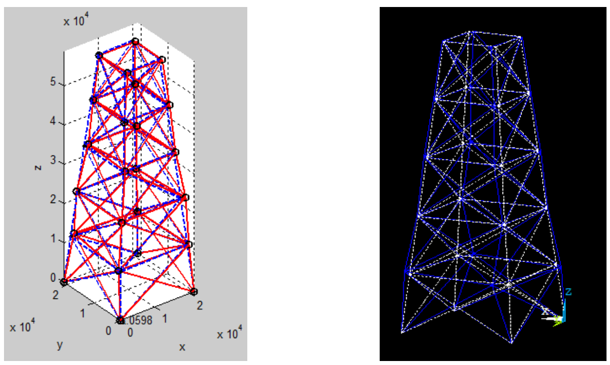
Fig 1: Comparison frequency of structure

Mode Shape of structure:



a) Matlab *b) Ansys*

Fig 2: Mode Shape of structure-mode 3.



a) Matlab *b) Ansys*

Fig 3: Mode Shape of structure-mode 10.

5.Conclusions.

Modal analysis is used to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component while it is being designed. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions.

The modal analysis of steel frame is shown in figure 2 and figure 3. The material properties used in Matlab program are the same with the properties used in analysis by Ansys software, in additional property which is needed for finite element analysis that is Poisson's ratio. In this case 0.3 is chosen. The analysis is done for 3D element and the type of element used in this analysis is 3D Frame in Matlab and Beam 24 in Ansys.

Because the analysis is done in 3D element, the mode shapes are not only in one direction such the theoretical and experimental analysis. From the table 1, it can be seen that the value of frequencies from mode 1 to mode 10 of Matlab program and the value when done by Ansys are the same. These results also can be verified from mode shapes shown in figure 2 and figure 3.

In practical, the results obtained from finite element analysis, actually, should be almost the same with theoretical calculation but the results shows there is an error. In this paper, the result obtained Ansys analys is considered a theoretical calculation. The error may be caused by error due to selection of material property (Poisson's ratio), element selection, and the mesh (fine or coarse mesh) selection.

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